

# Safe and Robust Deep Learning

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# SafeAI @ ETH Zurich ([safeai.ethz.ch](https://safeai.ethz.ch))

Joint work with



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## Publications:

S&P'18: AI2: Safety and Robustness Certification of Neural Networks with Abstract Interpretation

NeurIPS'18: Fast and Effective Robustness Certification

POPL'19: An Abstract Domain for Certifying Neural Networks

ICLR'19: Boosting Robustness Certification of Neural Networks

ICML'18: Differentiable Abstract Interpretation for Provably Robust Neural Networks

ICML'19: DL2: Training and Querying Neural Network with Logic

## Systems:

ERAN: Generic neural network verifier

DiffAI: System for training provably robust networks

DL2: System for training and querying networks with logical constraints

# Deep learning systems

Self driving cars



<https://waymo.com/tech/>

Translation

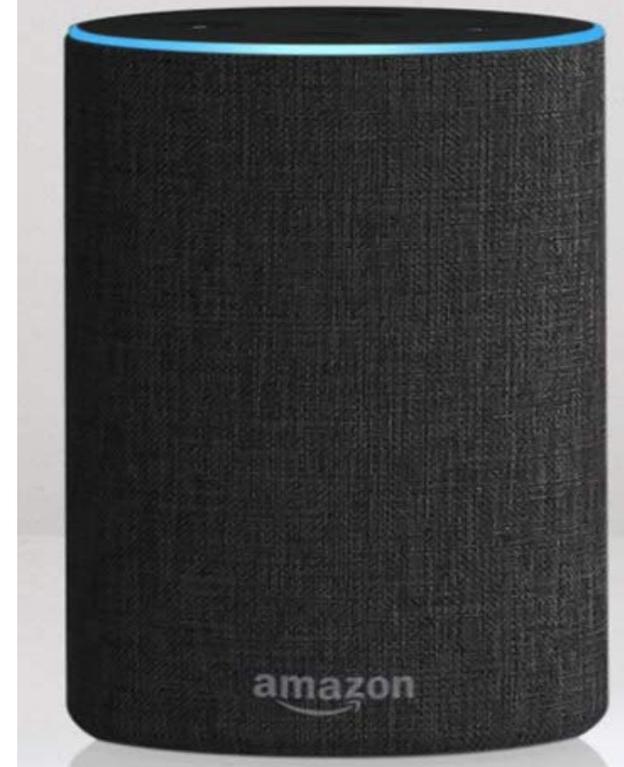
≡ Google Translate

🗨️ Text

📄 Documents

<https://translate.google.com>

Voice assistant



[https://www.amazon.com/  
Amazon-Echo-And-Alexa-Devices](https://www.amazon.com/Amazon-Echo-And-Alexa-Devices)

# Attacks on deep learning

The self-driving car incorrectly decides to turn right on Input 2 and crashes into the guardrail



(a) Input 1

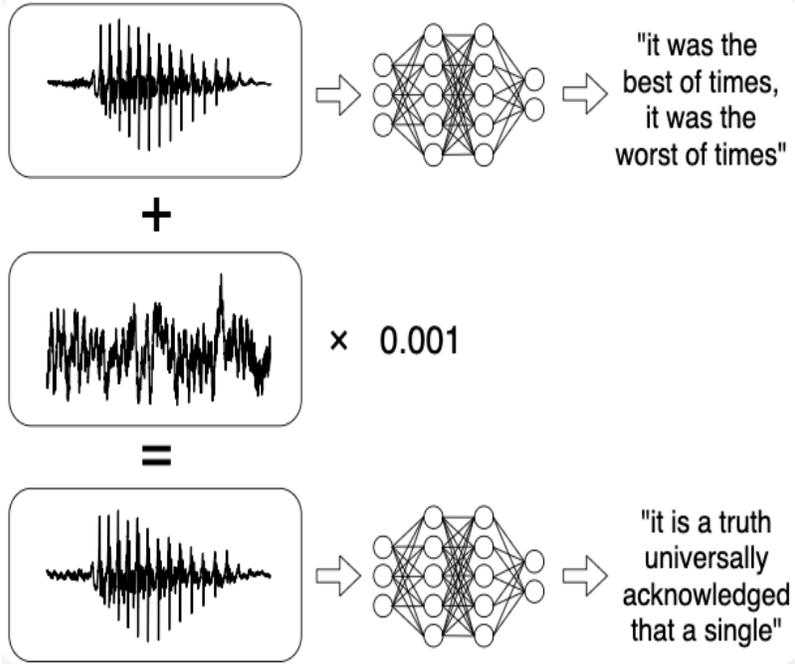
(b) Input 2 (darker version of 1)

The Ensemble model is fooled by the addition of an adversarial distracting sentence in blue.

**Article:** Super Bowl 50  
**Paragraph:** "Peyton Manning became the first quarterback ever to lead two different teams to multiple Super Bowls. He is also the oldest quarterback ever to play in a Super Bowl at age 39. The past record was held by John Elway, who led the Broncos to victory in Super Bowl XXXIII at age 38 and is currently Denver's Executive Vice President of Football Operations and General Manager. *Quarterback Jeff Dean had jersey number 37 in Champ Bowl XXXIV.*"  
**Question:** "What is the name of the quarterback who was 38 in Super Bowl XXXIII?"  
**Original Prediction:** John Elway  
**Prediction under adversary:** Jeff Dean

Adversarial Examples for Evaluating Reading Comprehension Systems, EMNLP'17

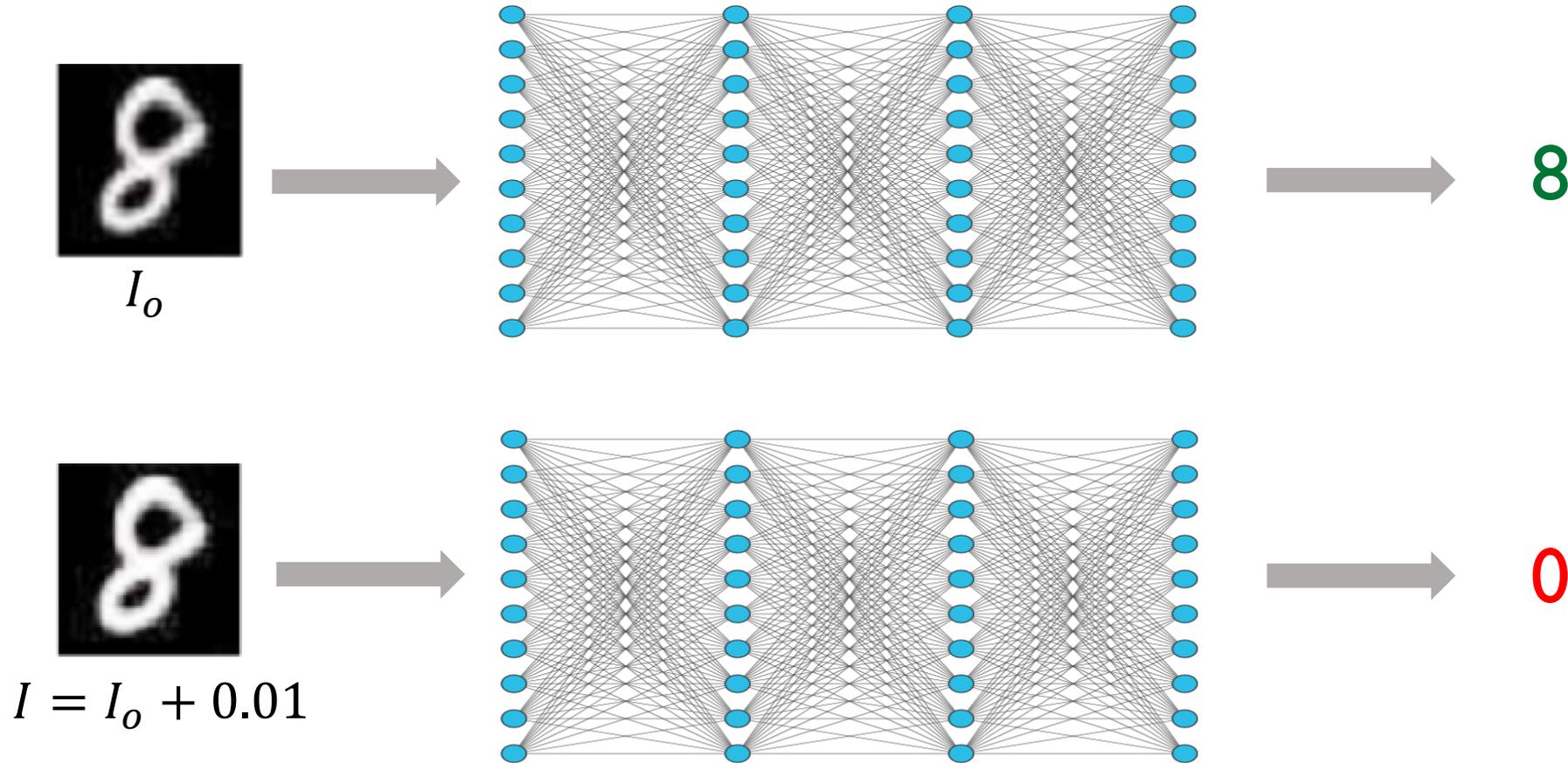
Adding small noise to the input audio makes the network transcribe any arbitrary phrase



Audio Adversarial Examples: Targeted Attacks on Speech-to-Text, ICML 2018

DeepXplore: Automated Whitebox Testing of Deep Learning Systems, SOSP'17

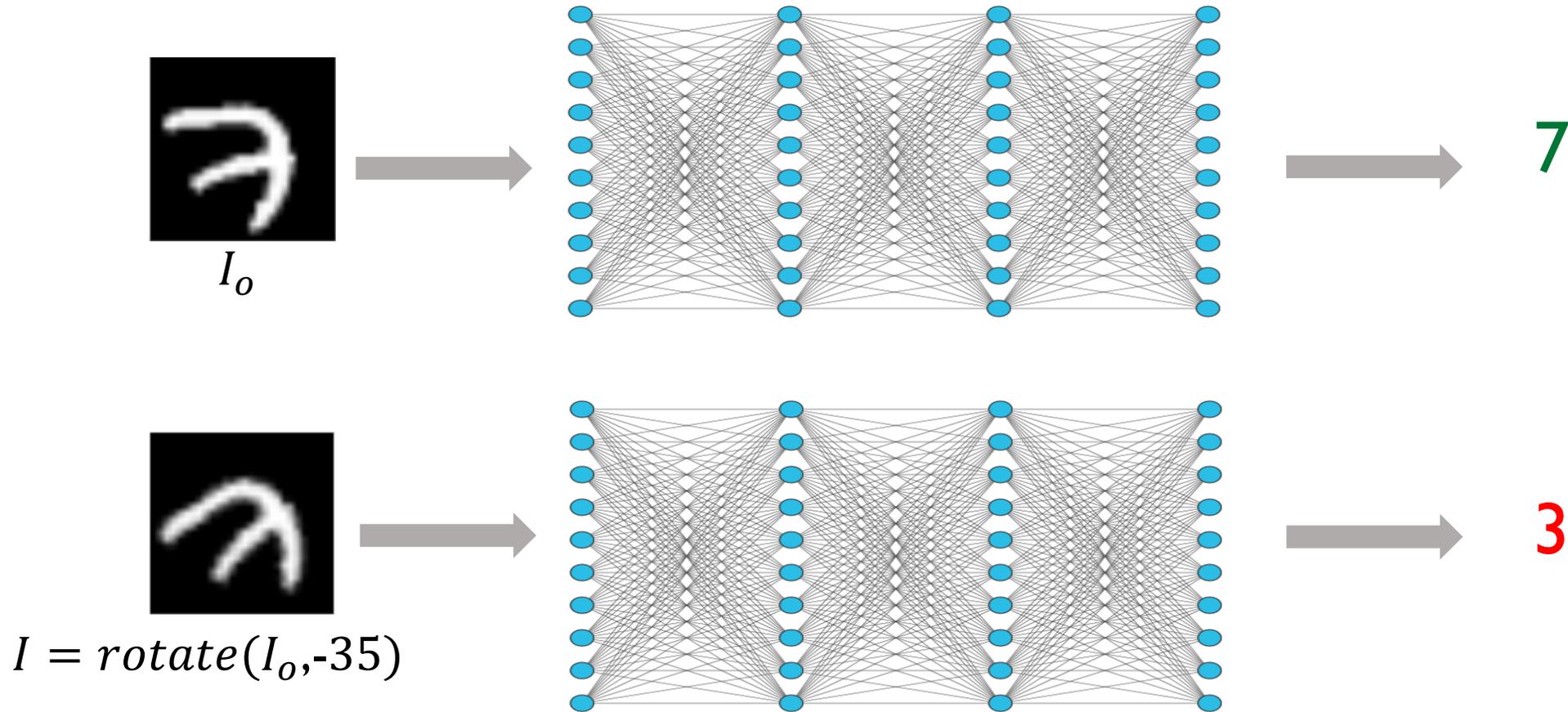
# Attacks based on intensity changes in images



**To verify absence of attack:**

$L_\infty$ -norm: consider all images  $I$  in the  $\epsilon$ -ball  $\mathcal{B}_{(I_0, \infty)}(\epsilon)$  around  $I_0$

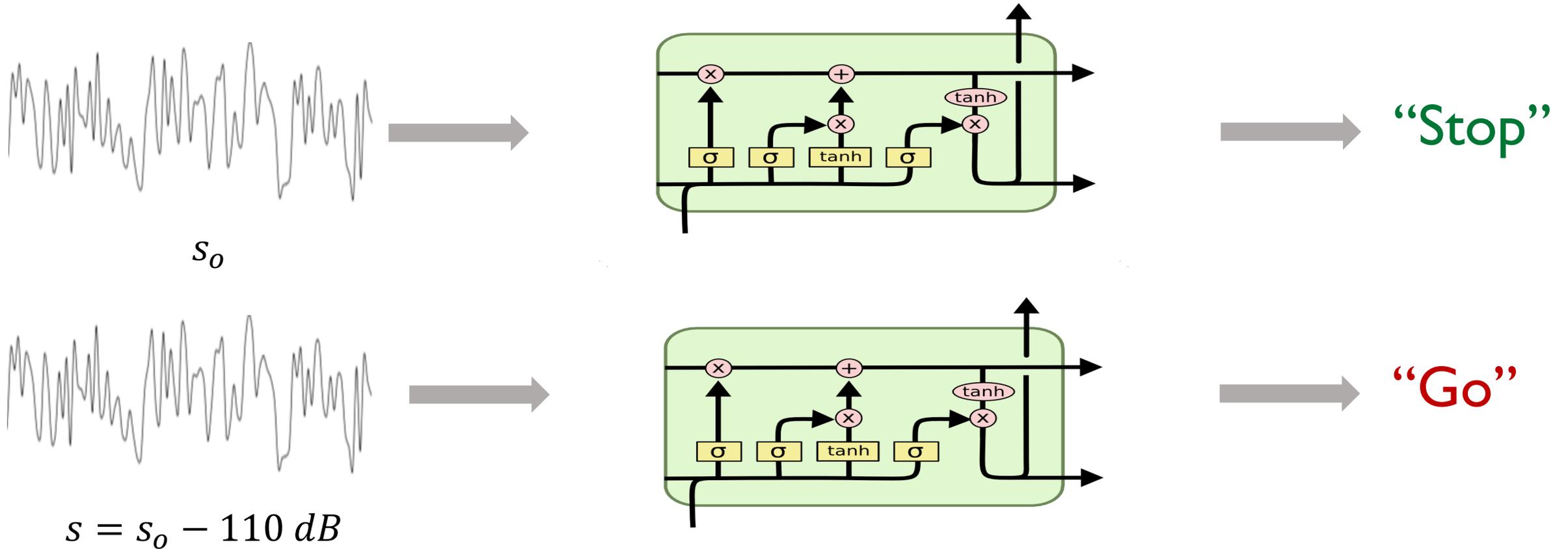
# Attacks based on geometric transformations



**To verify absence of attack:**

Consider all images  $I$  obtained by applying geometric transformations to  $I_0$

# Attacks based on intensity changes to sound



**To verify absence of attack:**

Consider all signals  $s$  in the  $\epsilon$ -ball  $\mathcal{B}_{(s_0, \infty)}(\epsilon)$  around  $s_0$

# Neural network verification: problem statement

**Given:** Neural Network  $f$ ,  
Input Region  $\mathcal{R}$   
Safety Property  $\psi$

**Prove:**  $\forall I \in \mathcal{R}$ ,  
prove that  $f(I)$  satisfies  $\psi$

## Example networks and regions:

### Image classification network $f$

Region  $\mathcal{R}$  based on changes to pixel intensity

Region  $\mathcal{R}$  based on geometric: e.g., *rotation*

### Speech recognition network $f$

Region  $\mathcal{R}$  based on added noise to audio signal

### Aircraft collision avoidance network $f$

Region  $\mathcal{R}$  based on input sensor values

Input Region  $\mathcal{R}$  can contain an infinite number of inputs, thus enumeration is infeasible

# Experimental vs. certified robustness

## Experimental robustness

**Tries to find violating inputs**

**Like testing, no full guarantees**

E.g. Goodfellow 2014, Carlini & Wagner 2016, Madry et al. 2017

## Certified robustness

**Prove** absence of violating inputs

**Actual verification guarantees**

E.g.: Reluplex [2017], Wong et al. 2018, AI2 [2018]

In this talk we will focus on certified robustness

# General approaches to network verification

**Complete** verifiers, but suffer from scalability issues:

SMT: Reluplex [CAV'17], MILP: MIPVerify [ICLR'19],

Splitting: Neurify [NeurIPS'18],...

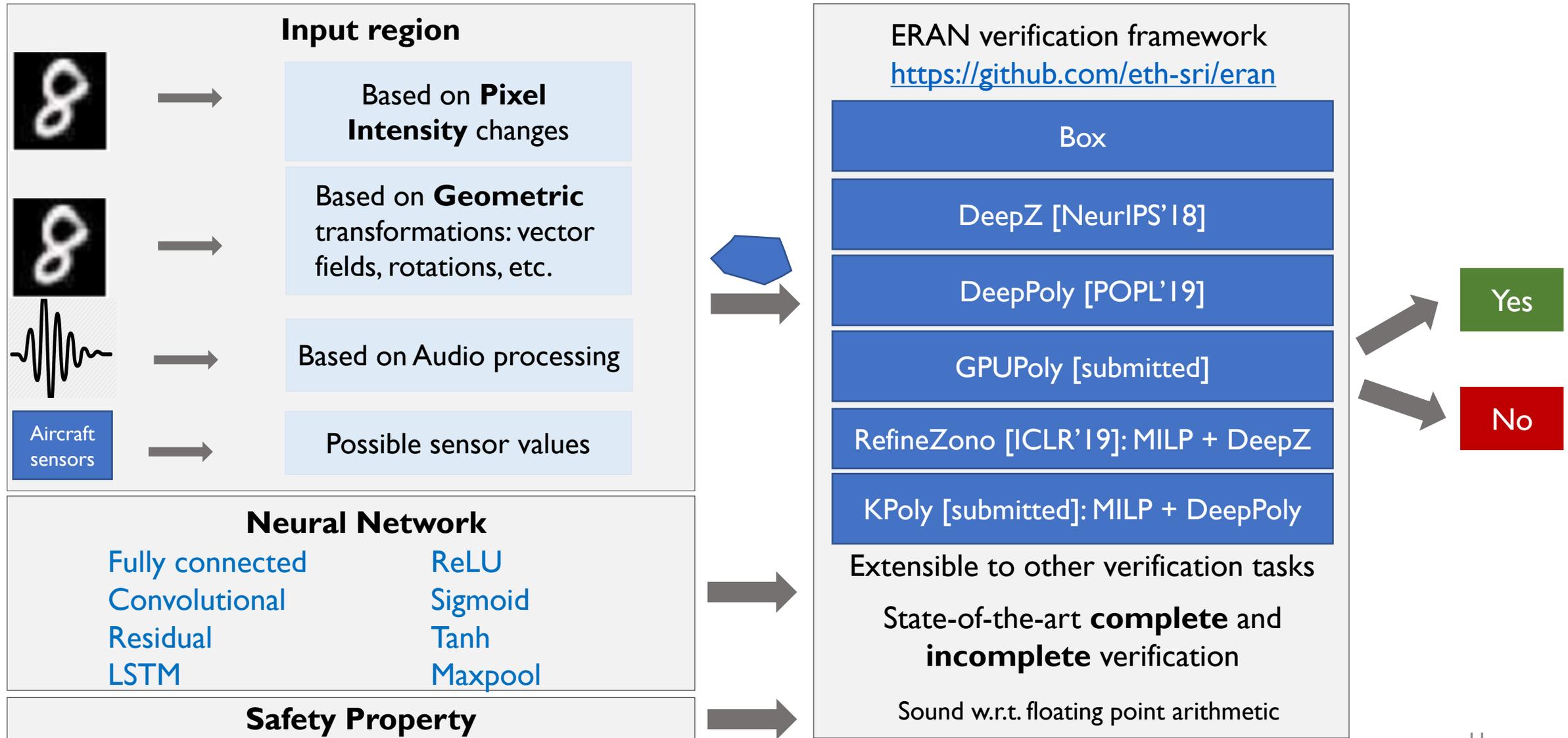
**Incomplete** verifiers, trade-off precision for scalability:

Box/HBox [ICML'18], SDP [ICLR'18], Wong et.al. [ICML'18], FastLin

[ICML'18], Crown [NeurIPS'18],...

**Key Challenge: scalable and precise automated verifier**

# Network verification with ERAN



# Complete and incomplete verification with ERAN

## Faster Complete Verification

Aircraft collision avoidance system (ACAS)		
Reluplex	Neurify	ERAN
> 32 hours	921 sec	227 sec

## Scalable Incomplete Verification

CIFAR10 ResNet-34		
$\epsilon$	%verified	Time (s)
0.03	66%	79 sec

# Geometric and audio verification with ERAN

## Geometric Verification

Rotation between  $-30^\circ$  and  $30^\circ$  on MNIST  
CNN with 4,804 neurons

$\epsilon$	%verified	Time(s)
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0.001	86	10 sec
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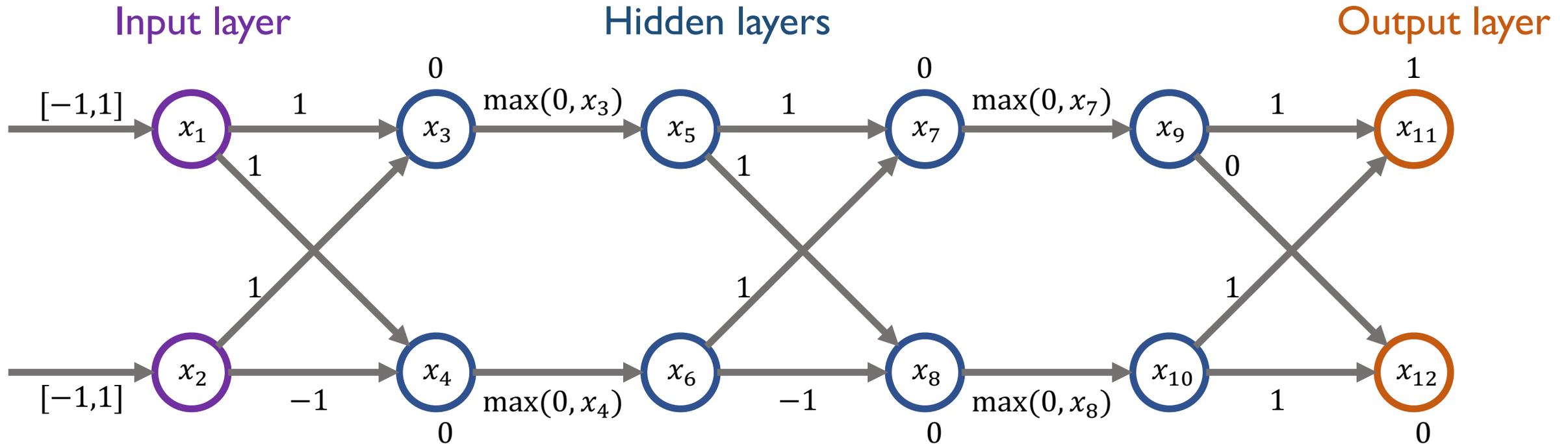
## Audio Verification

LSTM with 64 hidden neurons

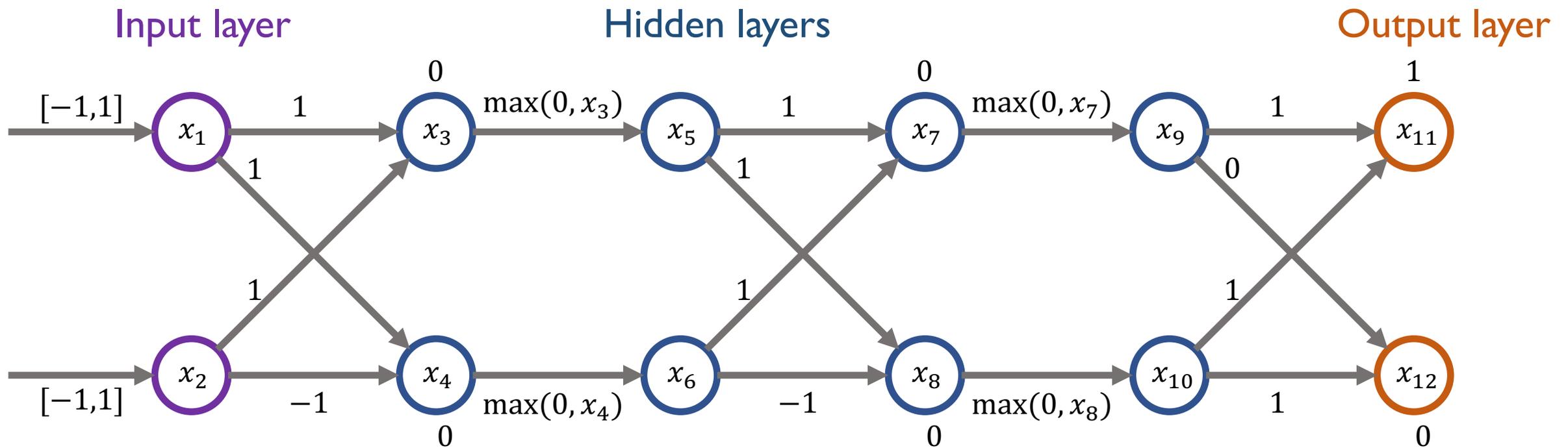
$\epsilon$	%verified	Time (s)
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-110 dB	90%	9 sec
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# Example: analysis of a toy neural network



We want to prove that  $x_{11} > x_{12}$  for all values of  $x_1, x_2$  in the input set



$$\min x_{11} - x_{12}$$

$$\begin{aligned} \text{s.t. : } & x_{11} = x_9 + x_{10} + 1, \quad x_{12} = x_{10}, \\ & x_9 = \mathbf{max}(0, x_7), \quad x_{10} = \mathbf{max}(0, x_8), \\ & x_7 = x_5 + x_6, \quad x_8 = x_5 - x_6, \\ & x_5 = \mathbf{max}(0, x_3), \quad x_6 = \mathbf{max}(0, x_4), \\ & x_3 = x_1 + x_2, \quad x_4 = x_1 - x_2, \\ & -1 \leq x_1 \leq 1, \quad -1 \leq x_2 \leq 1. \end{aligned}$$

Each  $x_j = \mathbf{max}(0, x_i)$  corresponds to  
 $(x_i \leq 0 \text{ and } x_j = 0)$  or  
 $(x_i > 0 \text{ and } x_j = x_i)$

Solver has to explore two paths per ReLU  
 resulting in **exponential** number of paths

Complete verification with solvers often does not scale

# Abstract interpretation



Patrick and Radhia Cousot  
Inventors

An elegant framework for approximating concrete behaviors

## Key Concept: Abstract Domain

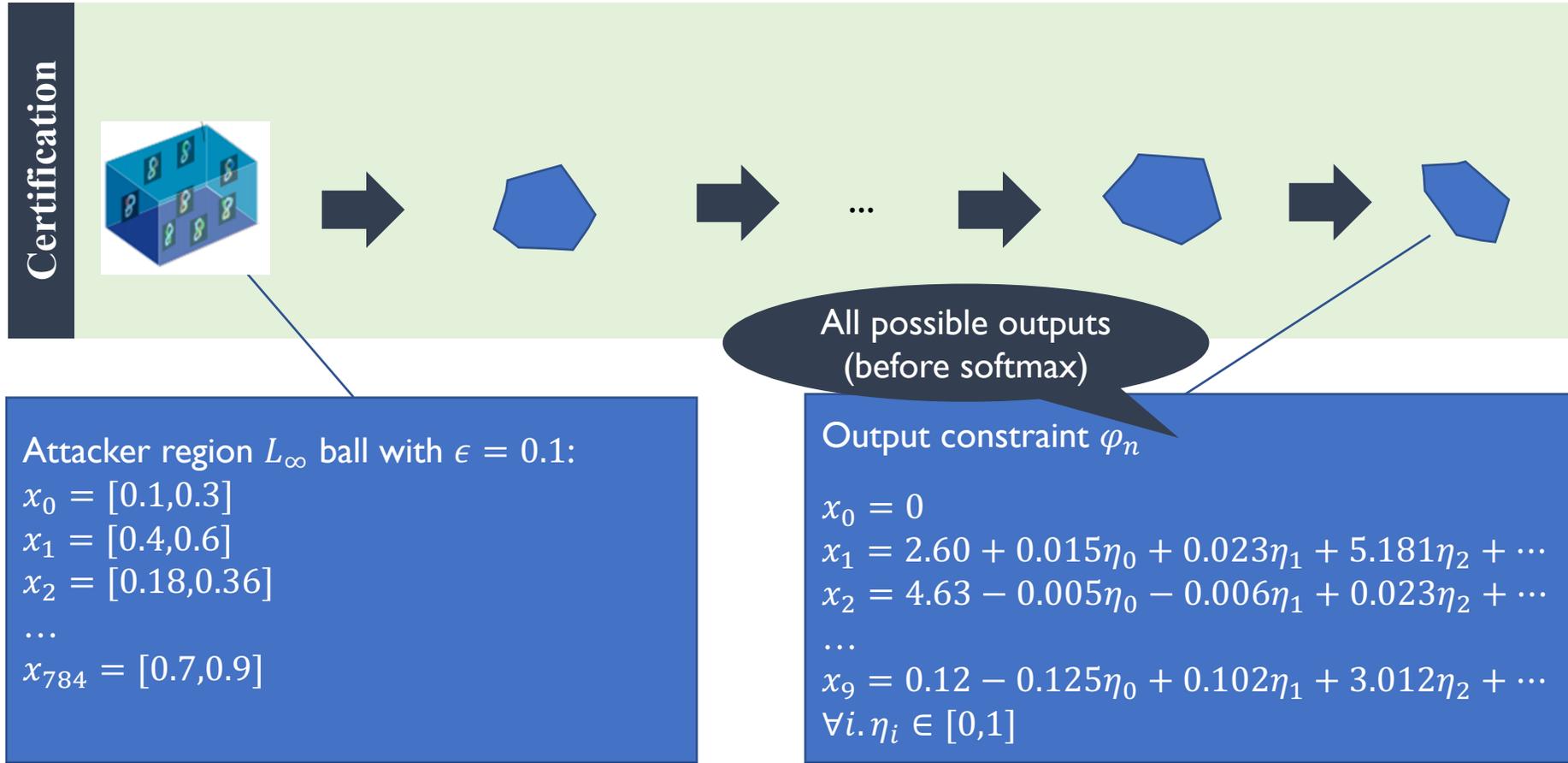
**Abstract element:** approximates set of concrete points

**Concretization function  $\gamma$ :** concretizes an abstract element to the set of points that it represents.

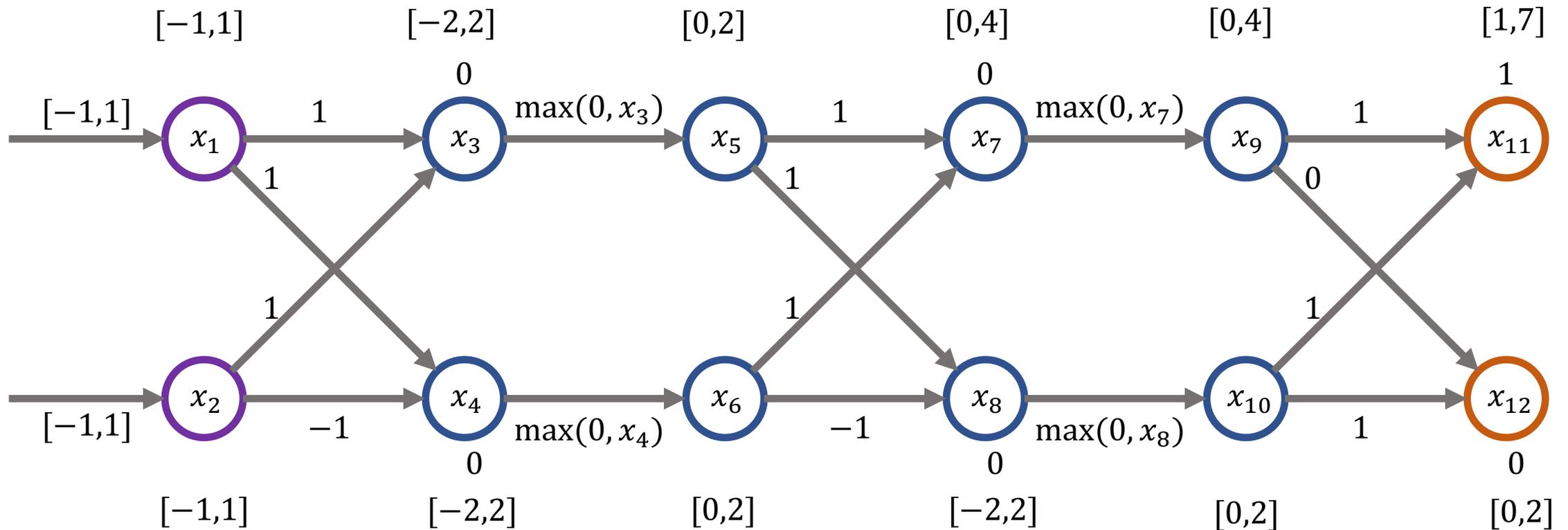
**Abstract transformers:** approximate the effect of applying concrete transformers e.g. affine, ReLU

Tradeoff between the precision and the scalability of an abstract domain

# Network verification with ERAN: high level idea



# Box approximation (scalable but imprecise)



Verification with the Box domain fails as it cannot capture relational information

# DeepPoly approximation [POPL'19]

**Shape:** associate a lower polyhedral  $a_i^{\leq}$  and an upper polyhedral  $a_i^{\geq}$  constraint with each  $x_i$

$$a_i^{\leq}, a_i^{\geq} \in \{x \mapsto v + \sum_{j \in [i-1]} w_j \cdot x_j \mid v \in \mathbb{R} \cup \{-\infty, +\infty\}, w \in \mathbb{R}^{i-1}\} \text{ for } i \in [n]$$

**Concretization of abstract element  $a$ :**

$$\gamma_n(a) = \{x \in \mathbb{R}^n \mid \forall i \in [n]. a_i^{\leq}(x) \leq x_i \wedge a_i^{\geq}(x) \geq x_i\}$$

**Domain invariant:** store auxiliary concrete lower and upper bounds  $l_i, u_i$  for each  $x_i$

$$\gamma_n(a) \subseteq \times_{i \in [n]} [l_i, u_i]$$

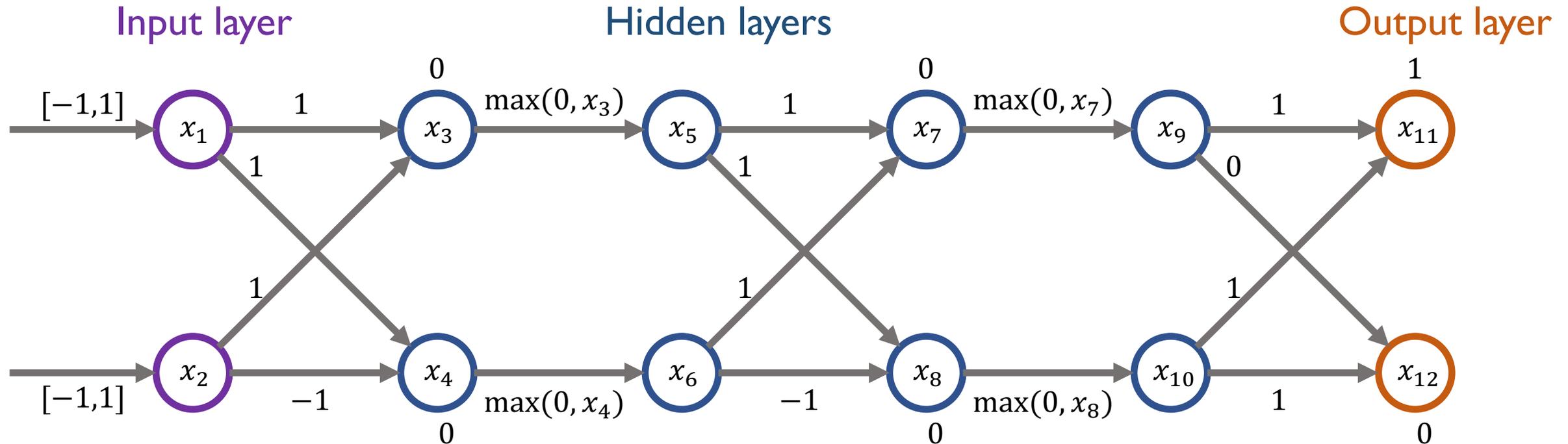
- less precise than Polyhedra, restriction needed to ensure scalability
- captures affine transformation precisely unlike Octagon, TVPI
- custom transformers for ReLU, sigmoid, tanh, and maxpool activations

$n$ : #neurons,  $m$ : #constraints

$w_{max}$ : max #neurons in a layer,  $L$ : # layers

Transformer	Polyhedra	Our domain
Affine	$O(nm^2)$	$O(w_{max}^2 L)$
ReLU	$O(\exp(n, m))$	$O(1)$

# Example: analysis of a toy neural network



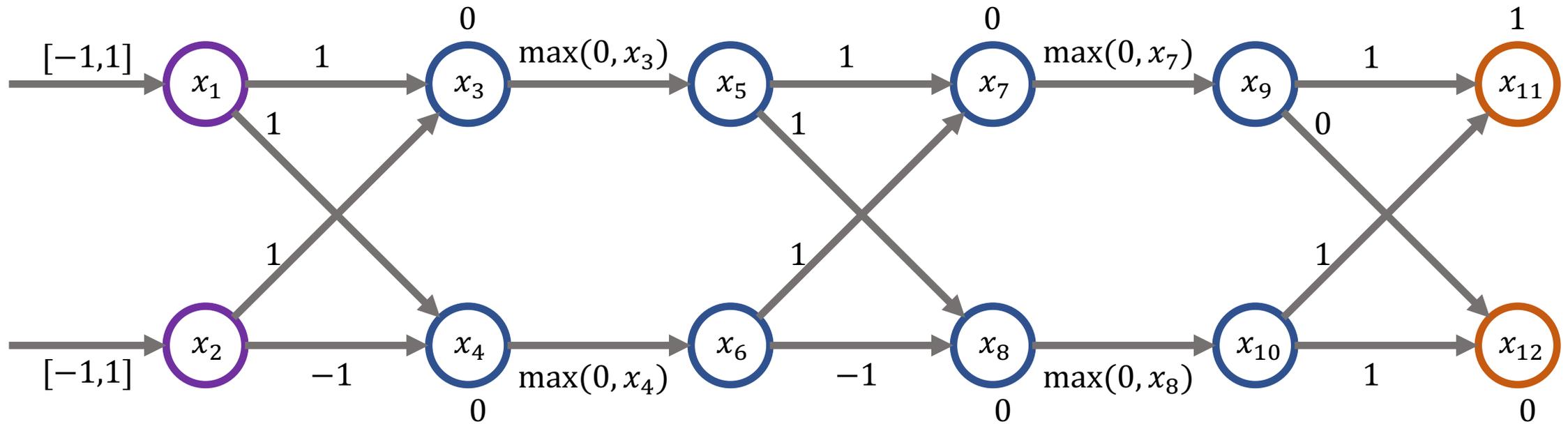
1. 4 constraints per neuron
2. Pointwise transformers  $\Rightarrow$  parallelizable.
3. Backsubstitution  $\Rightarrow$  helps precision.
4. Non-linear activations  $\Rightarrow$  approximate and minimize the area

$$\langle x_1 \geq -1, \quad \langle x_3 \geq x_1 + x_2,$$

$$x_1 \leq 1, \quad x_3 \leq x_1 + x_2,$$

$$l_1 = -1, \quad l_3 = -2,$$

$$u_1 = 1 \rangle \quad u_3 = 2 \rangle$$



$$\langle x_2 \geq -1, \quad \langle x_4 \geq x_1 - x_2,$$

$$x_2 \leq 1, \quad x_4 \leq x_1 - x_2,$$

$$l_2 = -1, \quad l_4 = -2,$$

$$u_2 = 1 \rangle \quad u_4 = 2 \rangle$$

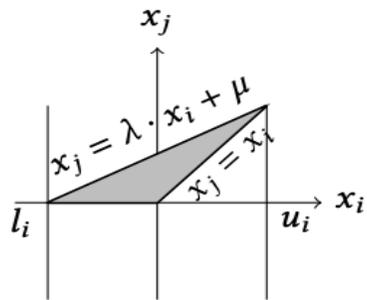
# ReLU activation

Pointwise transformer for  $x_j := \max(0, x_i)$  that uses  $l_i, u_i$

if  $u_i \leq 0, a_j^{\leq} = a_j^{\geq} = 0, l_j = u_j = 0,$

if  $l_i \geq 0, a_j^{\leq} = a_j^{\geq} = x_i, l_j = l_i, u_j = u_i,$

if  $l_i < 0$  and  $u_i > 0$

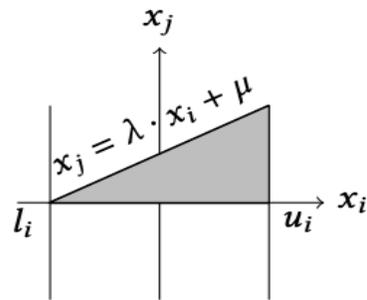


(a)

$$x_i \leq x_j, 0 \leq x_j,$$

$$x_j \leq u_i(x_i - l_i)/(u_i - l_i).$$

$$l_j = 0, u_j = u_i$$

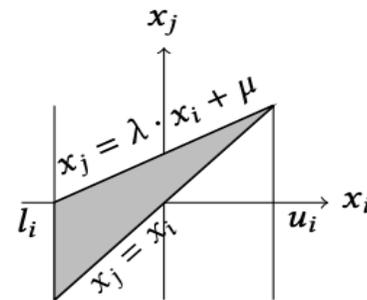


(b)

$$0 \leq x_j,$$

$$x_j \leq u_i(x_i - l_i)/(u_i - l_i),$$

$$l_j = 0, u_j = u_i$$



(c)

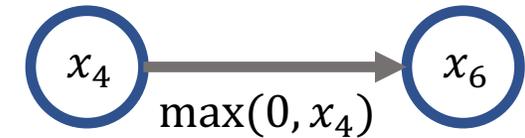
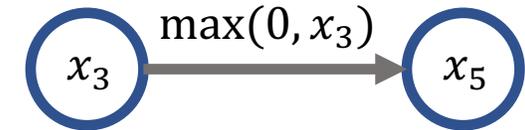
$$x_i \leq x_j,$$

$$x_j \leq u_i(x_i - l_i)/(u_i - l_i),$$

$$l_j = l_i, u_j = u_i$$

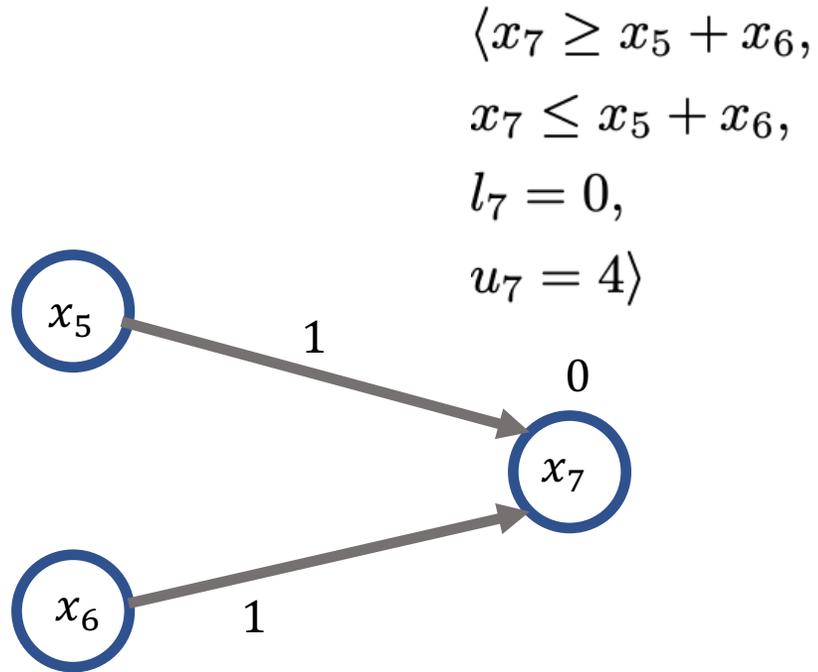
choose (b) or (c) depending on the area

$$\langle x_5 \geq 0, \\ x_5 \leq 0.5 \cdot x_3 + 1, \\ l_5 = 0, \\ u_5 = 2 \rangle$$



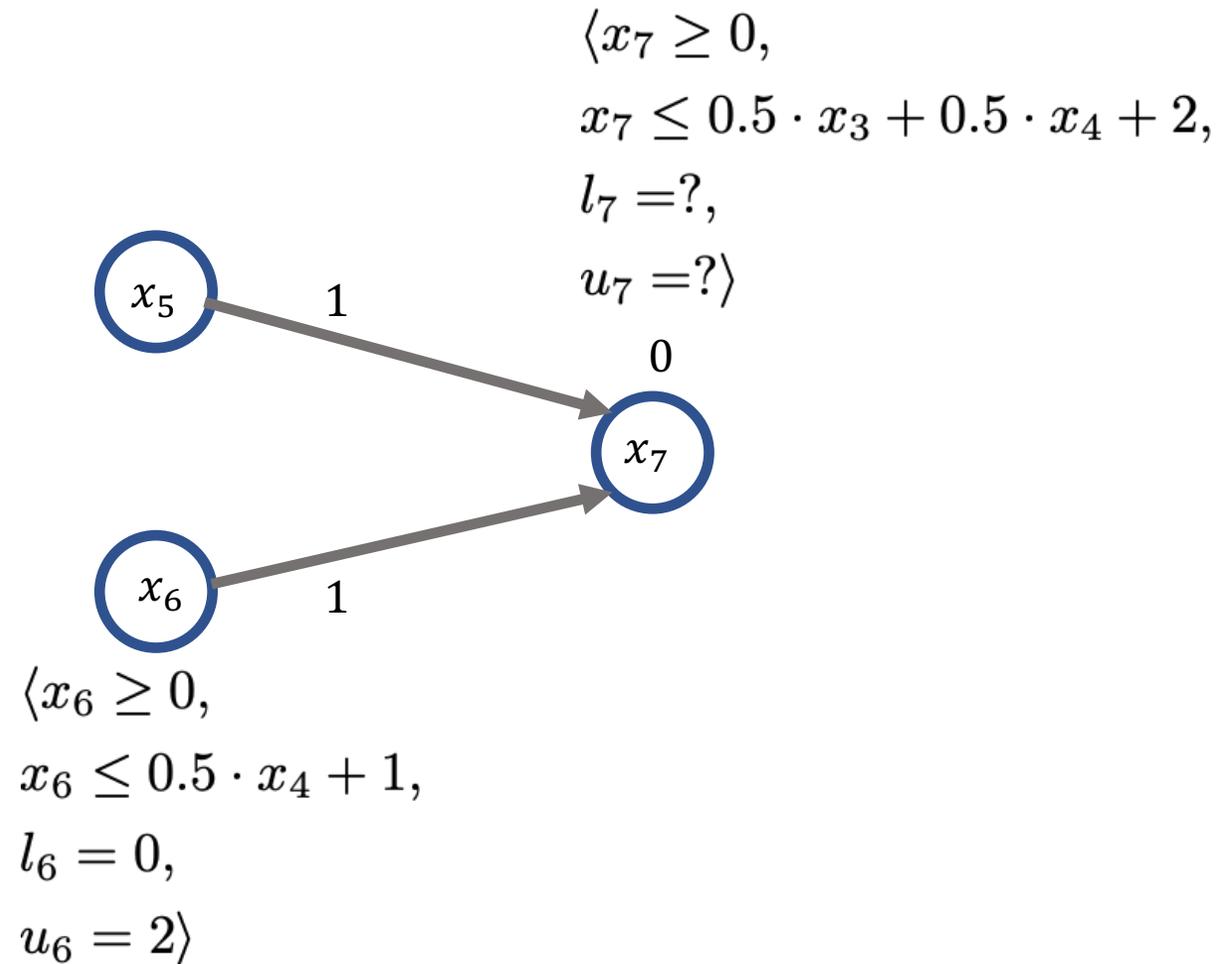
$$\langle x_6 \geq 0, \\ x_6 \leq 0.5 \cdot x_4 + 1, \\ l_6 = 0, \\ u_6 = 2 \rangle$$

# Affine transformation after ReLU



Imprecise upper bound  $u_7$  by substituting  $u_5, u_6$  for  $x_5$  and  $x_6$  in  $a_7^{\geq}$  23

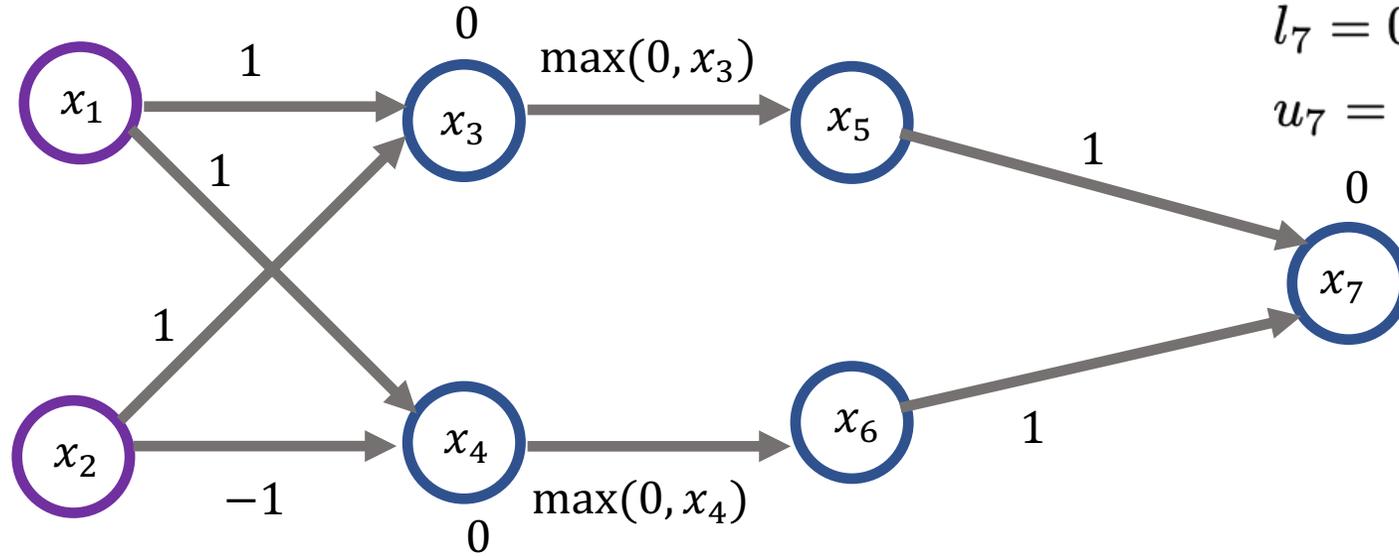
# Backsubstitution



$$\langle x_1 \geq -1, \\ x_1 \leq 1, \\ l_1 = -1, \\ u_1 = 1 \rangle$$

$$\langle x_5 \geq 0, \\ x_5 \leq 0.5 \cdot x_3 + 1, \\ l_5 = 0, \\ u_5 = 2 \rangle$$

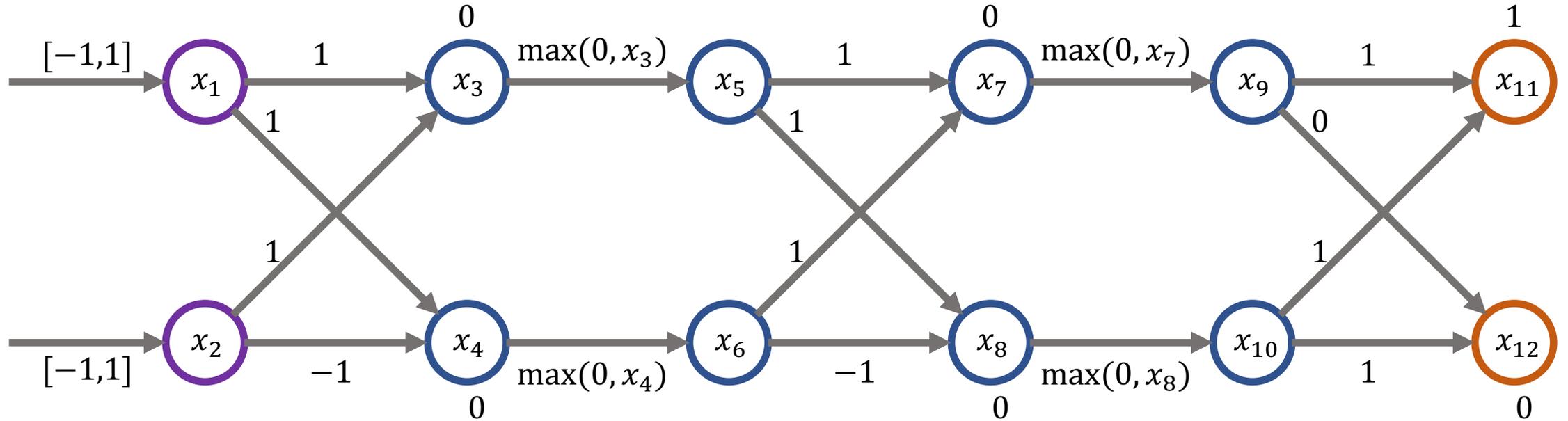
$$\langle x_7 \geq 0, \\ x_7 \leq x_1 + 2, \\ l_7 = 0, \\ u_7 = 3 \rangle$$



$$\langle x_2 \geq -1, \\ x_2 \leq 1, \\ l_2 = -1, \\ u_2 = 1 \rangle$$

$$\langle x_6 \geq 0, \\ x_6 \leq 0.5 \cdot x_4 + 1, \\ l_6 = 0, \\ u_6 = 2 \rangle$$

$$\begin{array}{llll}
\langle x_1 \geq -1, & \langle x_3 \geq x_1 + x_2, & \langle x_5 \geq 0, & \langle x_7 \geq x_5 + x_6, \langle x_9 \geq x_7, \langle x_{11} \geq x_9 + x_{10} + 1, \\
x_1 \leq 1, & x_3 \leq x_1 + x_2, & x_5 \leq 0.5 \cdot x_3 + 1, & x_7 \leq x_5 + x_6, x_9 \leq x_7, x_{11} \leq x_9 + x_{10} + 1, \\
l_1 = -1, & l_3 = -2, & l_5 = 0, & l_7 = 0, l_9 = 0, l_{11} = 1, \\
u_1 = 1 \rangle & u_3 = 2 \rangle & u_5 = 2 \rangle & u_7 = 3 \rangle u_9 = 3 \rangle u_{11} = 5.5 \rangle
\end{array}$$



$$\begin{array}{llll}
\langle x_2 \geq -1, & \langle x_4 \geq x_1 - x_2, & \langle x_6 \geq 0, & \langle x_8 \geq x_5 - x_6, \langle x_{10} \geq 0, & \langle x_{12} \geq x_{10}, \\
x_2 \leq 1, & x_4 \leq x_1 - x_2, & x_6 \leq 0.5 \cdot x_4 + 1, & x_8 \leq x_5 - x_6, x_{10} \leq 0.5 \cdot x_8 + 1, & x_{11} \leq x_{10}, \\
l_2 = -1, & l_4 = -2, & l_6 = 0, & l_8 = -2, l_{10} = 0, & l_{12} = 0, \\
u_2 = 1 \rangle & u_4 = 2 \rangle & u_6 = 2 \rangle & u_8 = 2 \rangle u_{10} = 2 \rangle & u_{12} = 2 \rangle
\end{array}$$

# Checking for robustness

Prove  $x_{11} - x_{12} > 0$  for all inputs in  $[-1,1] \times [-1,1]$

$$\begin{array}{ll} \langle x_{11} \geq x_9 + x_{10} + 1, & \langle x_{12} \geq x_{10}, \\ x_{11} \leq x_9 + x_{10} + 1, & x_{12} \leq x_{10}, \\ l_{11} = 1, & l_{12} = 0, \\ u_{11} = 5.5 \rangle & u_{12} = 0 \rangle \end{array}$$

Computing lower bound for  $x_{11} - x_{12}$  using  $l_{11}, u_{12}$  gives -1 which is an imprecise result

With backsubstitution, one gets 1 as the lower bound for  $x_{11} - x_{12}$ , proving robustness

# Abstract interpretation + solvers

Key Idea: refine abstract interpretation results by calling the solver

- Refine neuron bounds before ReLU transformer is applied  $\Rightarrow$  less area

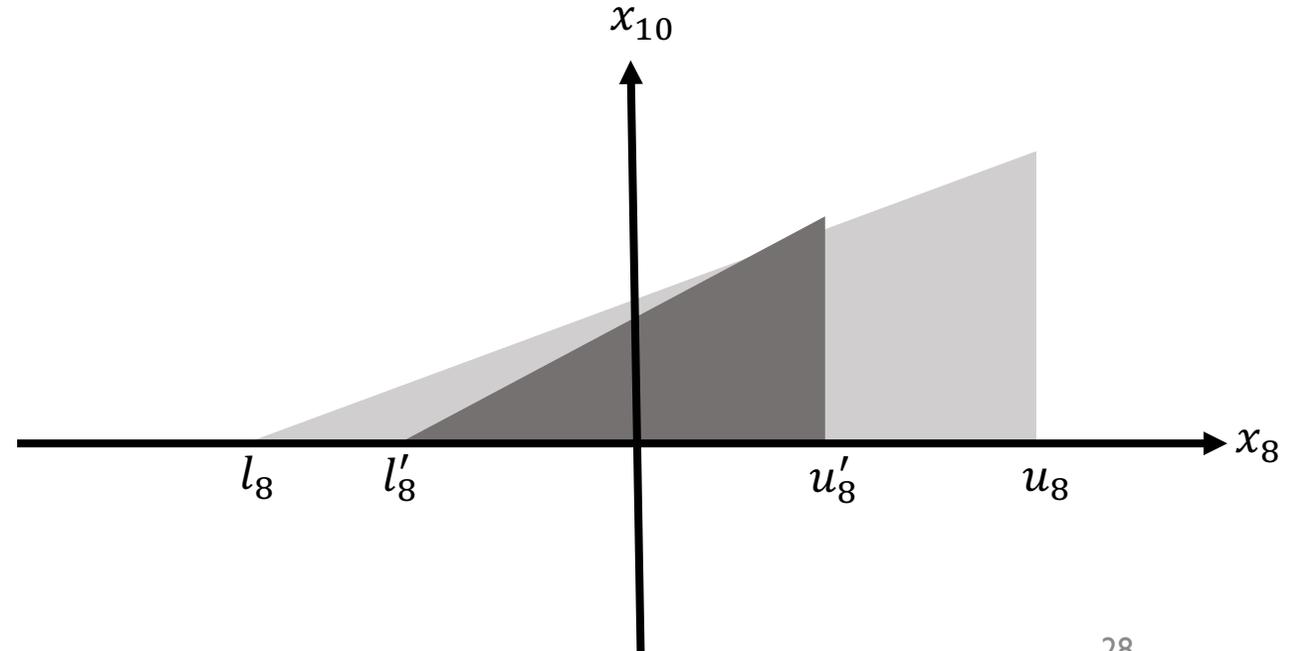
$$l'_8 := \min x_8$$

$$s.t. : x_8 = x_5 - x_6,$$

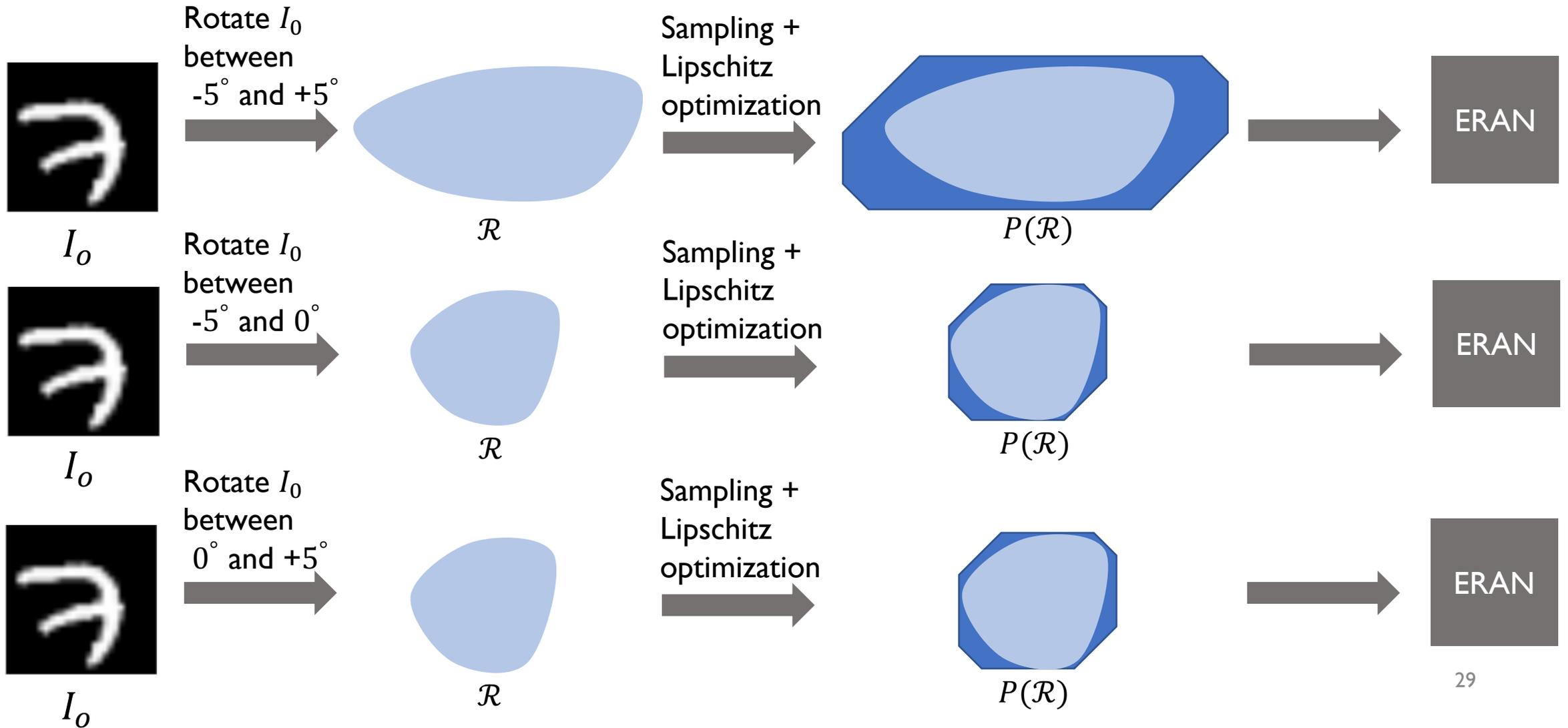
$$x_5 = \max(0, x_3), x_6 = \max(0, x_4),$$

$$x_3 = x_1 + x_2, x_4 = x_1 - x_2,$$

$$-1 \leq x_1 \leq 1, -1 \leq x_2 \leq 1.$$



# Verification against geometric attacks



# Medium sized benchmarks

Dataset	Model	Type	#Neurons	#Layers	Defense
MNIST	6 × 100	feedforward	610	6	None
	6 × 200	feedforward	1,210	6	None
	9 × 200	feedforward	1,810	9	None
	ConvSmall	convolutional	3,604	3	DiffAI
	ConvBig	convolutional	34,688	6	DiffAI
CIFAR10	ConvSmall	convolutional	4,852	3	Wong et al.
	ConvBig	convolutional	62,464	6	PGD

# Results on medium benchmarks (100 test images)

Dataset	Model	#correct	$\epsilon$	DeepPoly		kPoly	
				% 	time(s)	% 	time(s)
MNIST	$6 \times 100$	99	0.026	21	0.3	44	151
	$6 \times 200$	99	0.015	32	0.5	56	387
	$9 \times 200$	97	0.015	29	0.9	54	1040
	ConvSmall	100	0.12	13	6.0	28	1018
	ConvBig	100	0.3	93	12.3	93	286
CIFAR10	ConvSmall	38	0.03	35	0.4	35	1.4
	ConvBig	65	0.008	39	49	40	2882

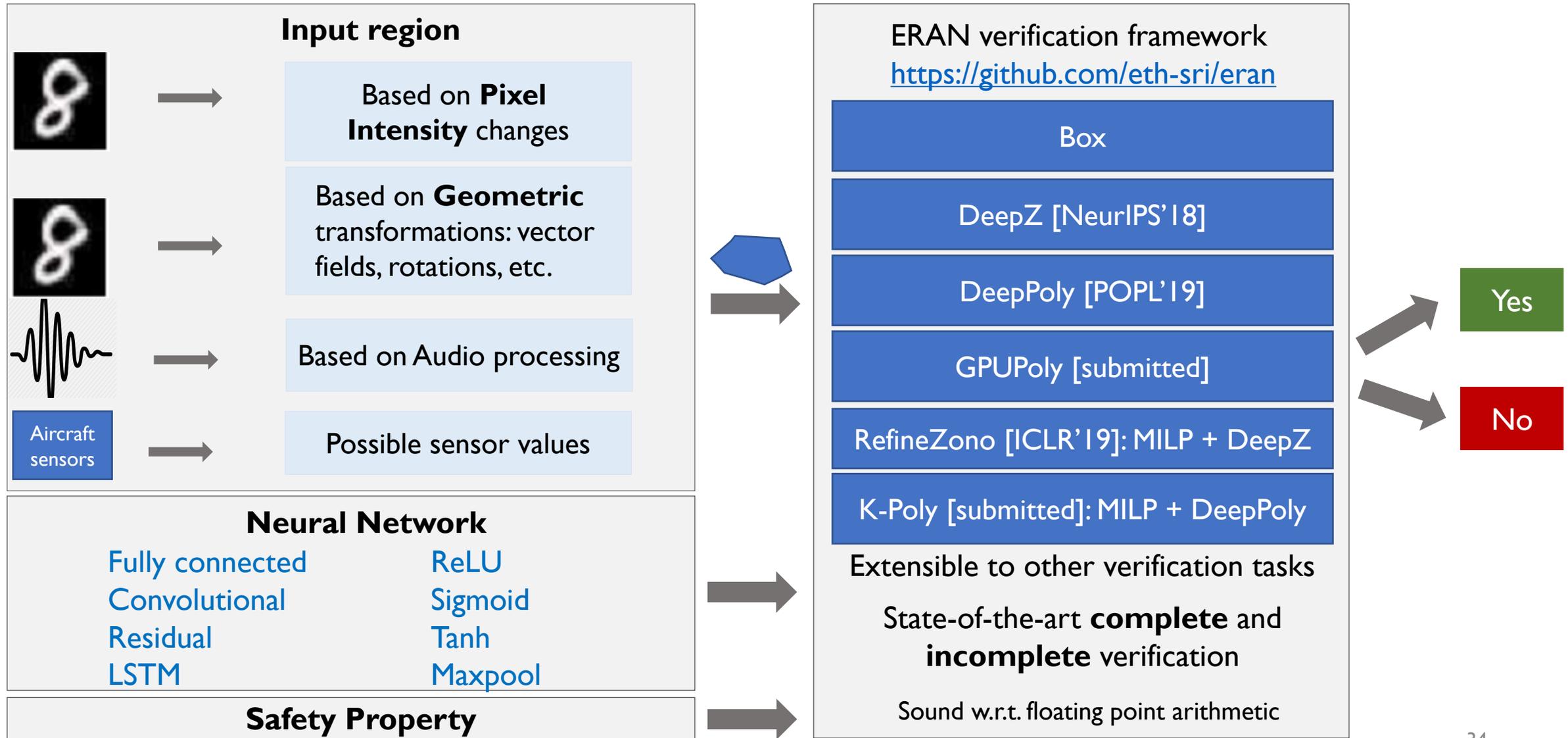
# Large benchmarks

Dataset	Model	Type	#Neurons	#Layers	Defense
CIFAR10	ResNetTiny	residual	311K	12	PGD
	ResNet18	residual	558K	18	PGD
	ResNetTiny	residual	311K	12	DiffAI
	SkipNet18	residual	558K	18	DiffAI
	ResNet18	residual	558K	18	DiffAI
	ResNet34	residual	967K	34	DiffAI

# Results on large benchmarks (500 test images)

Model	Training	#correct	$\epsilon$	Hbox[ICML'18]	GPU Poly		
				% 	time(s) %  time(s)		
ResNetTiny	PGD	391	0.002	0	0.3	322	30
ResNet18	PGD	419	0.002	0	6.8	324	1400
ResNetTiny	DiffAI	184	0.03	118	0.3	127	7.6
SkipNet18	DiffAI	168	0.03	130	6.1	140	57
ResNet18	DiffAI	193	0.03	129	6.3	139	37
ResNet34	DiffAI	174	0.03	103	16	114	79

# Network verification with ERAN



# In-progress work in verification/training (sample)

**Verification Precision:** More precise convex relaxations by considering multiple ReLUs

**Verification Scalability:** GPU-based custom abstract domains for handling large nets

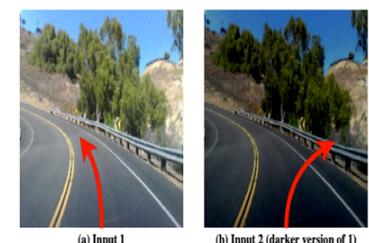
**Theory:** Proof on Existence of Accurate and Provable Networks with Box

**Provable Training:** Procedure for training Provable and Accurate Networks

**Applications:** e.g., reinforcement learning, geometric, audio, sensors

# Attacks on Deep Learning

The self-driving car incorrectly decides to turn right on Input 2 and crashes into the guardrail



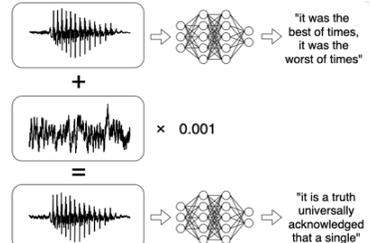
DeepXplore: Automated Whitebox Testing of Deep Learning Systems, SOSP'17

The Ensemble model is fooled by the addition of an adversarial distracting sentence in blue.

Article: Super Bowl 50  
 Paragraph: "Peyton Manning became the first quarterback ever to lead two different teams to multiple Super Bowls. He is also the oldest quarterback ever to play in a Super Bowl at age 39. The past record was held by John Elway, who led the Broncos to victory in Super Bowl XXXIII at age 38 and is currently Denver's Executive Vice President of Football Operations and General Manager. Quarterback Jeff Dean had jersey number 37 in Champ Bowl XXXIV."  
 Question: "What is the name of the quarterback who was 38 in Super Bowl XXXIII?"  
 Original Prediction: John Elway  
 Prediction under adversary: Jeff Dean

Adversarial Examples for Evaluating Reading Comprehension Systems, EMNLP'17

Adding small noise to the input audio makes the network transcribe any arbitrary phrase



Audio Adversarial Examples: Targeted Attacks on Speech-to-Text, ICML 2018

# Neural Network Verification: Problem statement

Given: Neural Network  $f$ ,  
 Input Region  $\mathcal{R}$   
 Safety Property  $\psi$

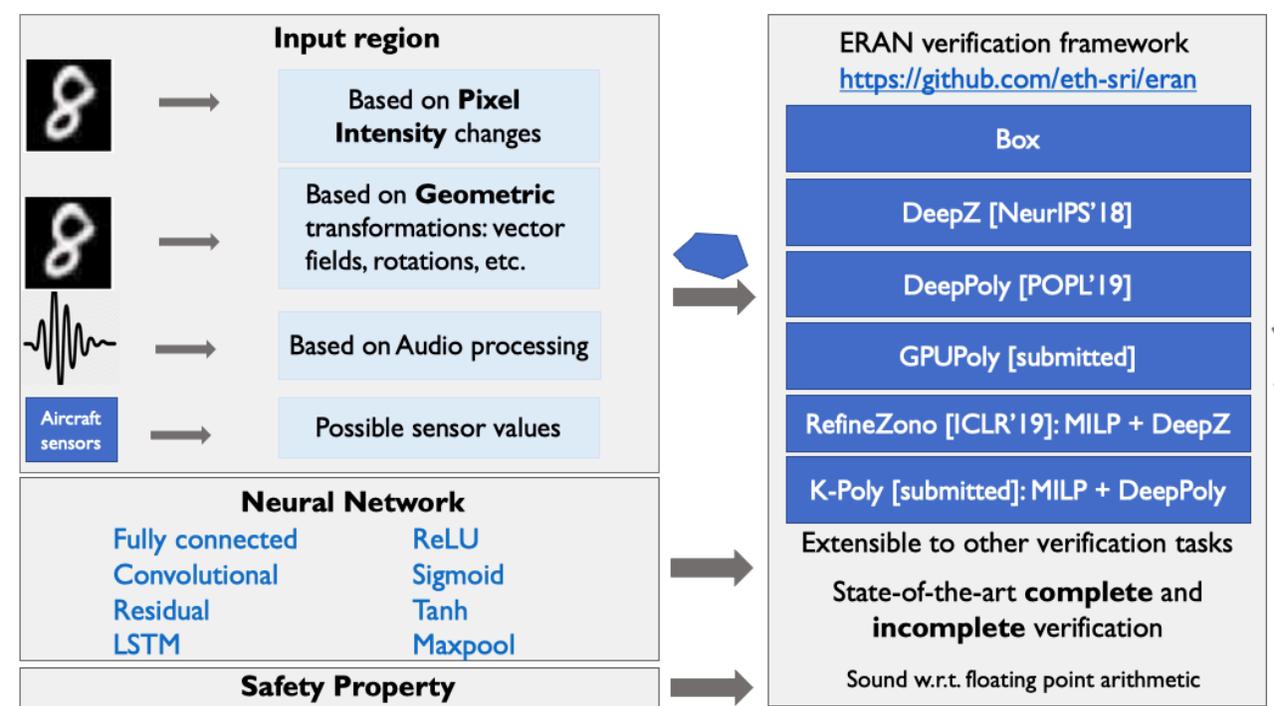
Prove:  $\forall I \in \mathcal{R}$ ,  
 prove that  $f(I)$  satisfies  $\psi$

Example networks and regions:

- Image classification network  $f$**   
 Region  $\mathcal{R}$  based on changes to pixel intensity  
 Region  $\mathcal{R}$  based on geometric: e.g., rotation
- Speech recognition network  $f$**   
 Region  $\mathcal{R}$  based on added noise to audio signal
- Aircraft collision avoidance network  $f$**   
 Region  $\mathcal{R}$  based on input sensor values

Input Region  $\mathcal{R}$  can contain an infinite number of inputs, thus enumeration is infeasible

# Network Verification with ERAN



# Complete and Incomplete Verification with ERAN

Faster Complete Verification

Aircraft collision avoidance system (ACAS)			
	Reluplex	Neurify	ERAN
	> 32 hours	921 sec	227 sec

Scalable Incomplete Verification

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