Logic-based verification and explanation of NNs

Nina Narodytska, VMware research

How to train your verifiable Binarized NNs?

Nina Narodytska, VMware research

joint work with Hongce Zhang (summer @ VMware), Aarti Gupta

Outline

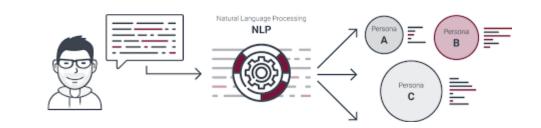
Introduction

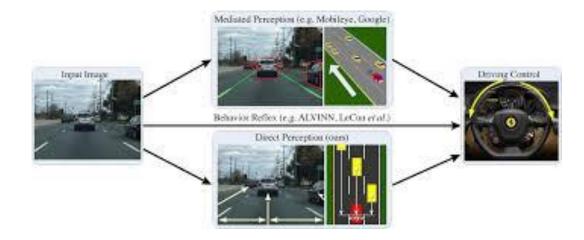
- Logic-based analysis of NNs
- Scalability of verification techniques
 - Analysis of bottlenecks
 - Experimental evaluation
- Conclusions

Introduction

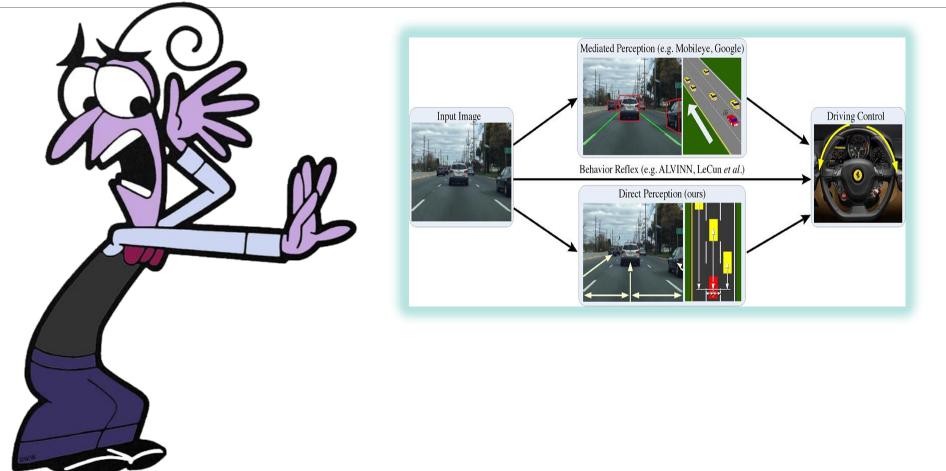
ML models







Verification of ML models



Verification of ML models

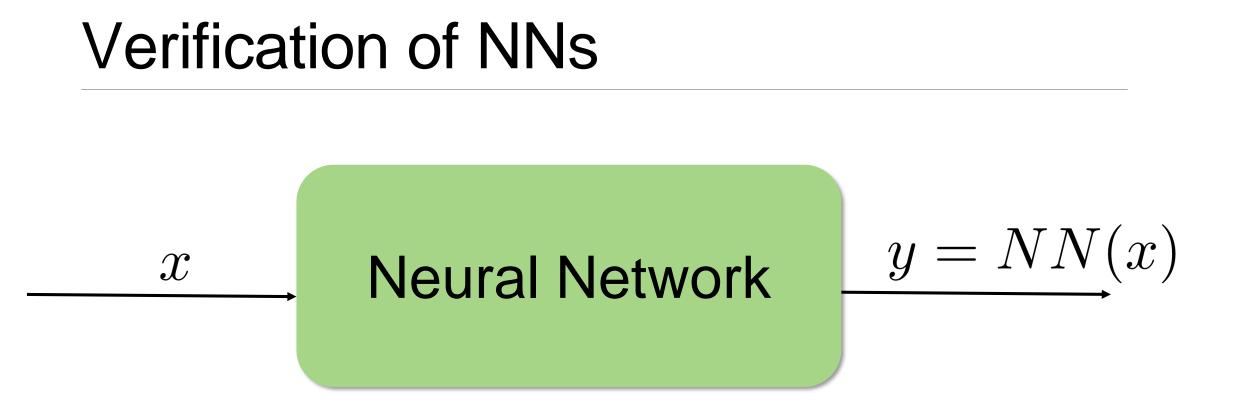


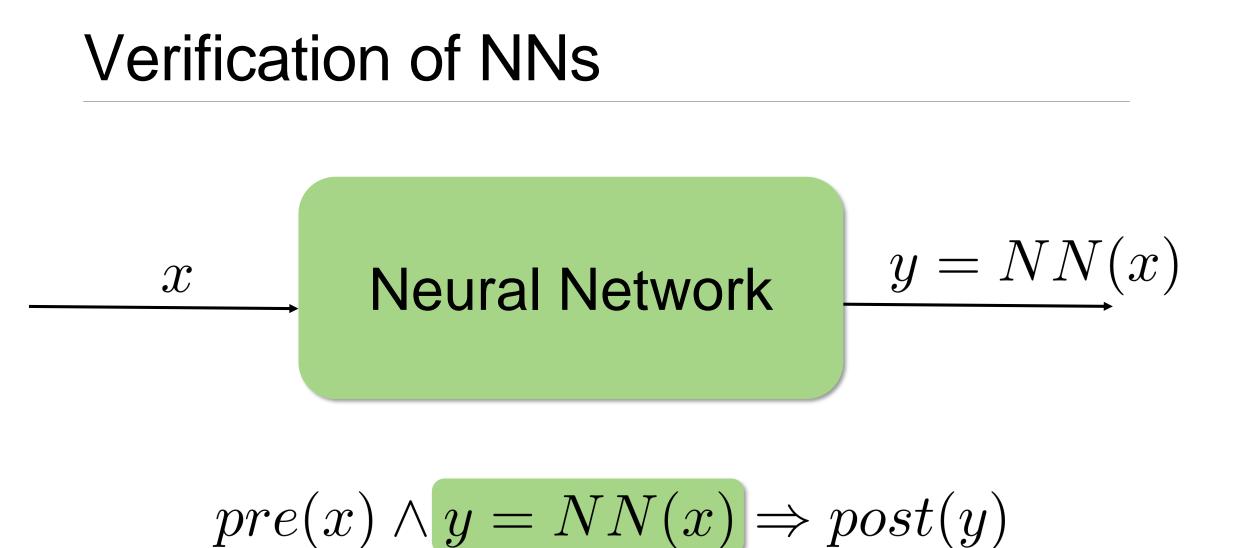
Verification of ML models



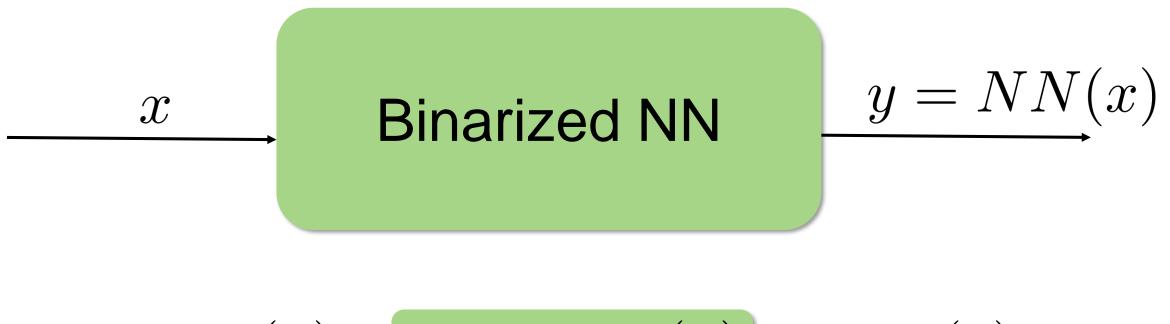


Verification of NNs





Verification of Binarized NNs



$$pre(x) \land y = BNN(x) \Rightarrow post(y)$$

Binarized Neural Networks: Training Deep Neural Networks with Weights and Activations Constrained to +1 or -1 Matthieu Courbariaux, Itay Hubara, Daniel Soudry, Ran El-Yaniv, Yoshua Bengio

Why are BNNs important?

Why BNNs?

Binarized neural networks: Training deep **neural networks** with weights and activations constrained to+ 1 or-1

<u>M Courbariaux</u>, <u>I Hubara</u>, <u>D Soudry</u>, <u>R El-Yaniv</u>... - arXiv preprint arXiv ..., 2016 - arxiv.org We introduce a method to train Binarized Neural Networks (BNNs)-neural networks with binary weights and activations at run-time. At training-time the binary weights and activations are used for computing the parameters gradients. During the forward pass, BNNs drastically ... \therefore \mathfrak{DD} Cited by 925 Related articles All 9 versions \gg

Binarized neural networks

<u>I Hubara, M Courbariaux, D Soudry</u>... - Advances in **neural** ..., 2016 - papers.nips.cc We introduce a method to train Binarized Neural Networks (BNNs)-neural networks with binary weights and activations at run-time. At train-time the binary weights and activations are used for computing the parameter gradients. During the forward pass, BNNs drastically ... \therefore \mathfrak{DD} Cited by 470 Related articles All 5 versions \gg

Xnor-net: Imagenet classification using binary convolutional neural networks <u>M Rastegari</u>, <u>V Ordonez</u>, <u>J Redmon</u>... - European Conference on ..., 2016 - Springer ... Because, at inference we only perform forward propagation with the binarized weights ... Similar to binarization in the forward pass, we can binarize \(g^{in}\) in the backward pass ... Our binarization technique is general, we can use any CNN architecture ... ☆ ワワ Cited by 1373 Related articles All 8 versions

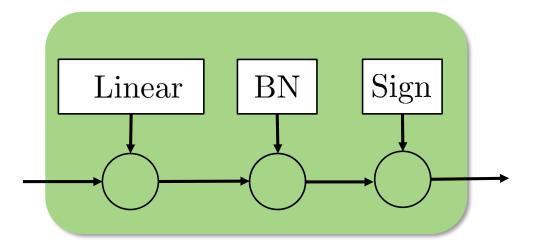
Compactness

- Only 1 bit per weight, {-1,1}
- Can be deployed on embedded devices

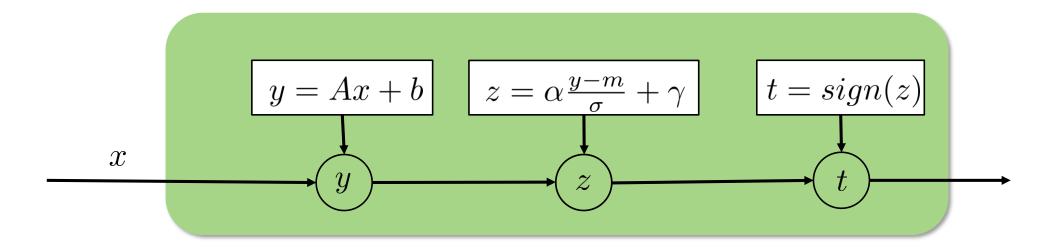
Inference efficiency

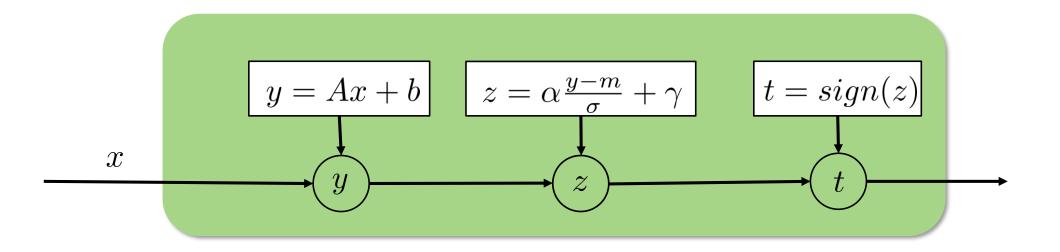
- fast binary matrix multiplication (7X speed up on GPU)
- "Accelerating Binarized Neural Networks: Comparison of FPGA, CPU, GPU, and ASIC" IEEE'2016

Structure of BNNs

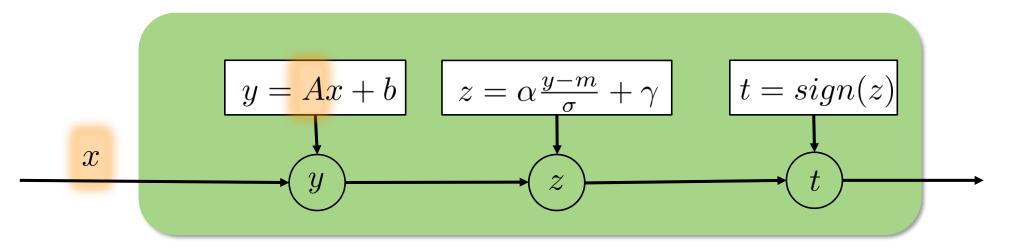


Binarized Neural Networks: Training Deep Neural Networks with Weights and Activations Constrained to +1 or -1 Matthieu Courbariaux, Itay Hubara, Daniel Soudry, Ran El-Yaniv, Yoshua Bengio

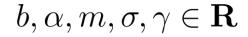




 $x, a_{i,j} \in \{-1, 1\}$ $b, \alpha, m, \sigma, \gamma \in \mathbf{R}$

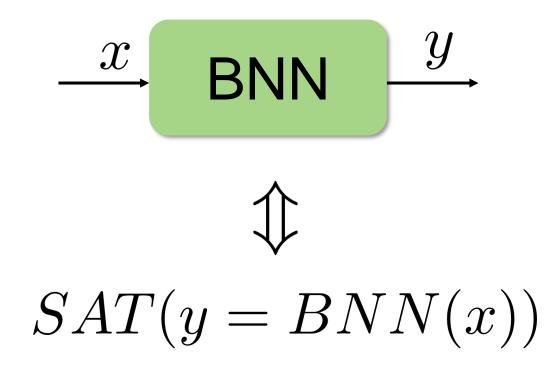


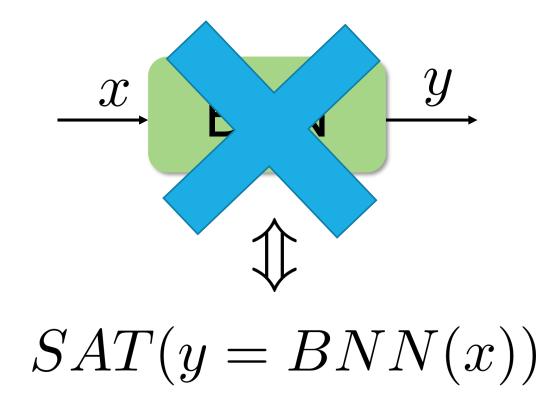
$$x, a_{i,j} \in \{-1, 1\}$$



BNNs and Logic reasoning





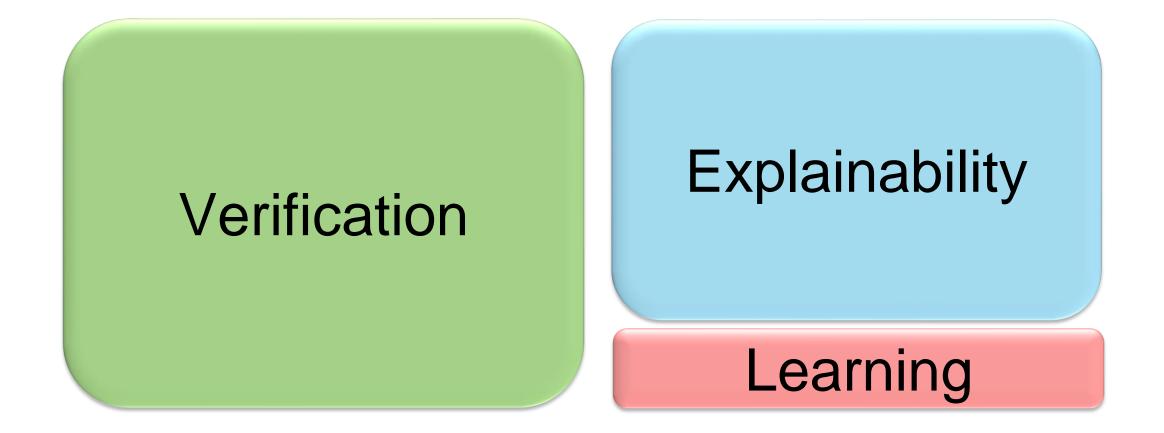


SAT(y = BNN(x))

BinBNN(x, y) :=

SAT(y = BNN(x))

Logic-based analysis of BNNs



Logic-based analysis of BNNs

Properties verification using SAT solvers

Quantitative reasoning using approximate methods

Knowledge compilation, e.g. BDD, SDD

Learning a network using optimization techniques

Nina Narodytska, Shiva Prasad Kasiviswanathan, Leonid Ryzhyk, Mooly Sagiv, and Toby Walsh. *Verifying properties of binarized deep neural networks AAAI'18* Elias B. Khalil, Amrita Gupta, Bistra Dilkina: *Combinatorial Attacks on Binarized Neural Networks ICLR'19*

$$pre(x) \land y = BNN(x) \Rightarrow post(y)$$

$pre(x) \land BinBNN(x, y) \Rightarrow post(y)$

$(x_1 = 0) \land BinBNN(x, y) \Rightarrow (y_1 = 0)$

$(x_1 = 0) \land BinBNN(x, y) \land (y_1 \neq 0)$

$(x_1 = 0) \land BinBNN(x, y) \land (y_1 \neq 0)$ \bigcirc SAT solver

Explainability

The Challenge of Crafting Intelligible Intelligence

By Daniel S. Weld, Gagan Bansal Communications of the ACM, June 2019, Vol. 62 No. 6, Pages 70-79 10.1145/3282486

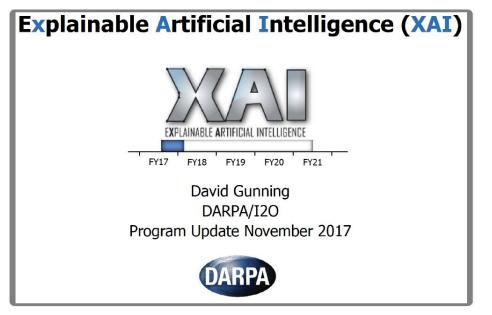
Comments (1)



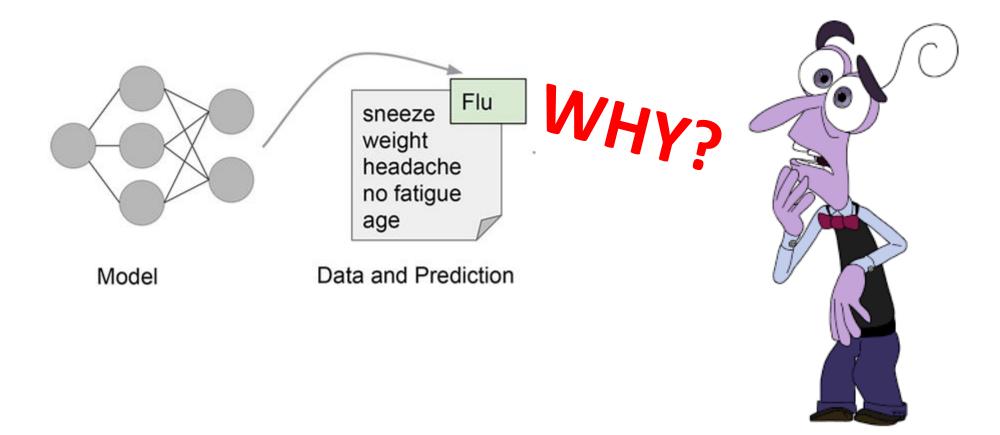


Artificial Intelligence (ai) systems have reached or exceeded human performance for many circumscribed tasks. As a result, they are increasingly deployed in mission-critical roles, such as credit scoring, predicting if a bail candidate will commit another crime, selecting the news we read on social networks, and selfdriving cars. Unlike other mission-critical software, extraordinarily complex AI systems are difficult to test: AI decisions are context specific and often based on thousands or millions of factors. Typically, AI behaviors are generated by searching vast action spaces or learned by the opaque optimization of mammoth neural networks operating over prodigious amounts of training data. Almost by definition, no clear-cut method can accomplish these AI tasks. European Union regulations on algorithmic decision-making and a "right to explanation"

Bryce Goodman,^{1*} Seth Flaxman,²



We summarize the potential impact that the European Union's new General Data Protection Regulation will have on the routine use of machine-learning algorithms. Slated to take effect as law across the European Union in 2018, it will place restrictions on automated individual decision making (that is, algorithms that make decisions based on user-level predictors) that "significantly affect" users. When put into practice, the law may also effectively create a right to explanation, whereby a user can ask for an explanation of an algorithmic decision that significantly affects them. We argue that while this law may pose large challenges for industry, it highlights opportunities for computer scientists to take the lead in designing algorithms and evaluation frameworks that avoid discrimination and enable explanation.



An **explanation** is a subset of input features so that changes to the rest of inputs do not affect the.

Alexey Ignatiev, Nina Narodytska, João Marques-Silva: AAAI'19: Abduction-Based Explanations for Machine Learning Models **Andy Shih and Arthur Choi and Adnan Darwiche :** IJCAI'18: A Symbolic Approach to Explaining Bayesian Network Classifiers

$$I = (x_1 = 0, x_2 = 0), y = 0$$

$$I = (x_1 = 0, x_2 = 0), y = 0$$

$$I' \models (y = NN(x)) \to (y = 0), I' \subset I$$

$$I = (x_1 = 0, x_2 = 0), y = 0$$

 $I' \models BinBNN(x, y) \rightarrow (y = 0), I' \subset I$

Quantitative reasoning

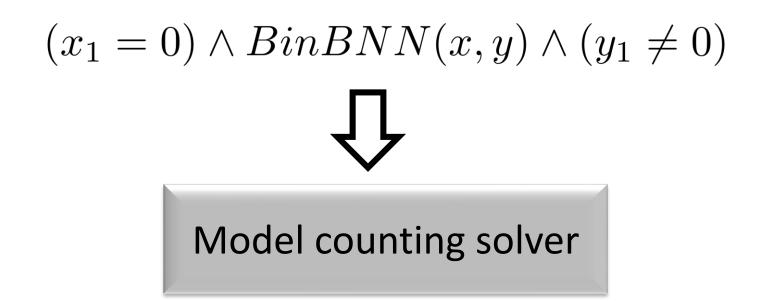
Nina Narodytska, Aditya A. Shrotri, Kuldeep S. Meel, Alexey Ignatiev, João Marques-Silva: *Assessing Heuristic Machine Learning Explanations with Model Counting SAT'19.*

Quantitative Verification of Neural Networks And its Security Applications Teodora Baluta, Shiqi Shen, Shweta Shinde, Kuldeep S. Meel, Prateek Saxena

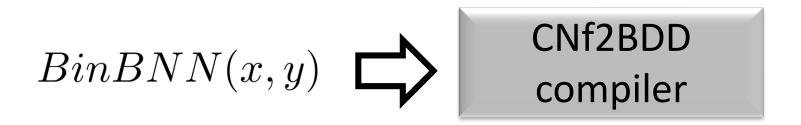
Quantitative reasoning

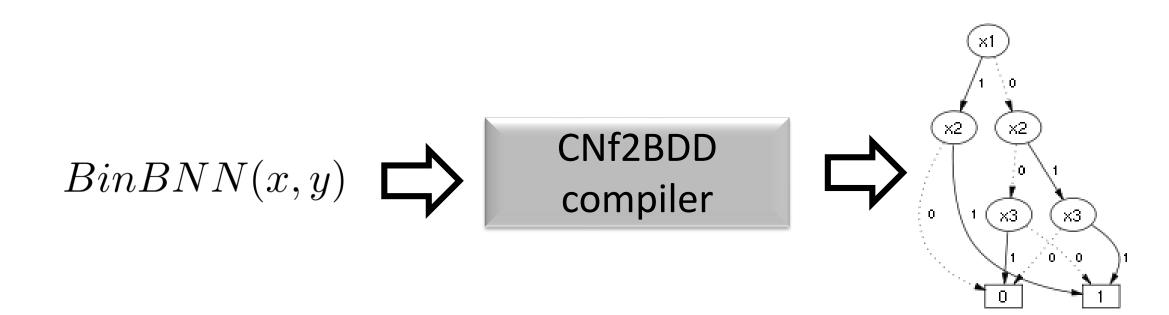
$(x_1 = 0) \land BinBNN(x, y) \land (y_1 \neq 0)$

Quantitative reasoning

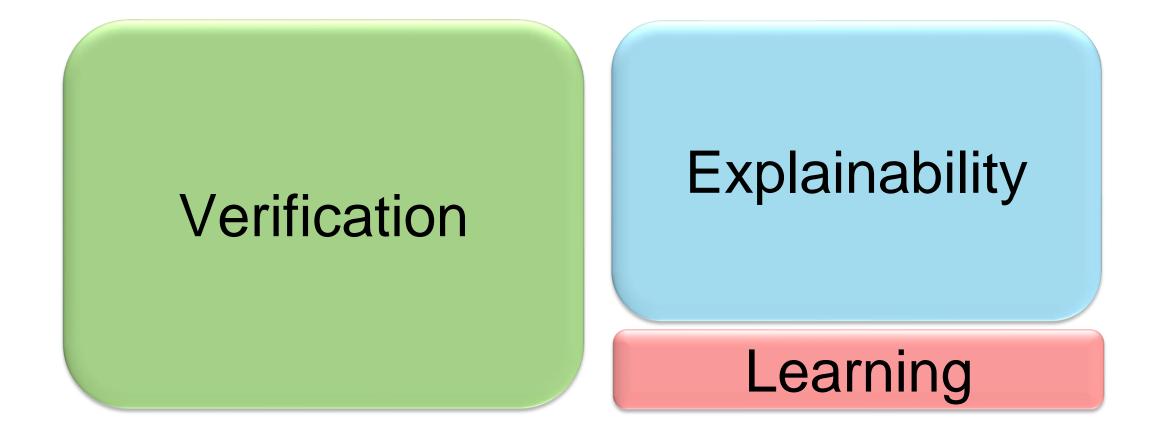


BinBNN(x, y)





Logic-based analysis of BNNs

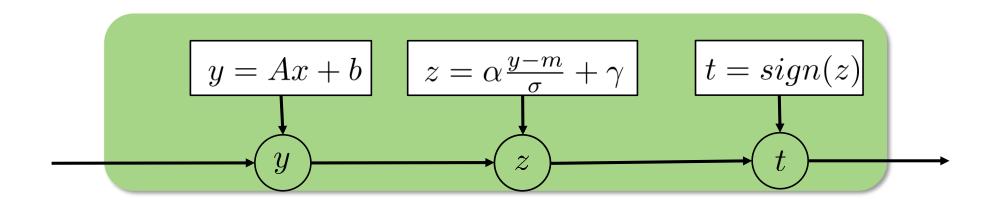


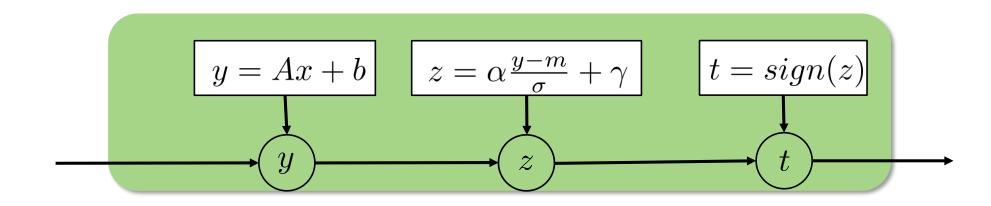
Work with small networks

Work with small networks

- Properties verification using SAT solvers
 - < 200K (robustness with a very small epsilon)</p>
- Quantitative reasoning using approximate methods
 < 51K
- Knowledge compilation, e.g. BDD, SDD
 < 10K

How can we improve scalability?





$$t_i = sign\left(\alpha \frac{(a_{i,1}x_1 + \dots + a_{i,n}x_n + b) - m}{\sigma} - \gamma\right)$$

$$t_i = sign\left(\alpha \frac{(a_{i,1}x_1 + \dots + a_{i,n}x_n + b) - m}{\sigma} - \gamma\right)$$

$$\left(\alpha \frac{(a_{i,1}x_1 + \dots + a_{i,n}x_n + b) - m}{\sigma} - \gamma \ge 0\right) \Leftrightarrow t_i = 1$$

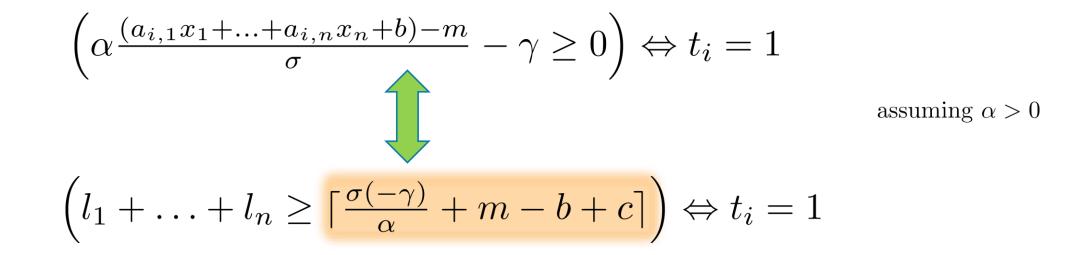
$$\left(\alpha \frac{(a_{i,1}x_1 + \dots + a_{i,n}x_n + b) - m}{\sigma} - \gamma \ge 0\right) \Leftrightarrow t_i = 1$$

assuming $\alpha > 0$

 $\left(\alpha \frac{(a_{i,1}x_1 + \dots + a_{i,n}x_n + b) - m}{\sigma} - \gamma \ge 0\right) \Leftrightarrow t_i = 1$ assuming $\alpha > 0$ $\left(l_1 + \dots + l_n \ge \left\lceil \frac{\sigma(-\gamma)}{\alpha} + m - b + c \right\rceil\right) \Leftrightarrow t_i = 1$

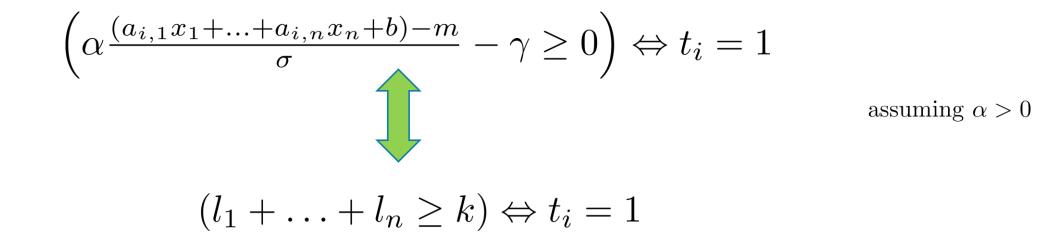
where

$$a_{i,j} = 1 \Rightarrow l_j = x_j, a_{i,j} = -1 \Rightarrow l_j = \bar{x}_j$$



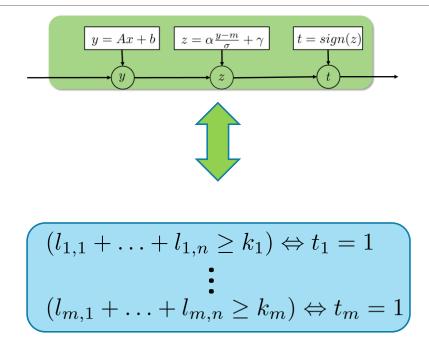
where

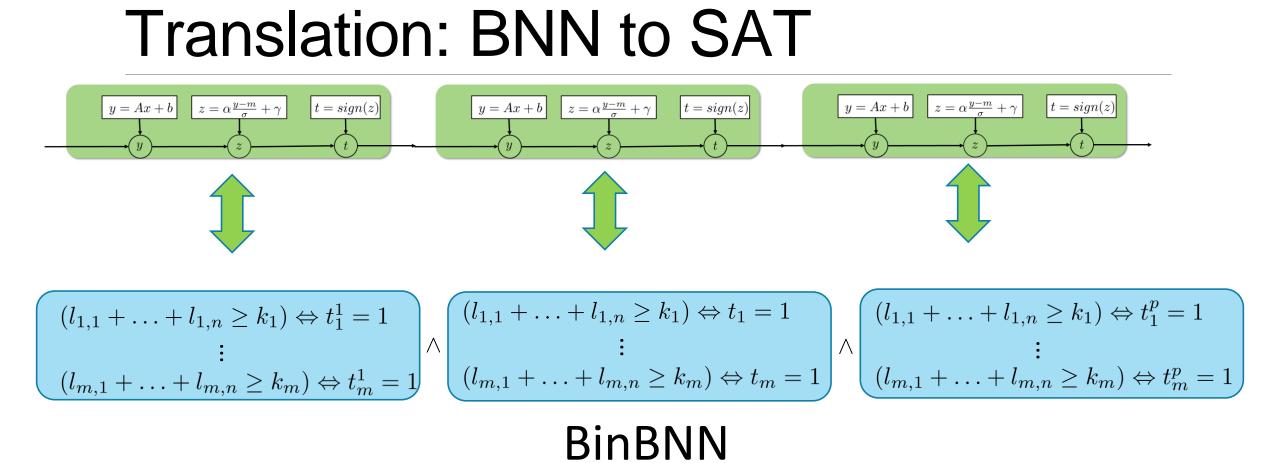
$$a_{i,j} = 1 \Rightarrow l_j = x_j, a_{i,j} = -1 \Rightarrow l_j = \bar{x}_j$$



where

$$a_{i,j} = 1 \Rightarrow l_j = x_j, a_{i,j} = -1 \Rightarrow l_j = \bar{x}_j$$





Why are BinBNNs hard to solve?

Birds view: a large formula

$$\begin{array}{c} (l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1 \\ \vdots \\ (l_{m,1} + \ldots + l_{m,n} \ge k_m) \Leftrightarrow t_m = 1 \end{array} \land \begin{array}{c} (l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1 \\ \vdots \\ (l_{m,1} + \ldots + l_{m,n} \ge k_m) \Leftrightarrow t_m = 1 \end{array} \land \begin{array}{c} (l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1 \\ \vdots \\ (l_{m,1} + \ldots + l_{m,n} \ge k_m) \Leftrightarrow t_m = 1 \end{array} \land \begin{array}{c} (l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1 \\ \vdots \\ (l_{m,1} + \ldots + l_{m,n} \ge k_m) \Leftrightarrow t_m = 1 \end{array}$$

. . .

Birds view: a large formula

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+ a structure aware solver

. . .

Birds view: a large formula

$$\begin{array}{c} (l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1 \\ \vdots \\ (l_{m,1} + \ldots + l_{m,n} \ge k_m) \Leftrightarrow t_m = 1 \end{array} \land \begin{array}{c} (l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1 \\ \vdots \\ (l_{m,1} + \ldots + l_{m,n} \ge k_m) \Leftrightarrow t_m = 1 \end{array} \land \begin{array}{c} (l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1 \\ \vdots \\ (l_{m,1} + \ldots + l_{m,n} \ge k_m) \Leftrightarrow t_m = 1 \end{array} \land \begin{array}{c} (l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1 \\ \vdots \\ (l_{m,1} + \ldots + l_{m,n} \ge k_m) \Leftrightarrow t_m = 1 \end{array}$$

+ a structure aware solver

. . .

Macroview: a large formula for a block



Macroview: a large block

 $(l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1$ $(l_{2,1} + \ldots + l_{2,n} \ge k_2) \Leftrightarrow t_2 = 1$ $(l_{3,1} + \ldots + l_{3,n} \ge k_3) \Leftrightarrow t_3 = 1$

Macroview: a large block

 $(l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1$ $(l_{2,1} + \ldots + l_{2,n} \ge k_2) \Leftrightarrow t_2 = 1$ $(l_{3,1} + \ldots + l_{3,n} \ge k_3) \Leftrightarrow t_3 = 1$

+ a nice shape of a matrix

Macroview: a large block

 $(l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1$ $(l_{2,1} + \ldots + l_{2,n} \ge k_2) \Leftrightarrow t_2 = 1$ $(l_{3,1} + \ldots + l_{3,n} \ge k_3) \Leftrightarrow t_3 = 1$

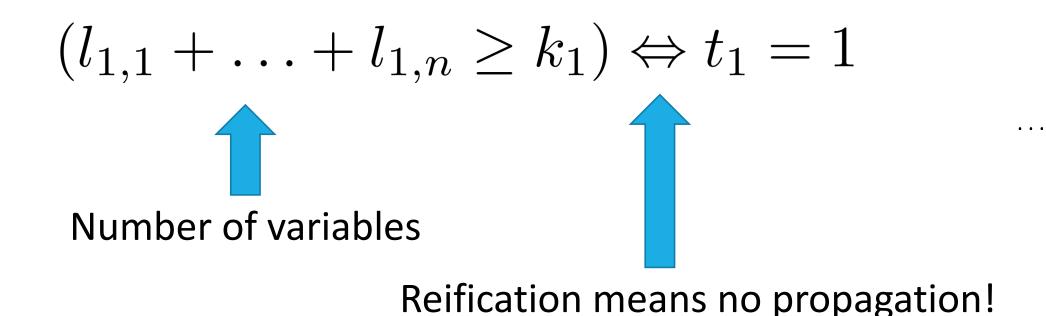
+ a nice shape of a matrix

Microview: a large constraint

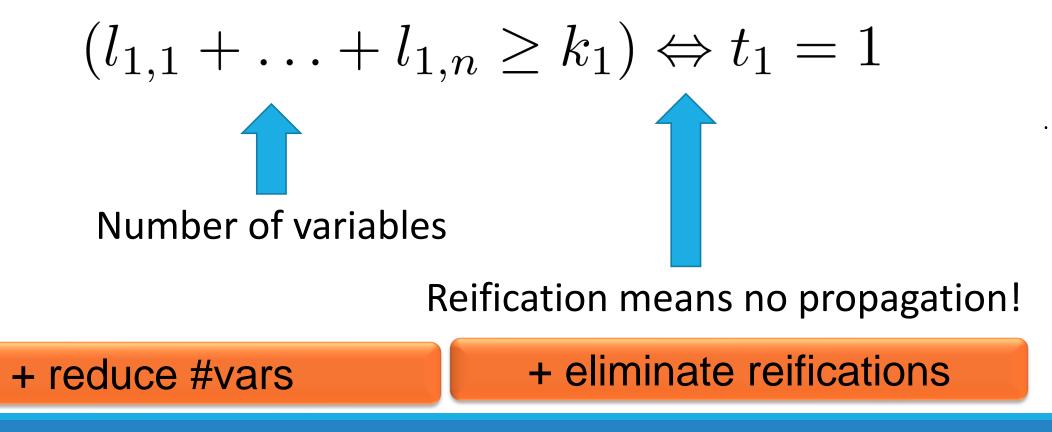
$(l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1$

. . .

Microview: a large constraint



Microview: a large constraint





+ reduce #vars

+ eliminate reifications

+ a nice shape of a matrix

+ a structure aware solver



+ reduce #vars

+ eliminate reifications

+ a nice shape of a matrix

+ a structure aware solver

Wish list



+ eliminate reifications

+ a nice shape of a matrix

+ a structure aware solver

BNN parameters and structure are not fixed*

We can train a BNN so that

+ reduce #vars

+ eliminate reifications

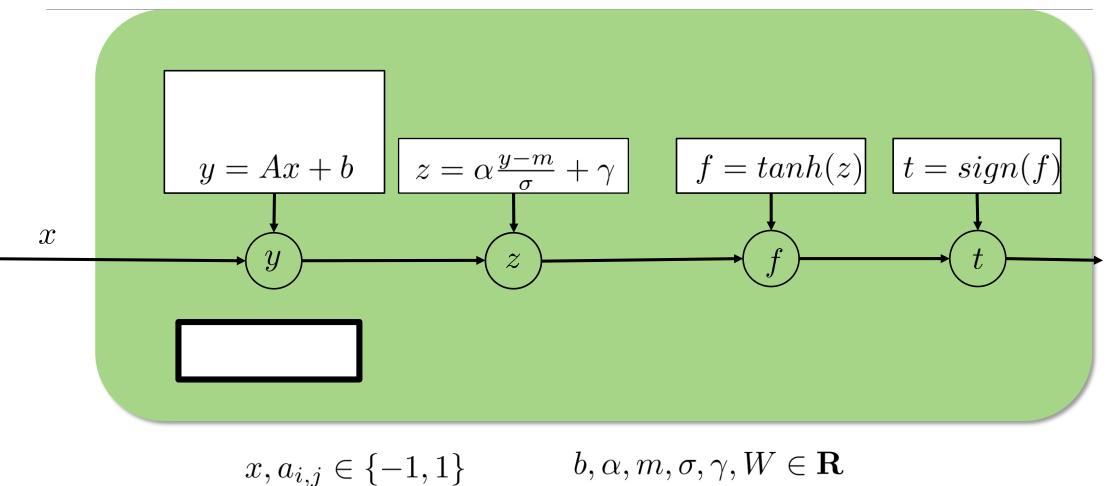
We can train a BNN so that

+ reduce #vars

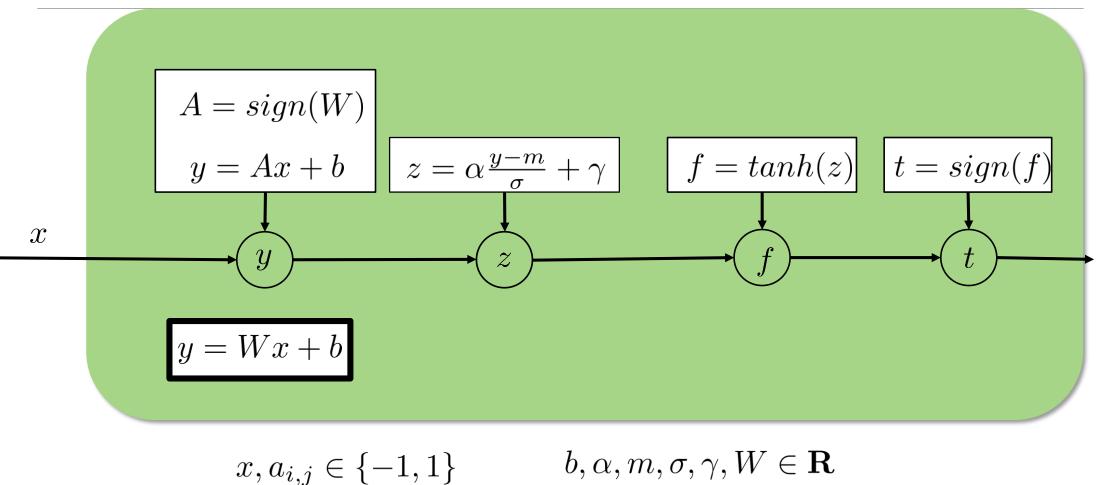
+ eliminate reifications

Training for Faster Adversarial Robustness Verification via Inducing ReLU Stability Kai Y. Xiao, Vincent Tjeng, Nur Muhammad (Mahi) Shafiullah, Aleksander Madry

Binarized Neural Networks



Binarized Neural Networks



Running example

Dataset:MNIST with backgroundProblem:Untargeted adversarial exampleswith ε in {1,3,5,10,15,25}Networks:BNNs with five FC layers



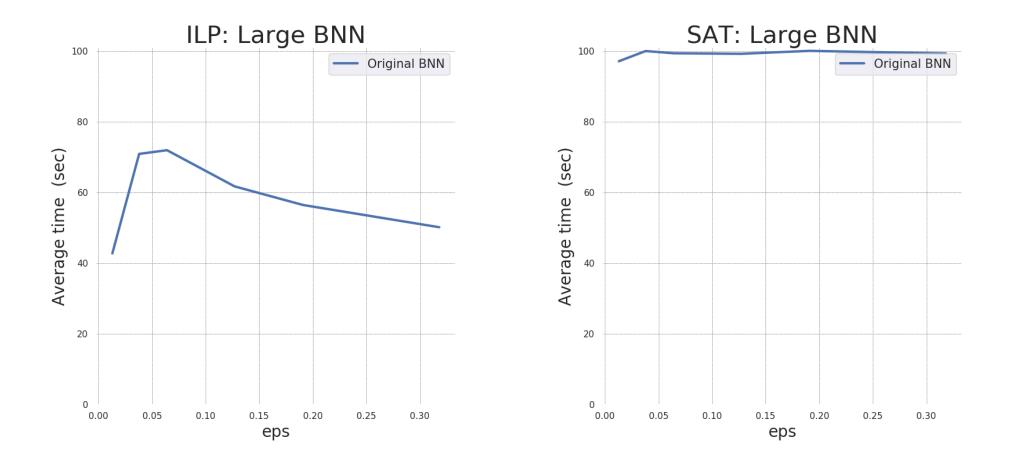
• "Small BNN" with 200K params

• "Large BNN" with 620K params

Running example

Train: From a pretrain full precision network
 Inputs: Normalized
 Results: average time to solve per ε
 out of 100 benchmarks
 Solvers: CPLEX, Glucose (PySAT convertor)

Baseline: verification of original BNNs



BNN+: Improved Binary Network Training

Sajad Darabi, Mouloud Belbahri, Matthieu Courbariaux, Vahid Partovi Nia

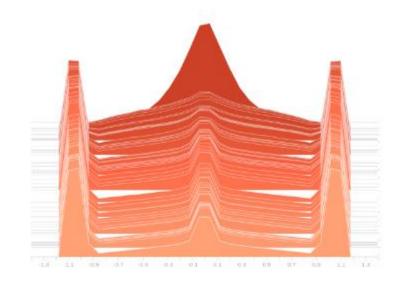


Figure 2: Progression of the weights training in BNN (Hubara et al., 2016). As training progresses the weights create three modes: at -1, 0, and at +1.

BNN+: Improved Binary Network Training

Sajad Darabi, Mouloud Belbahri, Matthieu Courbariaux, Vahid Partovi Nia

$$(l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1$$

where

$$a_{i,j} = 1 \Rightarrow l_j = x_j, a_{i,j} = -1 \Rightarrow l_j = \bar{x}_j$$

$$(l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1$$

where

$$a_{i,j} = 1 \Rightarrow l_j = x_j,$$

 $a_{i,j} = 0 \Rightarrow l_j = 0,$
 $a_{i,j} = -1 \Rightarrow l_j = \bar{x}_j$

Train ternary NN where weights are -1,0,1

Trained Ternary Quantization, ICLR'16, Zhu at el

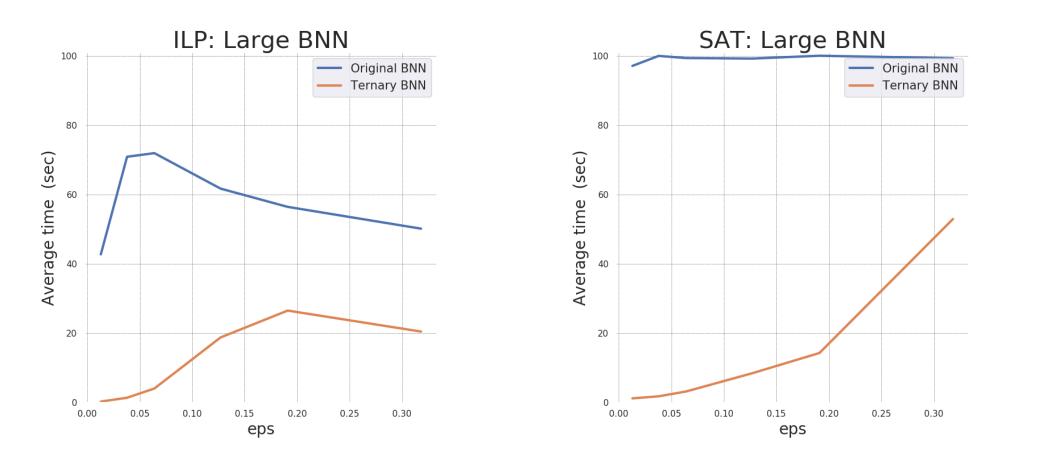
- 1. Train a BNN
- 2. Build a distribution of absolute values of weights
- 3. Select a percentile (40%, 60%), t= 0.03
- 4. Train a ternary BNN with the two-sided threshold t

$$a_{i,j} = \begin{cases} 0 & \text{if } |w_{i,j}| \le t\\ sign(w_{i,j}) & \text{otherwise} \end{cases}$$

- 1. Train a BNN
- 2. Build a distribution of absolute values of weights
- 3. Select a percentile (40%, 60%), t= 0.03
- 4. Train a ternary BNN with the two-sided threshold t

Note: Transformation from BNN to SAT changes a bit

	Small BNN	Large BNN
Original BNN	200K (73.0%)	600K (74.0%)
Ternary BNN	26K (75.2%)	40K (78.2%)



97

$$(l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1$$

where

$$a_{i,j} = 1 \Rightarrow l_j = x_j,$$

$$a_{i,j} = 0 \Rightarrow l_j = 0,$$

$$a_{i,j} = -1 \Rightarrow l_j = \bar{x}_j$$

$$(l_{1,1} + \ldots + l_{1,n} \ge k_1) \Leftrightarrow t_1 = 1$$

where

$$a_{i,j} = 1 \Rightarrow l_j = x_j,$$

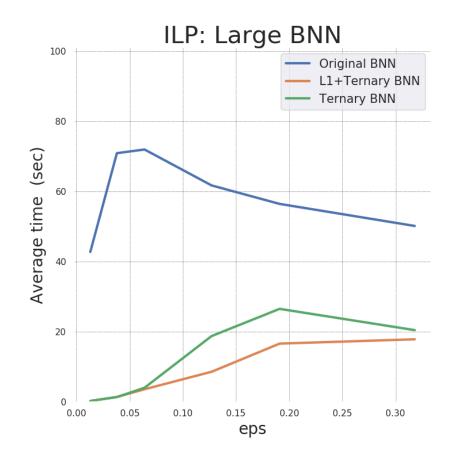
$$a_{i,j} = 0 \Rightarrow l_j = 0,$$

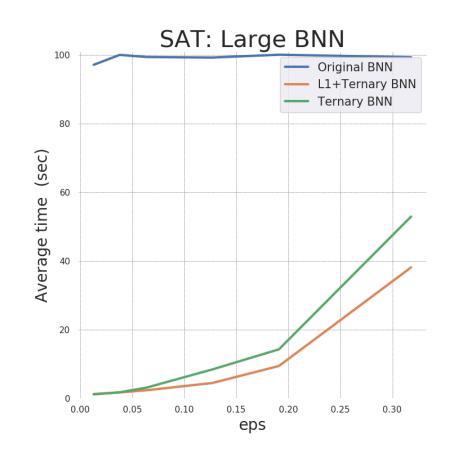
$$a_{i,j} = -1 \Rightarrow l_j = \bar{x}_j$$

Add L1 regularization

	Small BNN	Large BNN
Original BNN	200K (73.0%)	600K (74.0%)
Ternary BNN	26K (75.2%)	40K (78.2%)
L1 + Ternary BNN	24K (75.3%)	36K (78.4%)

L1 + Ternary BNN





$(l_{1,1} + \ldots + l_{1,n} - k_1 \ge 0) \Leftrightarrow t_1 = 1$

$$(l_{1,1} + \ldots + l_{1,n} - k_1 \ge 0) \Leftrightarrow t_1 = 1$$

 $LB_{(l_{1,1}+\ldots+l_{1,n}-k_1)} \ge 0$

$$(l_{1,1} + \ldots + l_{1,n} - k_1 \ge 0) \Leftrightarrow t_1 = 1$$

$$LB_{(l_{1,1}+\ldots+l_{1,n}-k_1)} \ge 0 \qquad t_1 = 1$$

$(l_{1,1} + \ldots + l_{1,n} - k_1 \ge 0) \Leftrightarrow t_1 = 1$

$$(l_{1,1} + \ldots + l_{1,n} - k_1 \ge 0) \Leftrightarrow t_1 = 1$$

 $UB_{(l_{1,1}+\ldots+l_{1,n}-k_1)} < 0$

$$(l_{1,1} + \ldots + l_{1,n} - k_1 \ge 0) \Leftrightarrow t_1 = 1$$

$$UB_{(l_{1,1}+\ldots+l_{1,n}-k_1)} < 0 \qquad t_1 = 0$$

Encourage LB and UB of a neurons to take the same sign:

$$sign(UB_{i,j}) = sign(LB_{i,j})$$

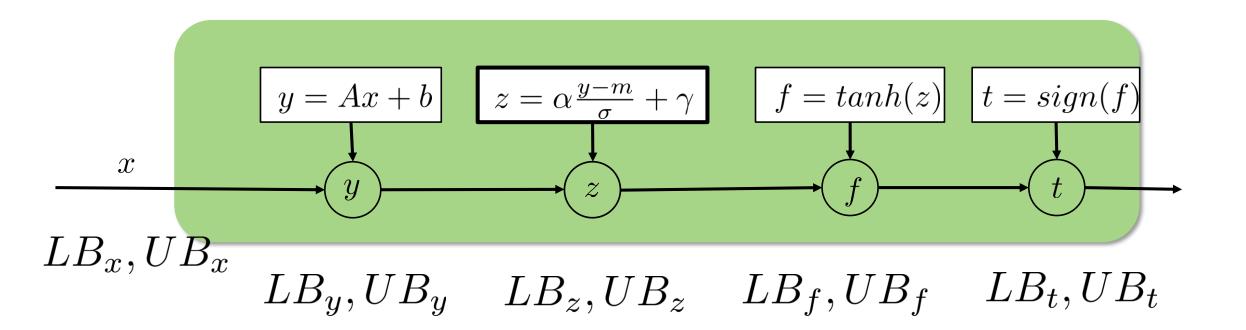
We add a term to the loss function:

$$sign(UB_{i,j}) * sign(LB_{i,j})$$

We add a (approximation) term to the loss function:

$$\frac{-sign(UB_{i,j}) * sign(LB_{i,j})}{-tanh(1 + UB_{ij}LB_{ij})}$$

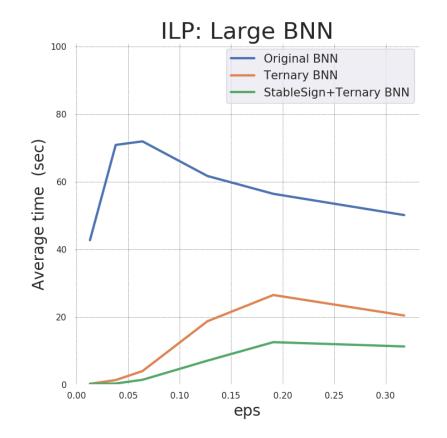
Training for Faster Adversarial Robustness Verification via Inducing ReLU Stability Kai Y. Xiao, Vincent Tjeng, Nur Muhammad (Mahi) Shafiullah, Aleksander Madry

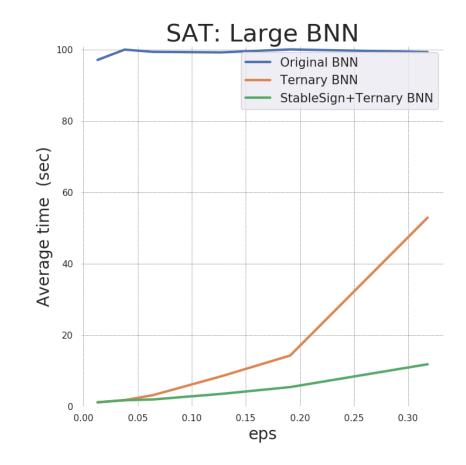


StableSign+Ternary quantization

	Small BNN	Large BNN
Original BNN	200K (73.0%)	600K (74.0%)
Ternary BNN	26K (75.2%)	40K (78.2%)
StableSign + Ternary BNN	25K (76.7%) ~20% stable	38K (78.4%) ~40% stable

StableSign+Ternary quantization

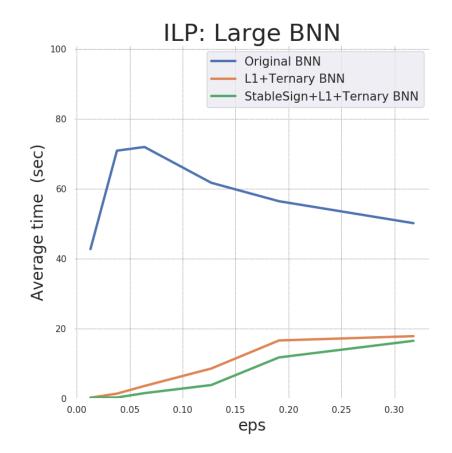


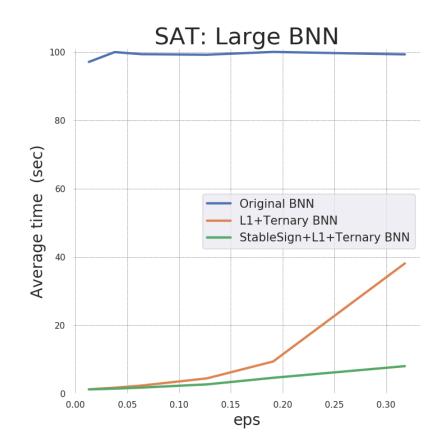


StableSign+ L1+Ternary quantization

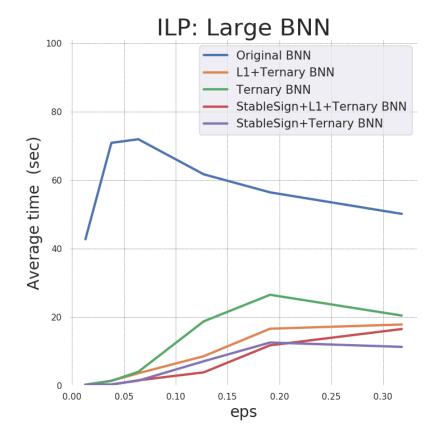
	Small BNN	Large BNN
Original BNN	200K (73.0%)	600K (74.0%)
L1 + Ternary BNN	24K (75.3%)	36K (78.4%)
StableSign + L1 + Ternary BNN	23K (76.6%) ~20% stable	34K (80.4%) 40% stable

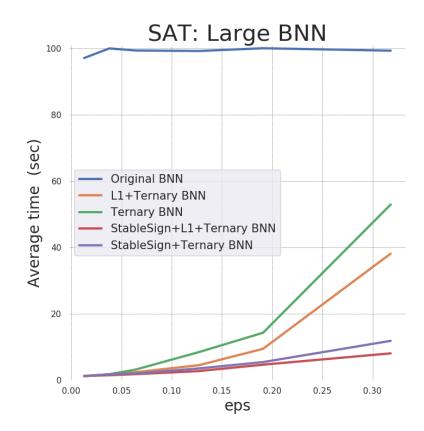
StableSign+ L1+Ternary quantization





Summary





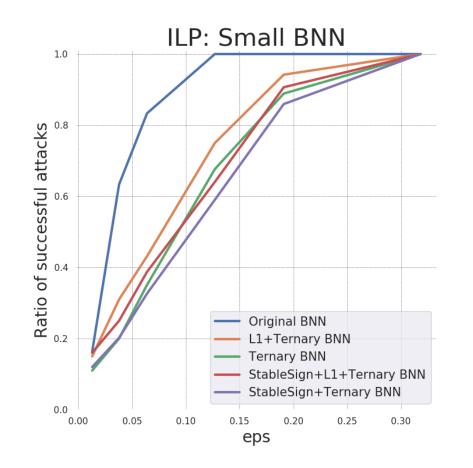
Running example

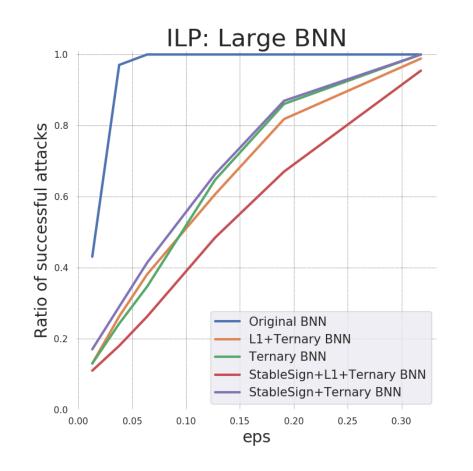
Additional analysis

Running example: Accuracy

	Small BNN	Large BNN
Original BNN	73.0%	74.0%
Ternary BNN	75.2%	78.2%
StableSign+Ternary BNN	76.7%	78.4%
L1+Ternary BNN	75.3%	78.4%
StableSign+L1+Ternary BNN	76.6%	80.0%

Resistance to attacks





Resistance to attacks (median)

Models	Large BNN	
	eps = 5	eps = 10
Original BNN	?	?
Ternary BNN	1%	1%
StableSign+Ternary BNN	1.2%	1.9%
L1+Ternary BNN	2.7%	1.9%
StableSign+L1+Ternary BNN	7%	1.3%

Conclusion

Proposed a method to train a (easier) verifiable BNN.

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Proposed a method to train a (easier) verifiable BNN.

- > 10X reduction in the number of coefficients
- ~3 000 000 -> ~50 000 reduction in the #clauses

Thanks!