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Implicit Linear Systems

Errata

There are two errata of note:

1. There is an obvious but embarrassing typo in the statement of Theorem 10.1. There should be no accents on the matrices shown in Property 4, which should read

   Property 4. $[\, E_i - \lambda F_i \,]$ has full column rank for all $\lambda \in \mathbb{C}$.

2. There is an error in the derivation of Equation (8.24) on page 138, which should read as on the following page:

$$M_k\, s_{t_0} - \mathcal{J} W_k = Q(W_k, s_{t_0}) \tag{8.15}$$

where M_{t_1,t_0} has been re-written as M_k to emphasize that this matrix is a function of W_k and not s_{t_0}, and where $\mathcal{J} = \text{diag}\,(J, J, \cdots)$. Then the least-squares solution of (8.9) is given by solving

$$M_k^T M_k s_{t_0} = M_k^T \mathcal{J} W_k. \tag{8.16}$$

Let s_k be computed from W_k via the state-transition equation

$$s_k = \Phi_{k,t_0} s_{t_0}, \tag{8.17}$$

compute s_{t_0} from (8.16), and let $s_{t_0} + \delta s_{t_0}$, $s_k + \delta s_k$, $W_k + \delta W_k$ be the corresponding perturbed values. Then in (8.16),

$$(M_k + \delta M_k)^T (M_k + \delta M_k)(s_{t_0} + \delta s_{t_0}) = (M_k + \delta M_k)^T \mathcal{J}(W_k + \delta W_k), \tag{8.18}$$

from which the following can be obtained by ignoring second-order terms:

$$M_k^T M_k \delta s_{t_0} = M_k^T (\mathcal{J} \delta W_k - (\delta M_k) s_{t_0}) - (\delta M_k^T) Q. \tag{8.19}$$

The right-hand side of this equation is to be expressed in terms of δW_k. Because M_k is a linear funcion of W_k alone, write the term $M_k s_{t_0}$ as

$$M_k s_{t_0} = \begin{bmatrix} E_0 \\ E_0 E_* \\ \vdots \\ E_0 E_*^k \end{bmatrix} x_0 + \begin{bmatrix} G_0 & & & \\ E_0 G_* & G_0 & & \\ \vdots & \ddots & \ddots & \\ E_0 E_*^{k-1} G_* & \cdots & E_0 G_* & G_0 \end{bmatrix} W_k + \mathcal{J} W_k \tag{8.20}$$

and denote the coefficient matrix of W_k by $\mathcal{G}_k$. Then the term $(\delta M_k) s_{t_0}$ becomes $(\mathcal{G}_k + \mathcal{J})\, \delta W_k$. Additionally, since δM_k is linear in δW_k, write the term $(\delta M_k^T) Q$ as $\mathcal{R}_k \delta W_k$. Then from (8.19),

$$\delta s_{t_0} = -(M_k^T M_k)^{-1} (M_k^T \mathcal{G}_k + \mathcal{R}_k)\, \delta W_k. \tag{8.21}$$

For the perturbation in s_k, write (8.17) as

$$s_k = \begin{bmatrix} E_*^k & 0 \\ 0 & I \end{bmatrix} s_{t_0} + \begin{bmatrix} E_*^{k-1} G_* & \cdots & E_*^0 G_* & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix} W_k \tag{8.22}$$

and denote the coefficient of W_k above by N_k to obtain $s_k + \delta s_k$ as

$$\Phi_{k,0} \delta s_{t_0} + N_k \delta W_k = (N_k - \Phi_{k,0} (M_k^T M_k)^{-1} (M_k^T \mathcal{G}_k + \mathcal{R}_k))\, \delta W_k = K_k \delta W_k, \tag{8.23}$$

where K_k is the coefficient matrix of δW_k.

Substituting this quantity into (3.17) gives

$$\kappa_k = \sup_{\delta W_k} \frac{\|W_k\|}{\|s_k\|} \frac{\|K_k \delta W_k\|}{\|\delta W_k\|} = \frac{\|W_k\|}{\|s_k\|} \|K_k\| \tag{8.24}$$

where a matrix norm subordinate to the vector norm $\|\cdot\|$ gives the value of the supremum.

Computing this number requires the solution of (8.9) for s_{t_0}, given W_k and $\{n_i\}$, $\{\zeta_k\}$, but once these are known, all required quantities are easily found. Thus in principle it is easy to compare the condition numbers of different sets of structural parameters to find the set that best models the measured data.