# E&CE 682: Multivariable Control Systems

Instructor: Prof. Daniel Miller (EIT3116; ext. 35215; miller@uwaterloo.ca)

Schedule: Lectures: Monday 2:30pm - 5:30pm (EIT3151)

**Calendar Description:** An introduction to control theory for linear time-invariant finite-dimensional systems from both the state-space and input-output viewpoints. State-space theory: the concepts of controllability, observability, stabilizability, and detectability; the pole-assignment theorem; observers and dynamic compensation; LQR regulators. Input-output theory: the ring of polynomials and the field of rational functions; the algebra of polynomial and rational matrices; coprime factorization of transfer matrices; Youla parametrization. Introduction to optimal control.

**Text:** I will provide coursenotes on LEARN; I will place some relevant textbooks on reserve in the library.

# Grading:

- There will be exercises spread throughout the notes. Depending on the size of the class, I will mark some of them to yield an assignment mark (20%).
- There will be an individual project worth 30%. It will entail modelling a physical system, designing a state estimator(s) and a controller(s), and carrying out some MATLAB simulations. Details will be provided in the second or third week to class.
- There will be a final exam worth 50%.

 $\ensuremath{\mathbf{Assumed Background: ECE380}}$  (or equivalent) and familiarity with basic linear algebra.

# **Detailed Description:**

# 1. Introduction to Linear Multivariable Systems

Some detailed examples include electrical systems and mechanical systems, including an inverted pendulum.

# 2. Mathematical Preliminaries

Vector spaces, matrices, matrix manipulations, norms, solving linear differential and difference vector equations.

# 3. Multivariable LTI Systems: An Input-Output Approach

State-space to transfer function, similarity transformations, discretizing a continuoustime system, poles and zeros of a multivariable system.

#### 4. Controllability

Controllability, reachability, PBH test, equivalence of pole placement and controllability, controllable canonical form.

# 5. Observers and Observability

Observability, detectability, PBH test, observers, separation principle.

# 6. LQR Optimal Control

Cost functions, dynamic programming, Riccati equation, optimal LQR feedback, properties of the optimal LQR feedback.

# 7. Youla Parametrization and its Applications

Matrix fractional descriptions of transfer functions, Bezout identity, parametrization of all stabilizing controllers, an introduction to  $L_1$ ,  $H_{\infty}$  and  $H_2$  optimal control.

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www.uwaterloo.ca/academicintegrity/ for more information.]

**Grievance:** A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4,

www.adm.uwaterloo.ca/infosec/Policies/policy70.htm.

When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

**Discipline:** A student is expected to know what constitutes academic integrity - check www.uwaterloo.ca/academicintegrity/

to avoid committing an academic offence, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about rules for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate Associate Dean. For information on categories of offences and types of penalties, students should refer to Policy 71, Student Discipline,

www.adm.uwaterloo.ca/infosec/Policies/policy71.htm.

For typical penalties check Guidelines for the Assessment of Penalties,

www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm.

**Appeals:** A decision made or penalty imposed under Policy 70 (Student Petitions and Grievances) (other than a petition) or Policy 71 (Student Discipline) may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72 (Student Appeals)

www.adm.uwaterloo.ca/infosec/Policies/policy72.htm.

Note for Students with Disabilities: The Office for Persons with Disabilities (OPD), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the OPD at the beginning of each academic term.