## 7 Problems

**1** For the discrete-time system with matrices $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{C} = [5, 1]$, $\mathbf{D} = 0$, and state $\mathbf{X}$,

(a) write the formula for the transfer matrix $\mathbf{H}(z)$;

(b) find the system matrices that result from the change of state variables $\mathbf{X} = \mathbf{S}\mathbf{X}'$, where $\mathbf{S} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}$;

(c) calculate the transfer matrix for the new system, and compare it to the one in (a).

**2** Determine whether each of the following matrices can be diagonalized by a similarity transformation $\mathbf{A} \rightarrow \mathbf{S}^{-1}\mathbf{A}\mathbf{S}$, and determine the resulting diagonal matrix ($\omega$ is an arbitrary real parameter):

(a) $\mathbf{A} = \begin{bmatrix} -5 & 1 \\ -3 & -1 \end{bmatrix}$, (b) $\mathbf{A} = \begin{bmatrix} -1 & -6 & 5 \\ -1 & 4 & -5 \\ -11 & -6 & 15 \end{bmatrix}$, (c) $\mathbf{A} = \begin{bmatrix} -2 & -\omega \\ \omega & -2 \end{bmatrix}$.

**3** Referring to Example 8, show that for any matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, if $\mathbf{U} + j\mathbf{V}$ is an eigenvector corresponding to eigenvalue $\alpha + j\omega$, then $\mathbf{U} - j\mathbf{V}$ is an eigenvector corresponding to eigenvalue $\alpha - j\omega$.

**4** Find a real similarity matrix to transform

$$\mathbf{A} = \begin{bmatrix} 0 & -5 & 1 \\ 1 & -4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

to real block-diagonal form as in Example 8.

**5** For the circuit of Figure P7.5 which has neither input nor output, write the state-

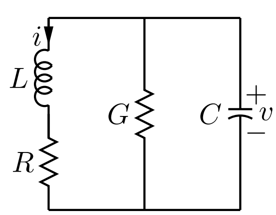


**Fig. P7.5** Linear circuit without input or output.

space equations, and for small $R$ and $G$ find a real similarity matrix to transform the system to the form given in Example 8.

**6** Calculate the Jordan form of the matrix $\mathbf{A} = \begin{bmatrix} 0 & -9 \\ 1 & 6 \end{bmatrix}$.

**7** The eigenvalues $\lambda_i$ must be computed to diagonalize a real matrix $\mathbf{A}$ and for many other purposes. A standard floating-point method is to find an orthogonal similarity transformation matrix $\mathbf{S}$ such that $\mathbf{S}^T\mathbf{A}\mathbf{S}$ is real and block-triangular, containing blocks on the diagonal that are either $1 \times 1$ or $2 \times 2$, with zeros below the diagonal blocks, to machine precision. Such a matrix is said to be in *real Schur form,* and its eigenvalues are easily computed from the diagonal blocks. Compute the eigenvalues of the real Schur matrix $\begin{bmatrix} -4 & 1 & 3 \\ 0 & -3 & -2 \\ 0 & 5 & 4 \end{bmatrix}$.

**8** Find $\mathbf{A}^{200}$ for $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.

**9** Find formulas for $e^{t\mathbf{A}}$ and $\mathbf{A}^k$, for arbitrary $t$ and $k$, and

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & -12 \\ 0 & 1 & -6 \end{bmatrix}.$$

**10** Given $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\alpha^2 & -2\alpha \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, find formulas for the exponential $e^{T\mathbf{A}}$ and for the integral $\left(\int_0^T e^{\tau\mathbf{A}} d\tau\right)\mathbf{B}$, and hence verify that with $\alpha = 3$ and $T = 0.1$, the system of Example 6 of Chapter 2 is the discretization of the system of Example 23 in the same chapter.

**11** In Section 3 of Chapter 2, a discretization of a time-continuous system with matrices $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ was obtained to have matrices $(\mathbf{F} = e^{T\mathbf{A}}, \mathbf{G} = \int_0^T e^{\tau\mathbf{A}} d\tau,$ $\mathbf{C}, \mathbf{D})$, where $T$ is the sampling interval. Given $\mathbf{F}$, $\mathbf{G}$, and $T$, show how to find $\mathbf{A}$ and $\mathbf{B}$, and comment on whether they are unique.