

# **Symbolic Execution**

Testing, Quality Assurance, and Maintenance  
Winter 2020

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based on slides by Prof. Johannes Kinder and others

# Symbolic Execution

Automatically explore program paths

- Execute program on “symbolic” input values
- “Fork” execution at each branch
- Record branching conditions

Constraint solver

- Decides path feasibility
- Generates test cases for paths and bugs

# History

Int. Conference on Reliable Software 1975

James C. King:

*A new approach to program testing*

Robert S. Boyer, Bernard Elspas, Karl N. Levitt:

*SELECT—a formal system for testing and debugging programs by symbolic execution*



Recent work on proving the correctness of programs by formal analysis [5] shows great promise and appears to be the ultimate technique for producing reliable programs. However, the practical accomplishments in this area fall short of a tool for routine use. Fundamental problems in reducing the theory to practice are not likely to be solved in the immediate future.

# History (2)

SAT / SMT solvers lead to boom in 2000s

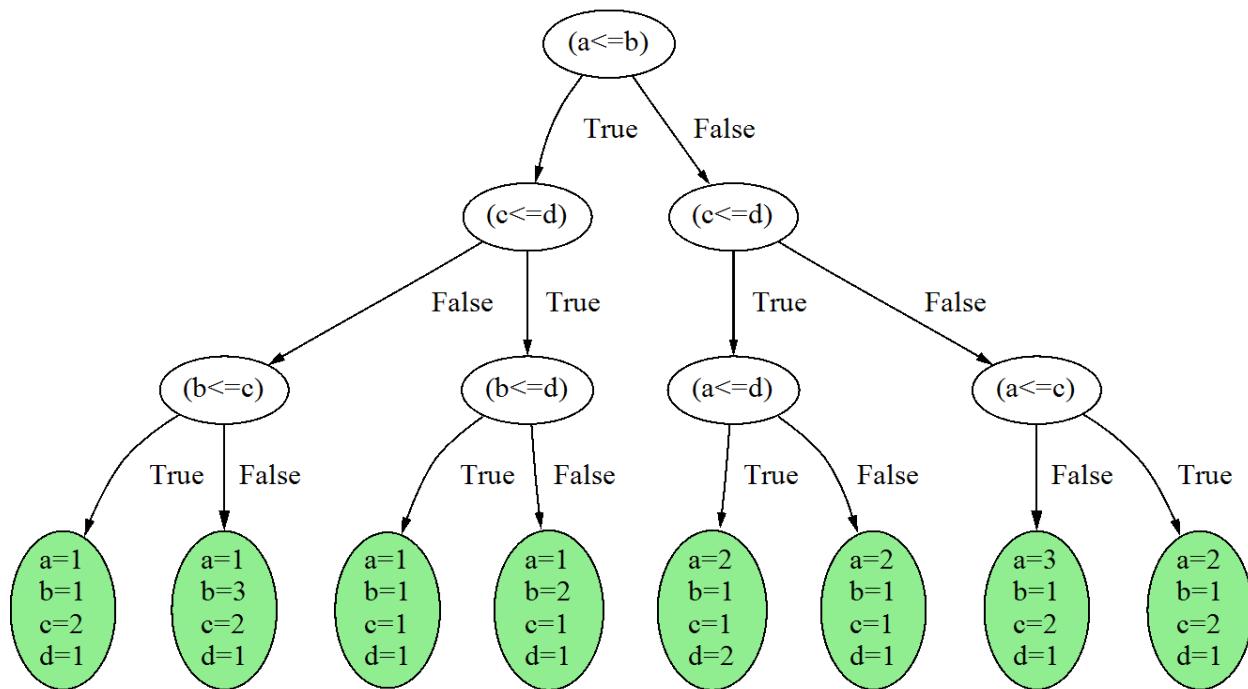
- Constraint solving becomes a commodity
- Makes classic algorithms viable in practice

Conceptual breakthroughs (Dynamic Symbolic Execution)

- Patrice Godefroid, Nils Klarlund, Koushik Sen: *DART: directed automated random testing.* PLDI 2005
- Cristian Cadar, Vijay Ganesh, Peter M. Pawlowski, David L. Dill, Dawson R. Engler: *EXE: automatically generating inputs of death.* CCS 2006

# Symbolic Execution Illustrated

```
int Max(int a, int b, int c, int d) {      int Max(int x, int y) {  
    return Max(Max(a, b), Max(c,d));        if (x <= y) return y;  
}  
}                                              else return x;
```



# Checking Path Feasibility

```
~  
4 (declare-fun a () Int)  
5 (declare-fun b () Int)  
6 (declare-fun c () Int)  
7 (declare-fun d () Int)  
8 (assert (< 0 a))  
9 (assert (< 0 b))  
10 (assert (< 0 c))  
11 (assert (< 0 d))  
12 (assert (≤ a b))  
13 (assert (> b c))  
14 (assert (> c d))  
15 (check-sat)  
16 (get-model)  
17
```



---

```
at  
model  
  (define-fun b () Int  
    3)  
  (define-fun a () Int  
    1)  
  (define-fun c () Int  
    2)  
  (define-fun d () Int  
    1)
```

$pc = \text{true}$

$x = X$

$r = 0$

```
1 int proc(int x) {  
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3     int r = 0  
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Symbolic  
program state

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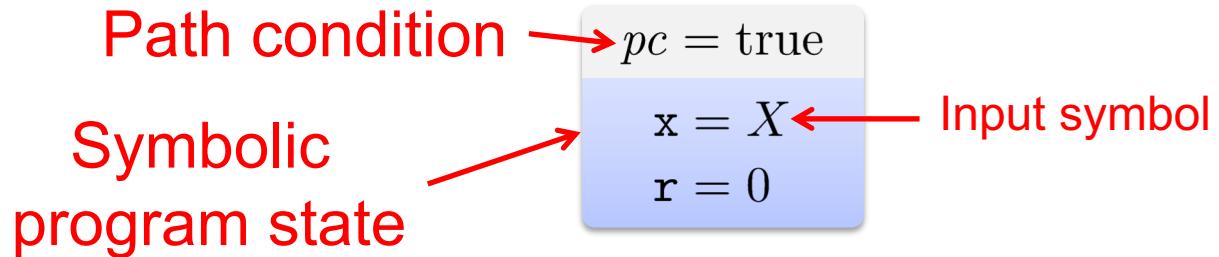
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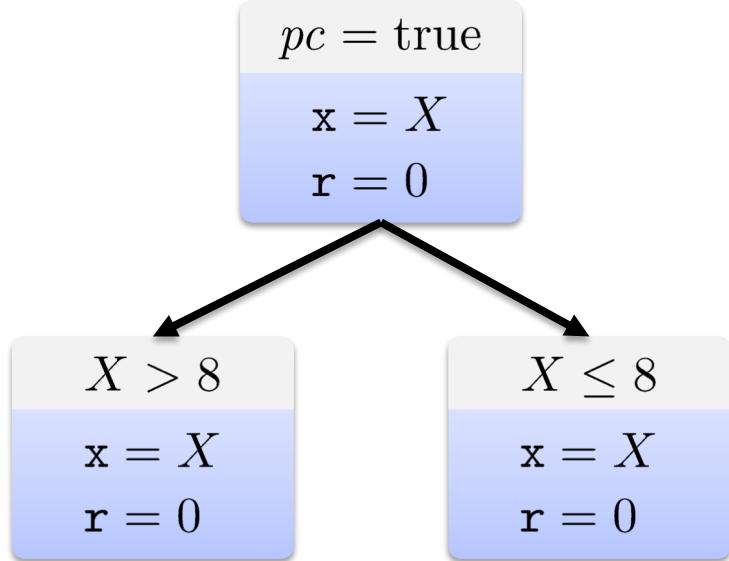
Input symbol

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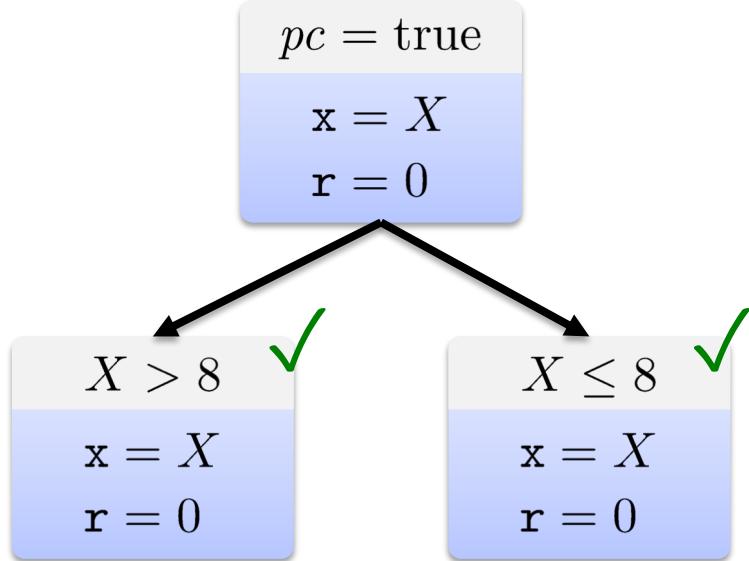
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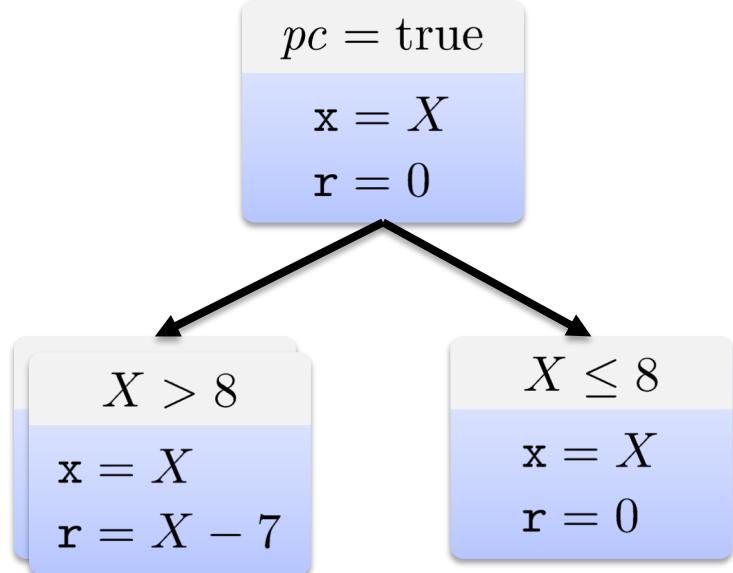


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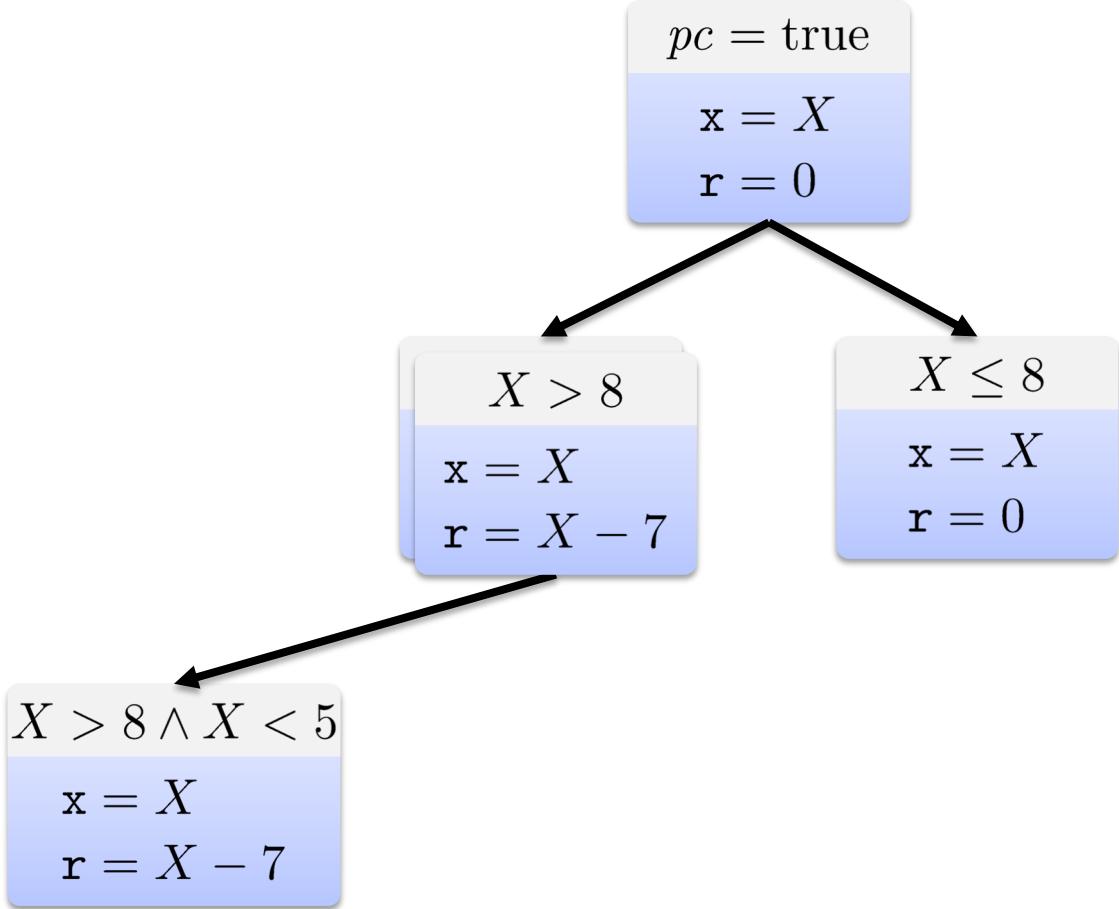


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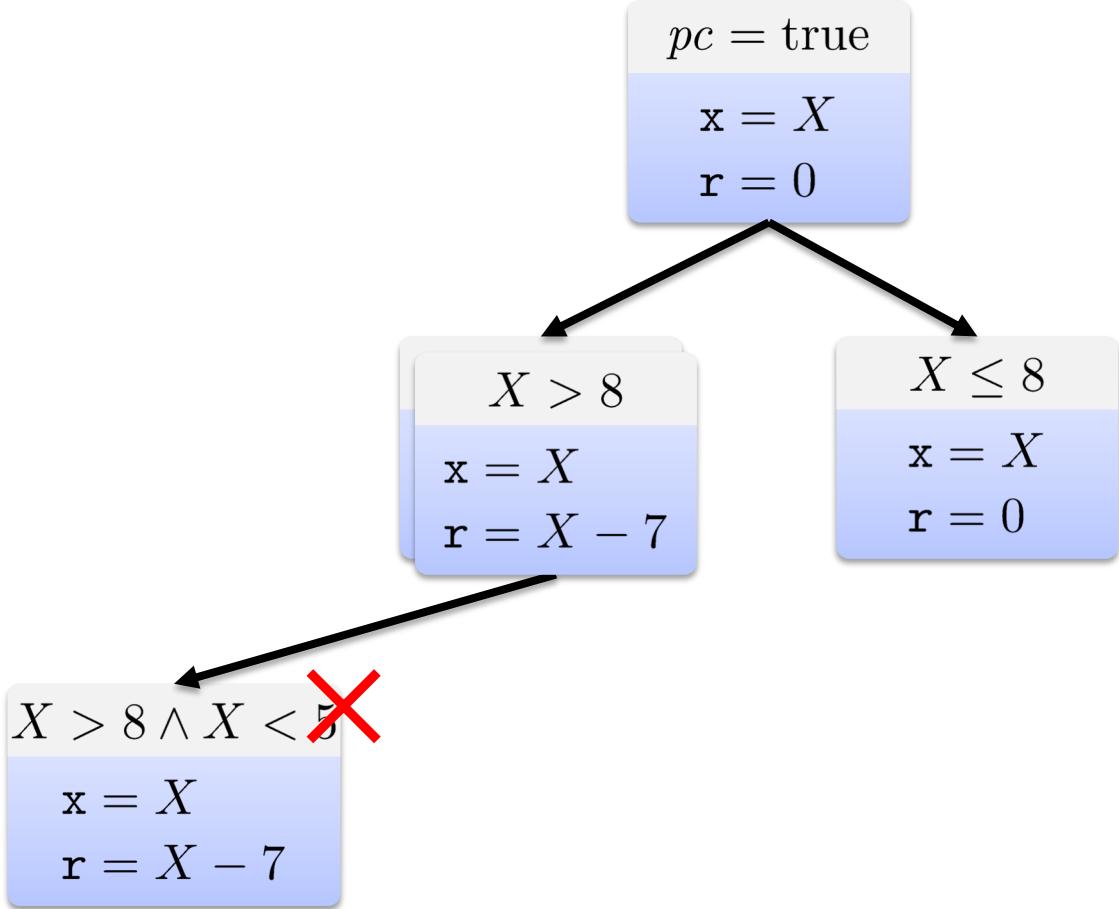
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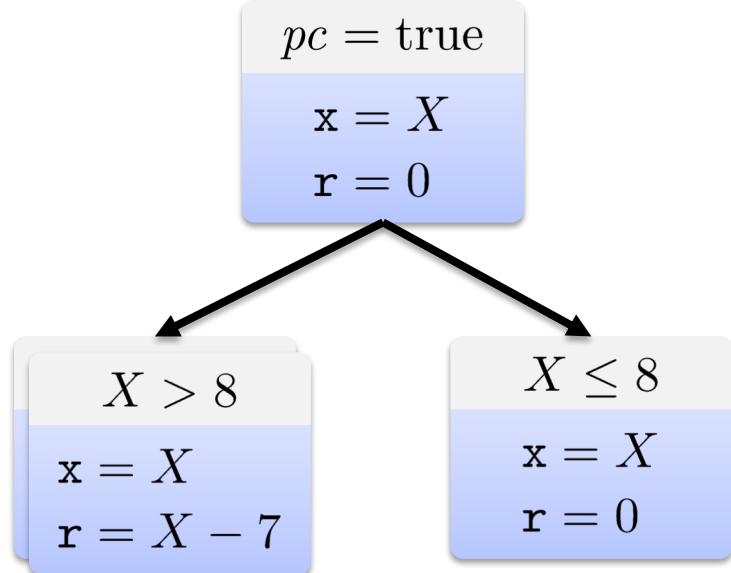


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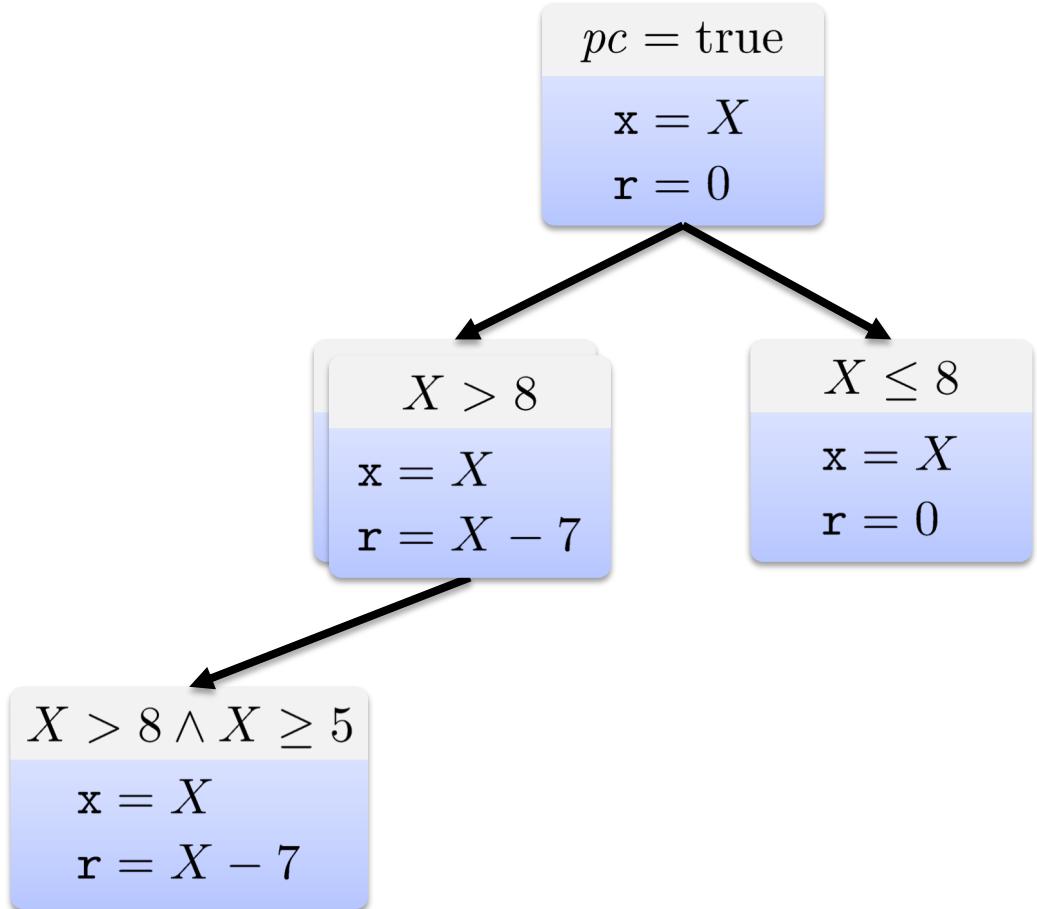


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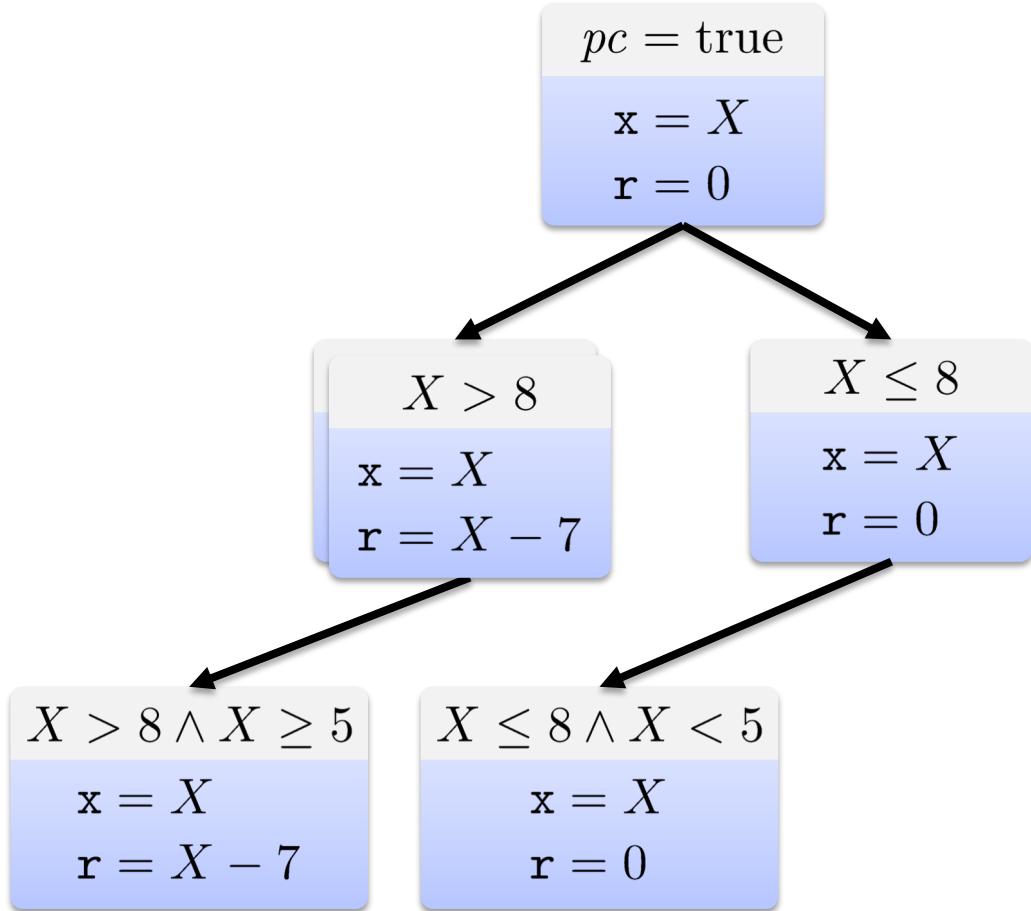
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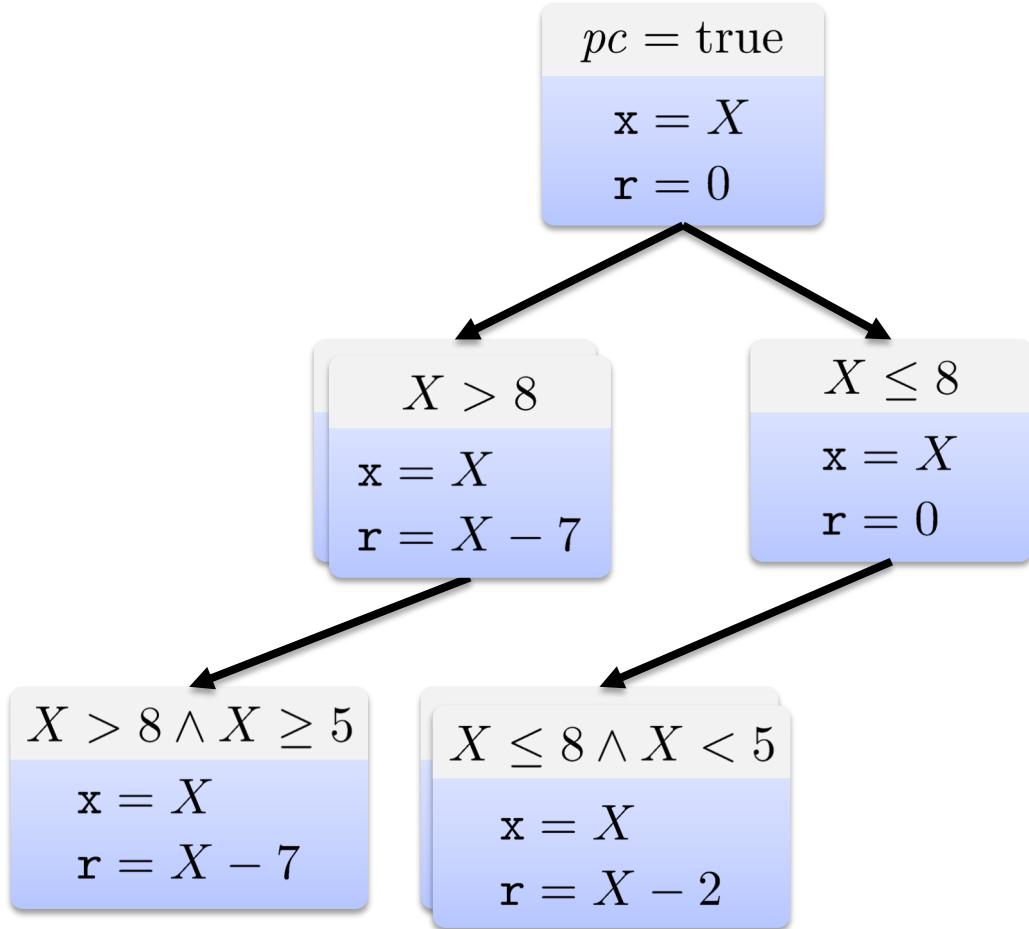
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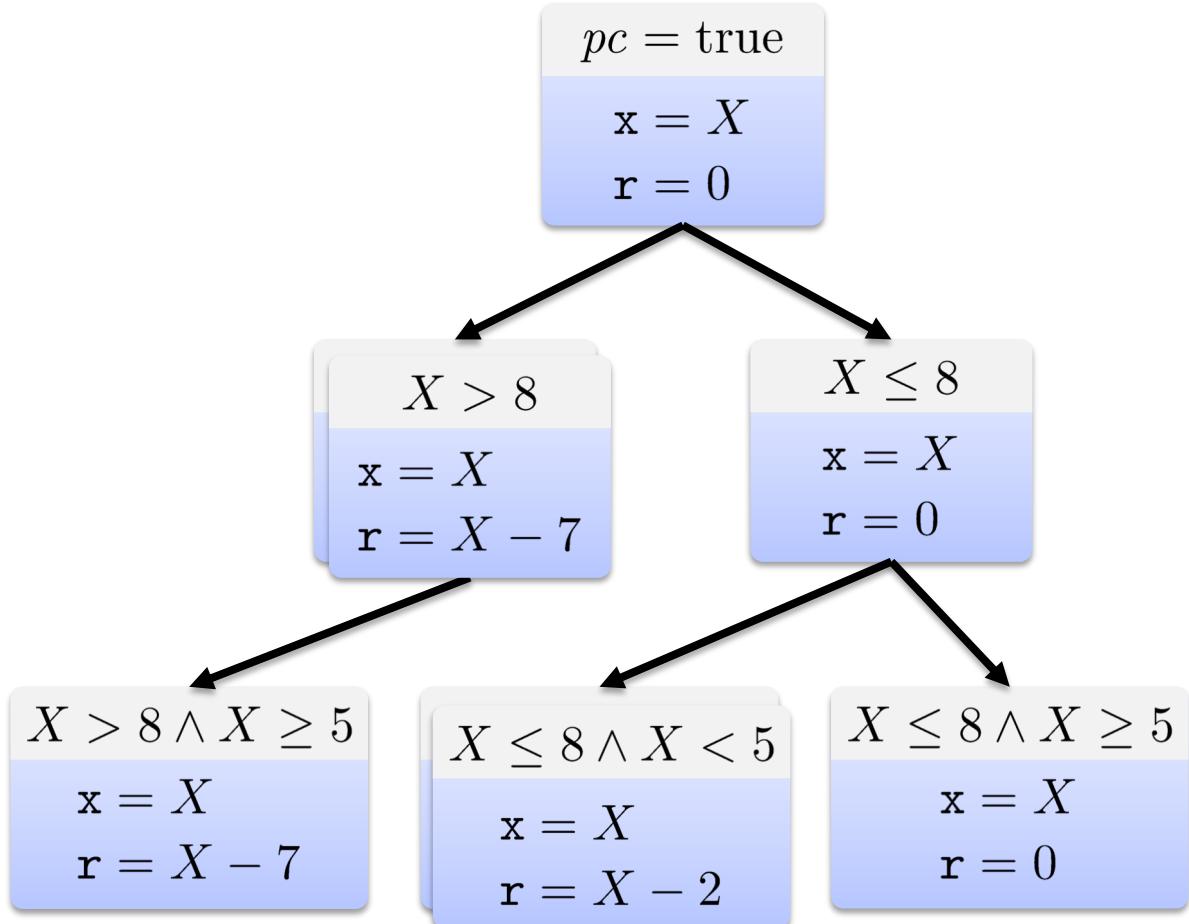
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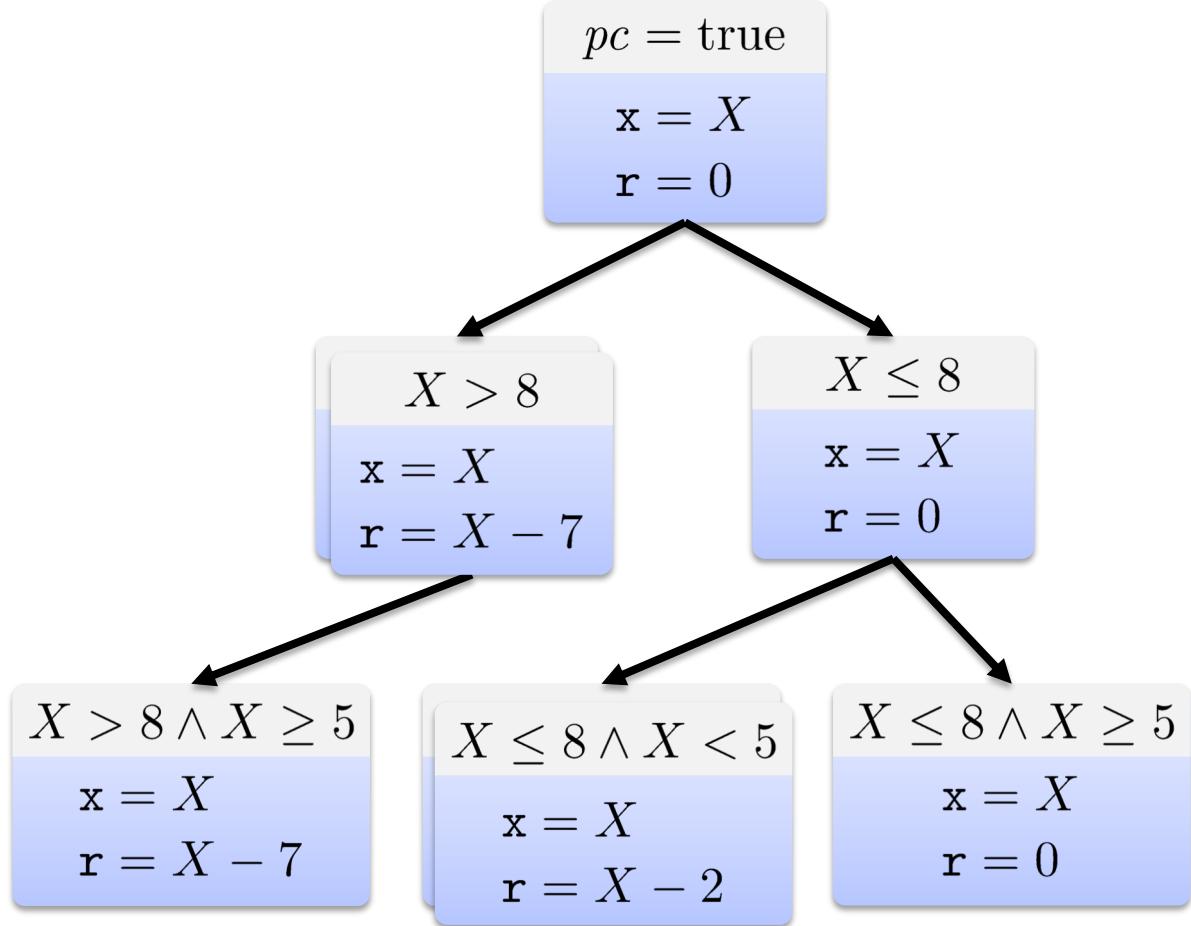
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```



Satisfying assignments:

$$X = 9$$

$$X = 4$$

$$X = 7$$

Test cases:

`proc(9)`

`proc(4)`

`proc(7)`

# Symbolic Execution

Analysis of programs by tracking symbolic rather than actual values

- a form of **Static Analysis**

Symbolic reasoning is used to reason about *all* the inputs that take the same path through a program

Builds constraints that characterize

- conditions for executing paths
- effects of the execution on program state

# Symbolic Execution

Uses symbolic values for input variables.

Builds constraints that characterize the conditions under which execution paths can be taken.

Collects **symbolic path conditions**

- a path condition for a path  $P$  is a formula  $PC$  such that  $PC$  is satisfiable if and only if  $P$  is executable

Uses theorem prover (**constraint solver**) to check if a path condition is satisfiable and the path can be taken.

# Symbolic State

A ***symbolic state*** is a pair  $S = (\text{Env}, \text{PC})$ , where

- $\text{Env} : L \rightarrow E$  is a mapping, called an ***environment***, from program variables to symbolic expressions (i.e., FOL terms)
- $\text{PC}$  is a FOL formula called a ***path condition***

A concrete state  $M : L \rightarrow Z$  satisfies a symbolic state  $S = (\text{Env}, \text{PC})$  iff

$$M \models (\text{Env}, \text{PC}) \text{ iff } \left( \left( \bigwedge_{v \in L} M(v) = \text{Env}(v) \right) \wedge \text{PC is SAT} \right)$$

Program semantics are extended to symbolic states

- each program statement updates symbolic variables and
- extends the path condition to reflect its operational semantics

# Example: Symbolic State Satisfiability

$$Env = \begin{cases} x \mapsto X \\ y \mapsto Y \end{cases} \quad PC = X > 5 \wedge Y < 3$$

$$[x \mapsto 10, y \mapsto 1] \models ?S \quad [x \mapsto 1, y \mapsto 10] \models ?S$$

$$Env = \begin{cases} x \mapsto X + Y \\ y \mapsto Y - X \end{cases} \quad PC = 2 * X - Y > 0$$

$$[x \mapsto 10, y \mapsto 1] \models ?S \quad [x \mapsto 1, y \mapsto 10] \models ?S$$

# Symbolic Evaluation/Execution

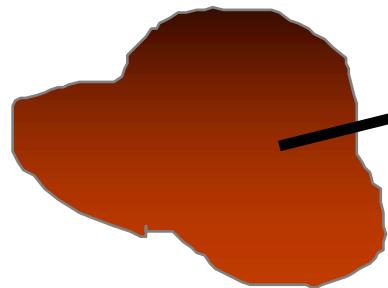
Symbolic execution creates a functional representation of a path in a Control Flow Graph of a program

For a path  $P_i$

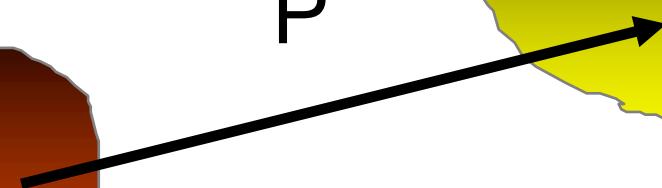
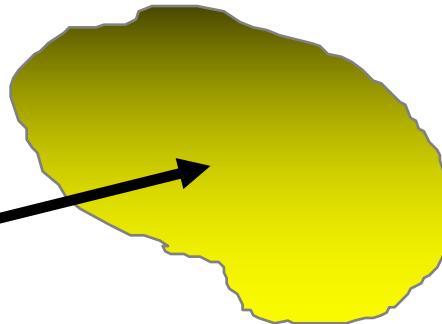
- $D[P_i]$  is the domain for path  $P_i$ 
  - the inputs that force the program to take path  $P_i$
- $C[P_i]$  is the computation for path  $P_i$ 
  - the result of executing the path

# Functional Representation of an Executable Component

$P : X \rightarrow Y$



P



P is composed of partial functions corresponding to the executable paths

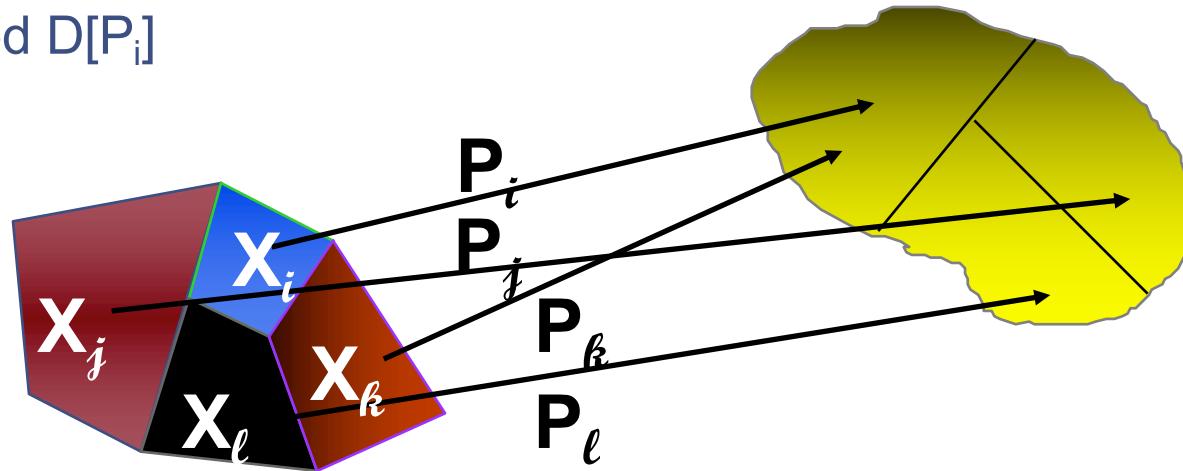
$$P = \{P_1, \dots, P_r\}$$

$$P_i : X_i \rightarrow Y$$

# Functional Representation of an Executable Component

$X_i$  is the domain of path  $P_i$

Denoted  $D[P_i]$



$$X = D[P_1] \cup \dots \cup D[P_r] = D[P]$$

$$D[P_i] \cap D[P_j] = \emptyset, i \neq j$$

# Exercise: Find a Violation

```
int x=0, y=0, z=0;  
if (a) {  
    x = -2;  
}  
if (b < 5) {  
    if (!a && c)  
        { y = 1; }  
    z = 2;  
}  
assert(x+y+z != 3);
```

a = α, b = β, c = γ  
x=0, y=0, z=0

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int x=0, y=0, z=0;  
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|  
a

```

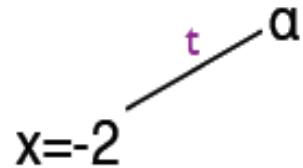
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|

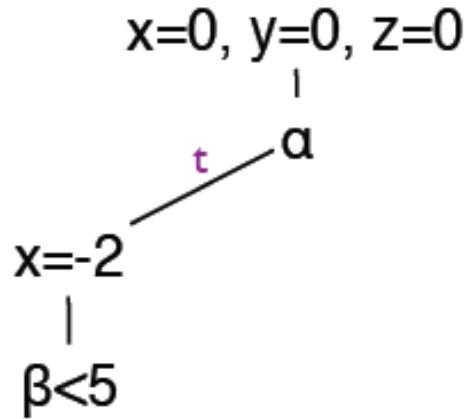


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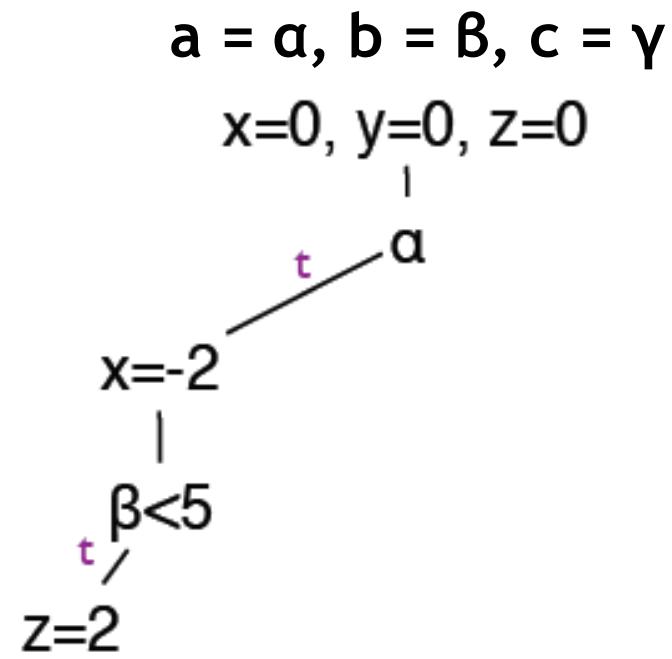
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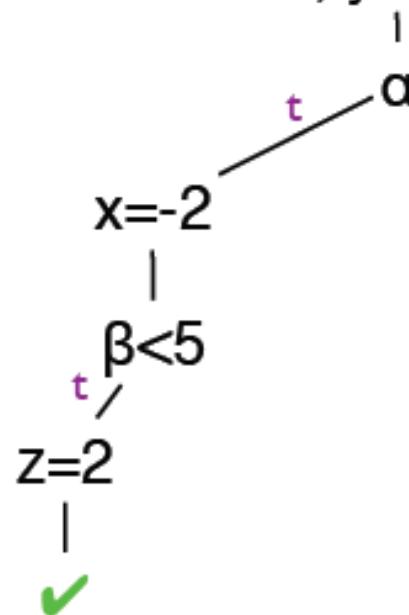
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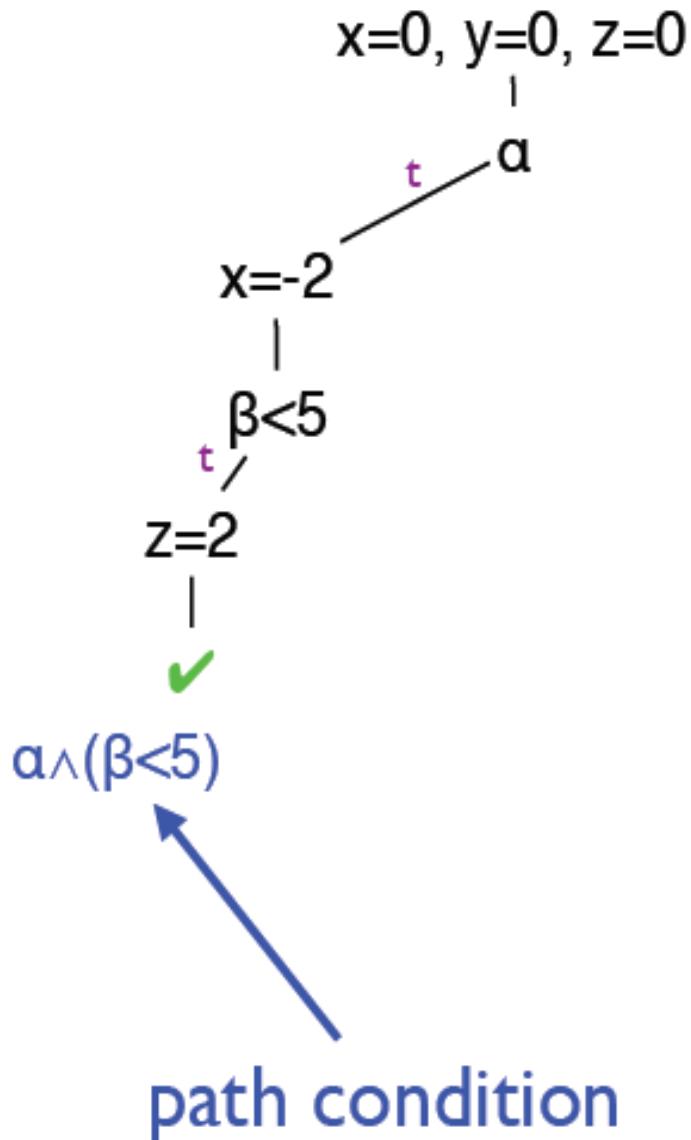
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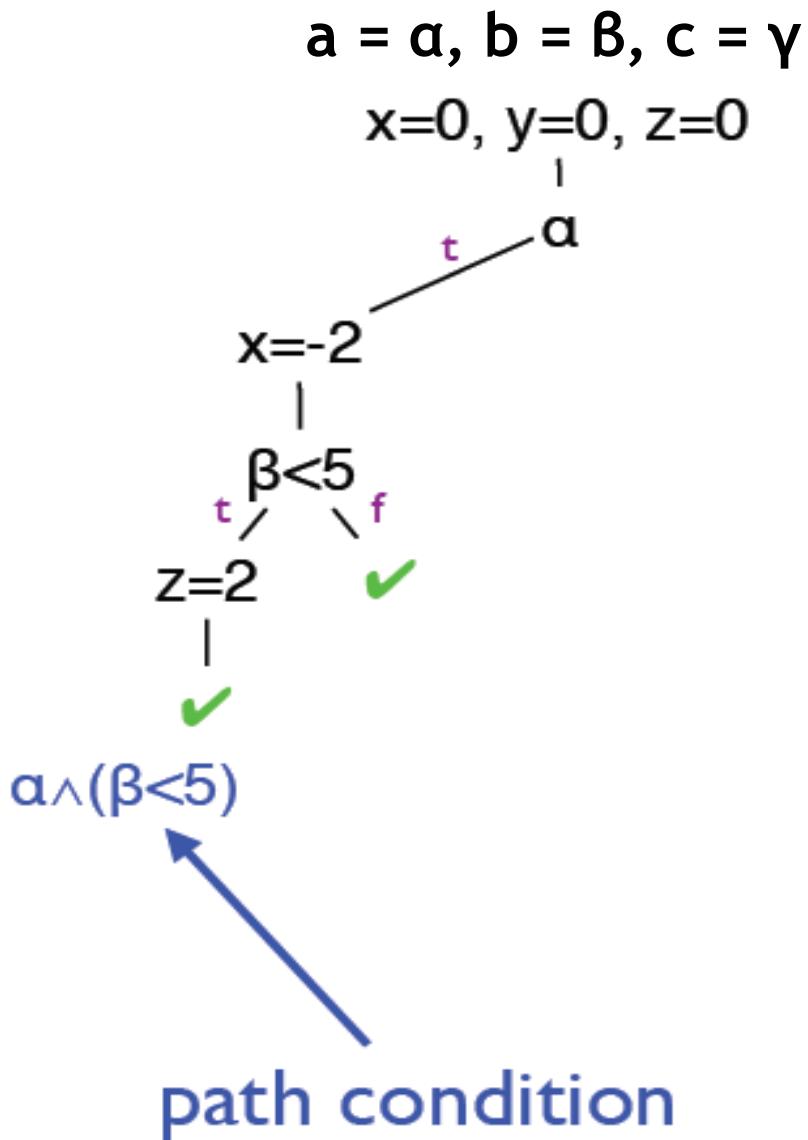
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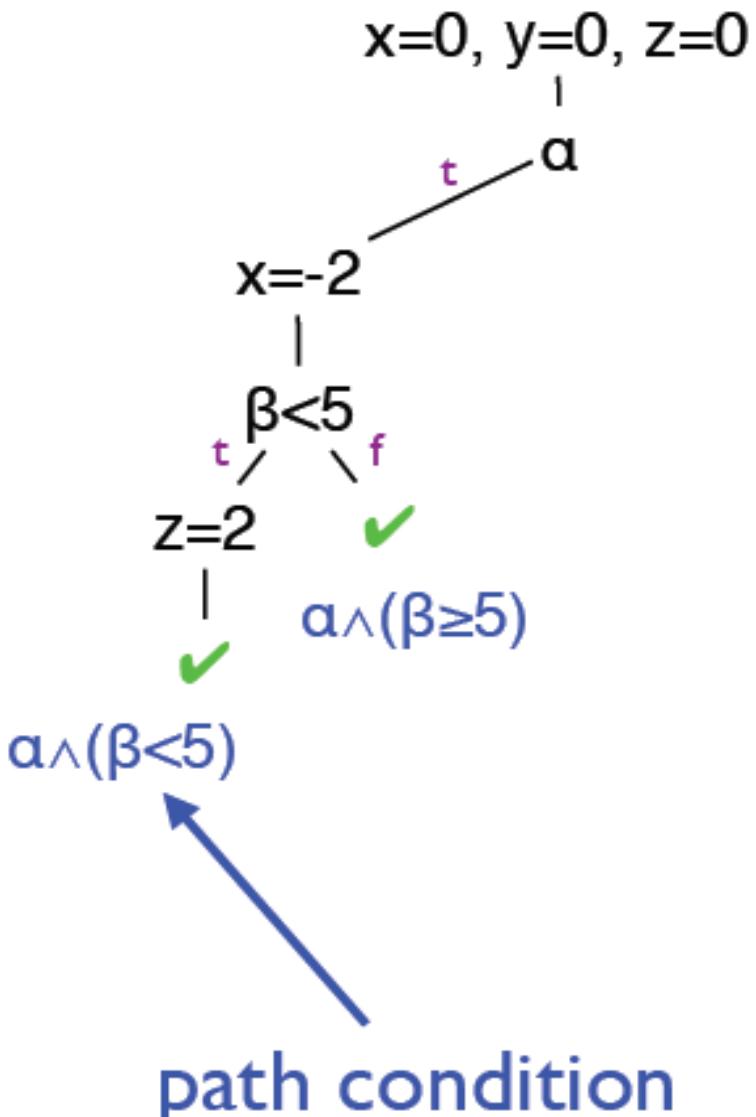


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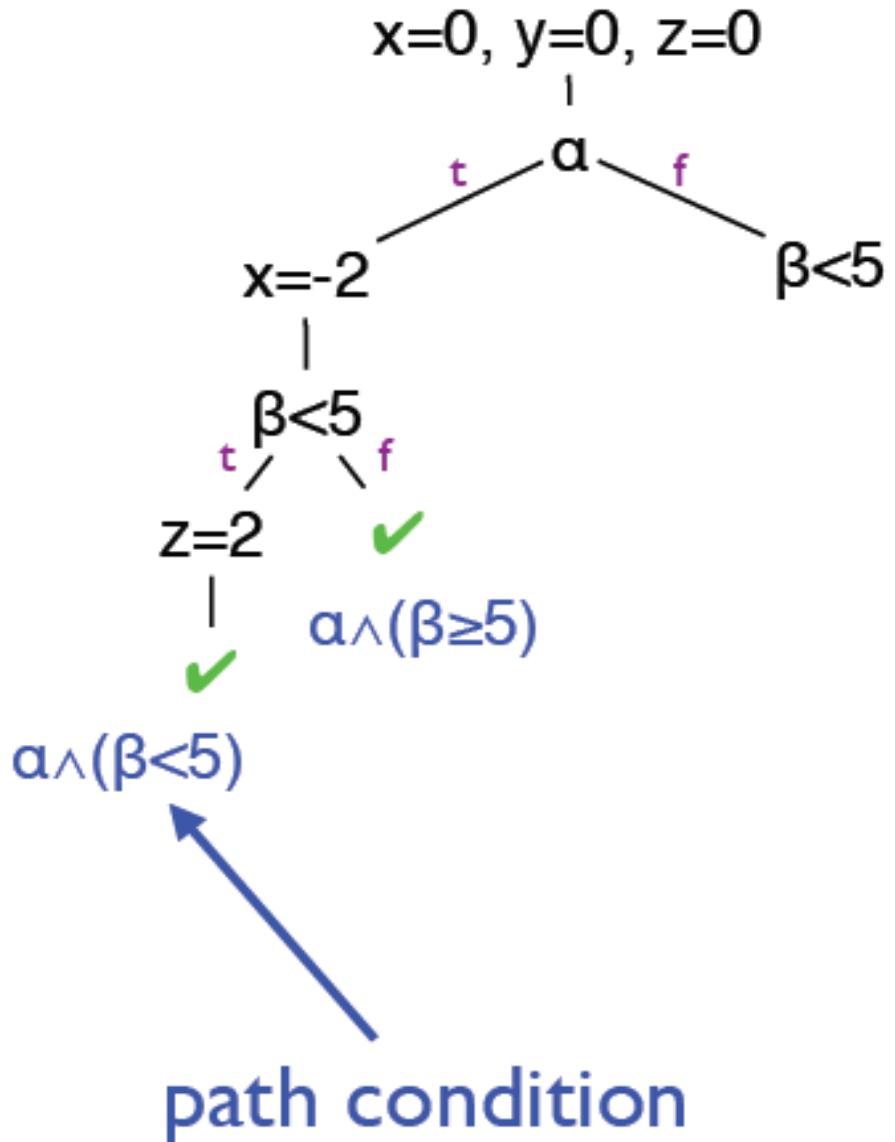


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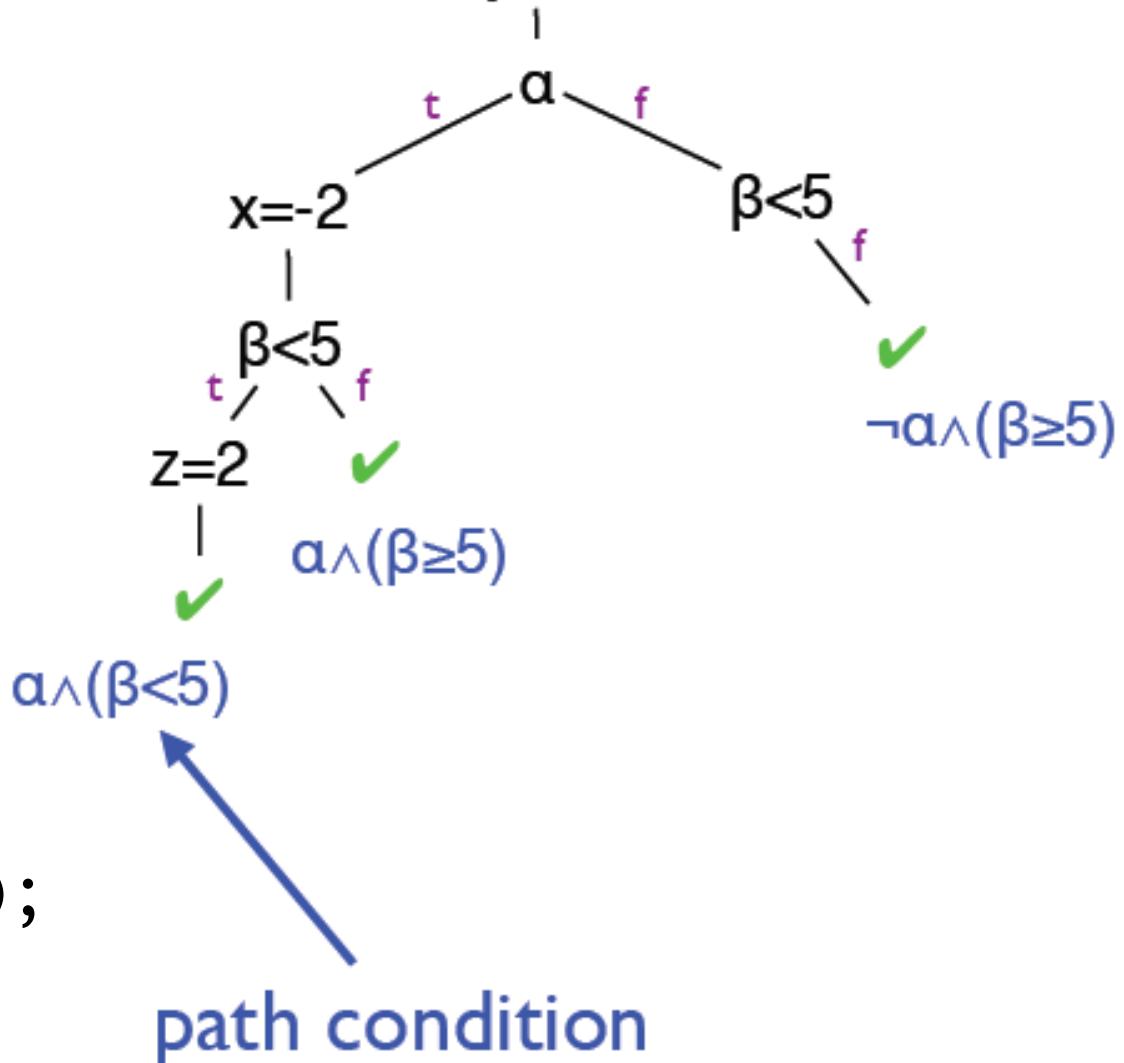
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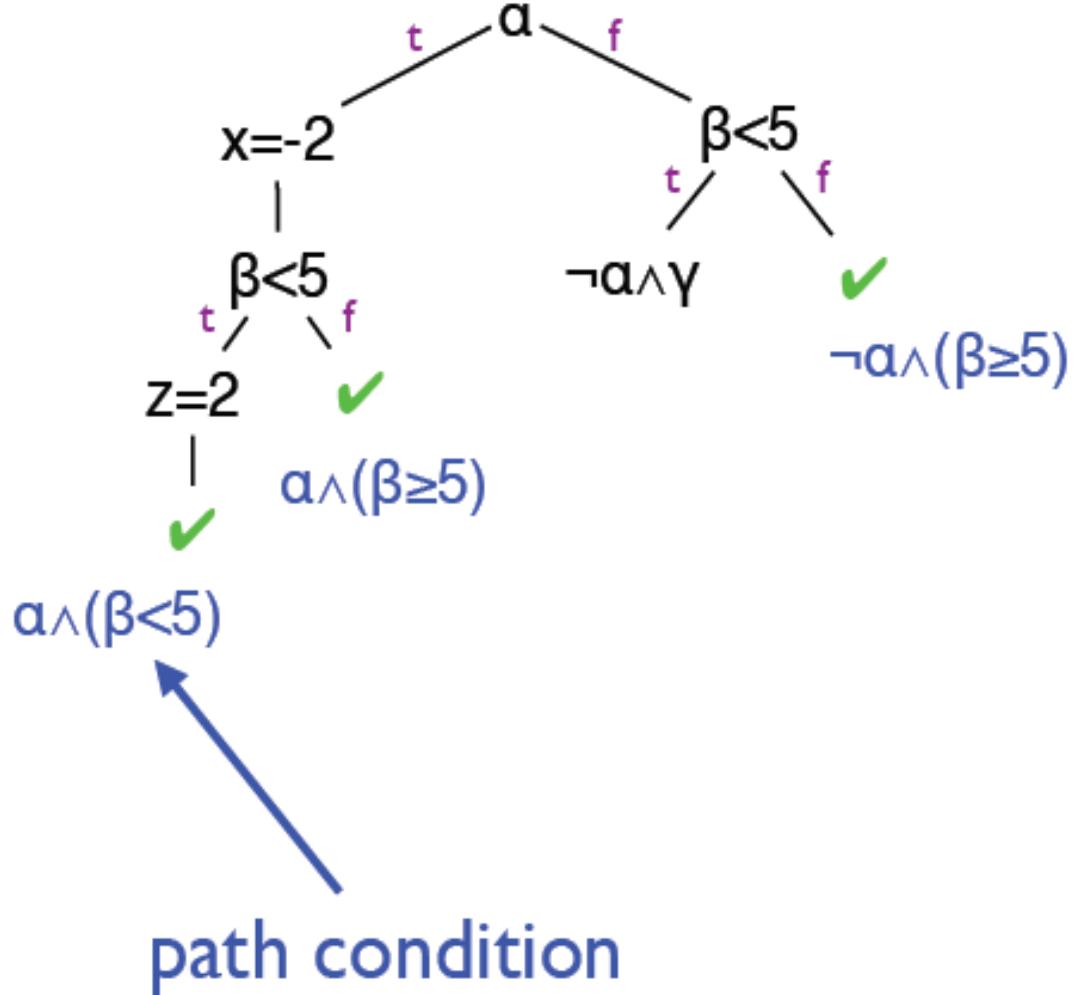


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$$\begin{aligned}
&a = \alpha, b = \beta, c = \gamma \\
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\end{aligned}$$

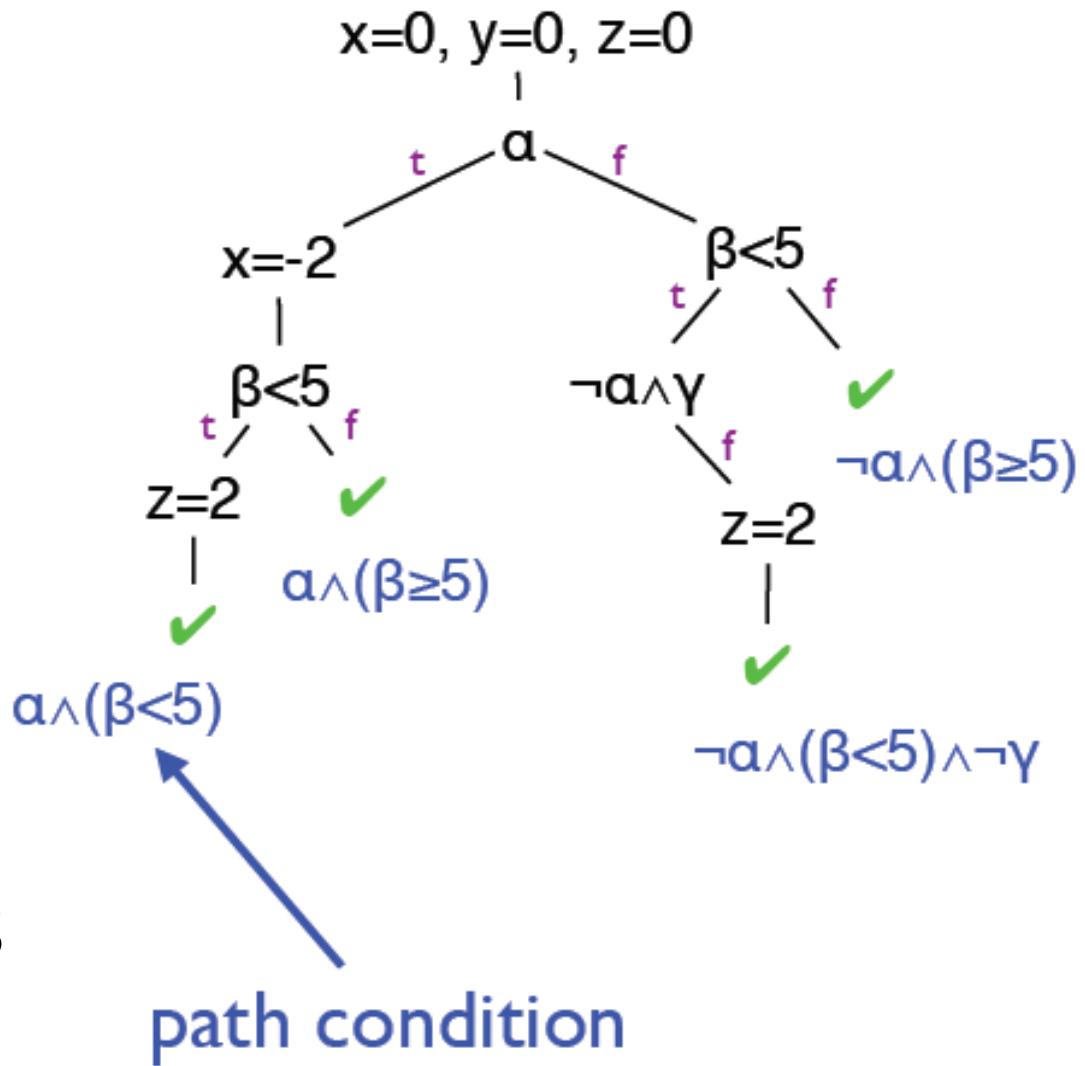


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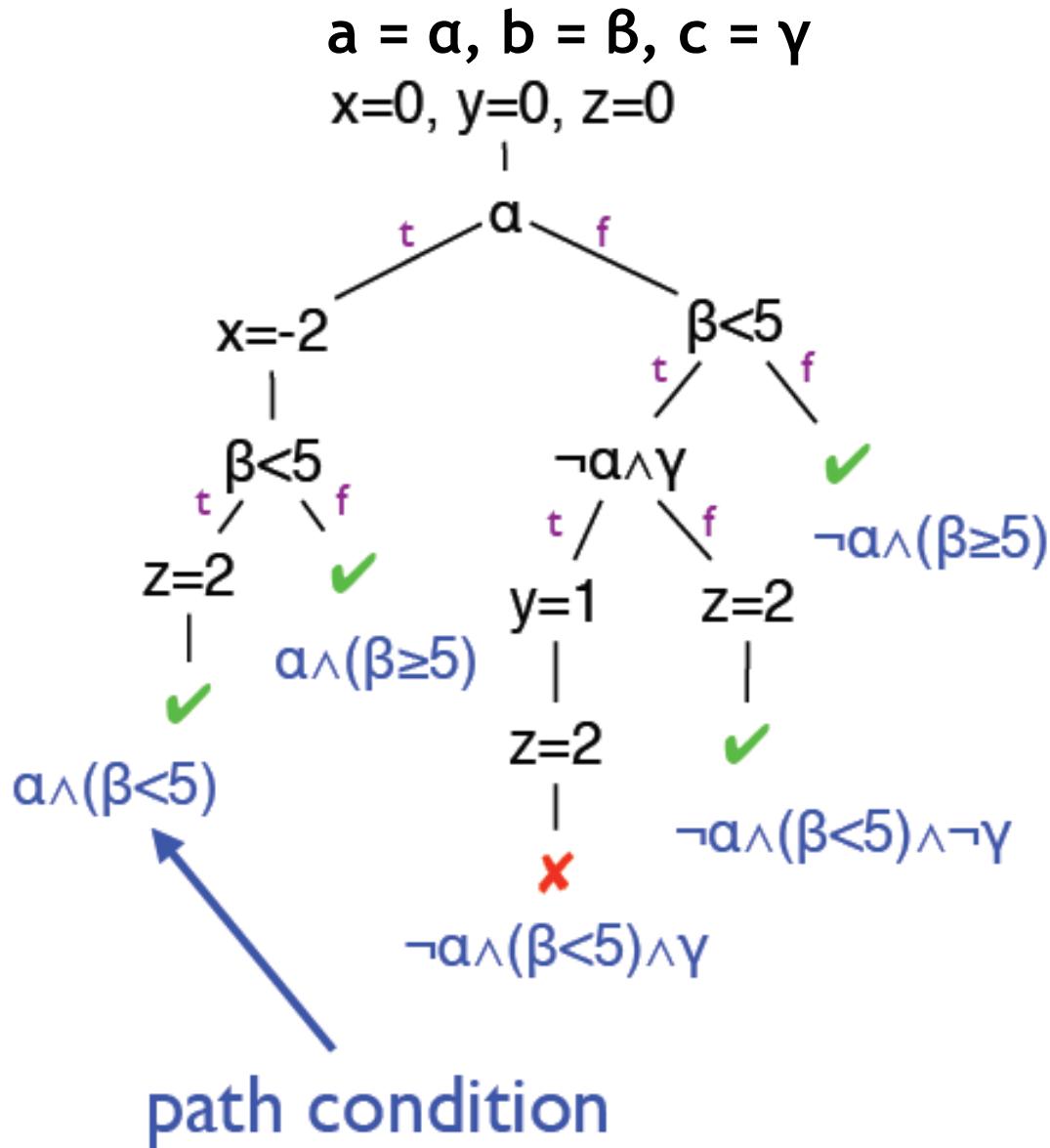
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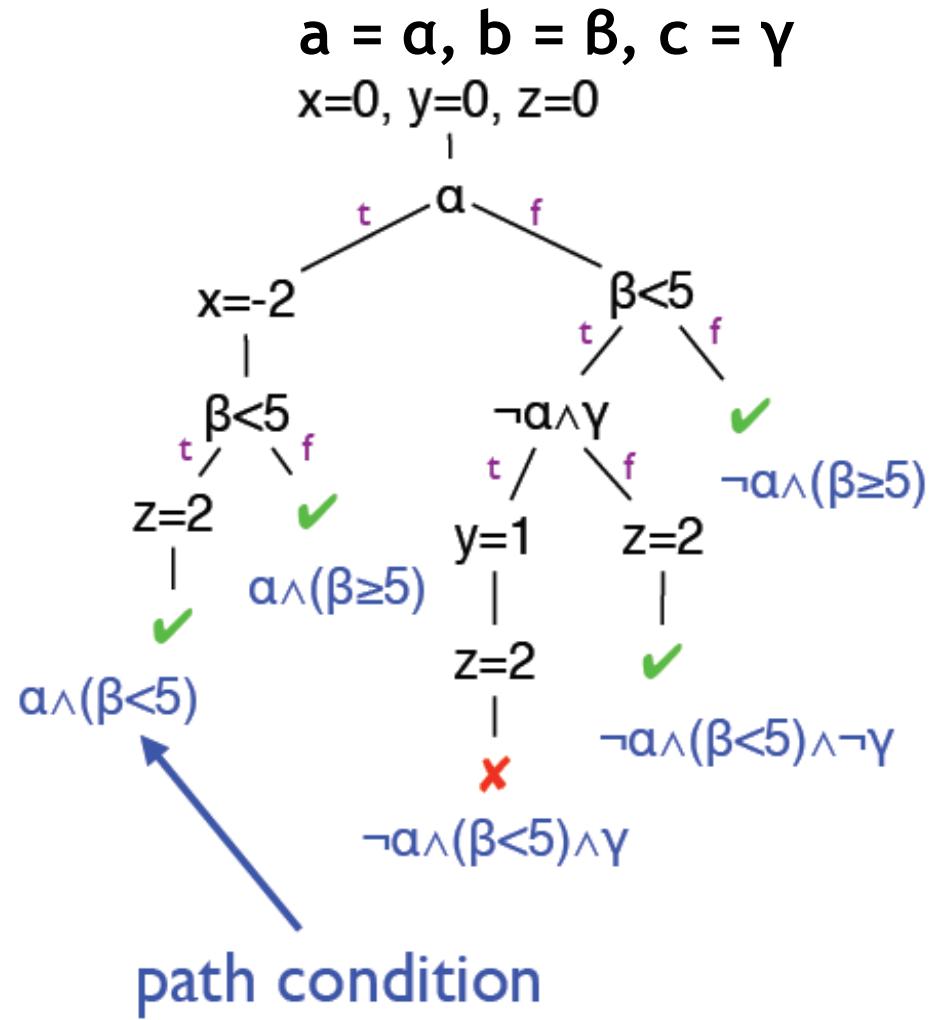
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```



a = false, b = 2, c = true -> x = 0, y = 1, z = 2 : Assert(0+1+2 != 3)

# Finding Bugs

Symbolic execution enumerates paths

- Runs into bugs that trigger whenever path executes
- Assertions, buffer overflows, division by zero, etc., require specific conditions

Error conditions

- Treat assertions as conditions
- Creates explicit error paths

`assert x != NULL`



`if (x == NULL)  
 abort();`

# Finding Bugs

Instrument program with properties

- Translate any safety property to reachability

Division by zero

$$y = 100 / x \rightarrow \text{assert } x \neq 0 \\ y = 100 / x$$

Buffer overflows

$$a[x] = 10 \rightarrow \text{assert } x \geq 0 \ \&& \ x < \text{len}(a)$$

Implementation is usually implicit

# Many problems remain

Code that is hard to analyze

Path explosion

- Complex control flow
- Loops
- Procedures

Environment (what are the inputs to the program under test?)

- pointers, data structures, ...
- files, data bases, ...
- threads, thread schedules, ...
- sockets, ...