Foundations: Syntax, Semantics, and Graphs

Testing, Quality Assurance, and Maintenance
Winter 2020

Prof. Arie Gurfinkel

based on slides by Ruzica Pizkac, Claire Le Goues, Lin Tan, Marsha Chechik, and others
Foundations

Syntax
- Syntax and BNF Grammar
- Abstract Syntax Trees (AST)

Semantics
- Natural Operational Semantics (a.k.a. big step)
- Structural Operational Semantics (a.k.a. small step)
- Judgements and derivations

Graphs
- Graph, cyclic, acyclic
- Nodes, edges, paths
- Trees, sub-graphs, sub-paths, ...
- Control Flow Graph (CFG)
SYNTAX
WHILE: A Simple Imperative Language

We will use a simple imperative language called WHILE

• the language is also sometimes called IMP

An example WHILE program:

{ p := 0; x := 1; n := 2 }; while x ≤ n do {
    x := x + 1;
    p := p + m
} ;
print_state

‘;’ is a connective, not terminator as in C!
WHILE: Syntactic Entities

- \( n \in \mathbb{Z} \) – integers
- \( \text{true, false} \in \mathbb{B} \) – Booleans
- \( x, y \in L \) – locations (program variables)
- \( e \in Aexp \) – arithmetic expressions
- \( b \in Bexp \) – Boolean expressions
- \( c \in Stmt \) – statements

Terminals are atomic entities that are completely defined by their tokens
- integers, Booleans, and locations are terminals

Non-Terminals are composed of one or more terminals
- determined by rules of the grammar
- Aexp, Bexp, and Stmt are non-terminals
WHILE: Syntax of Arithmetic Expressions

Arithmetic expressions (Aexp)
\[
e ::= n \quad \text{for } n \in \mathbb{Z} \\
| -n \quad \text{for } n \in \mathbb{Z} \\
| x \quad \text{for } x \in L \\
| e_1 \text{ aop } e_2 \\
| (\ ' e \ ')
\]

aop ::= ‘*’ | ‘/’ | ‘-’ | ‘+’

Notes:
- Variables are not declared before use
- All variables have integer type
- Expressions have no side-effects

BNF: https://en.wikipedia.org/wiki/Backus%E2%80%93Naur_form
WHILE: Syntax of Boolean Expressions

Boolean expressions (Bexp)
\[
b ::= \begin{array}{c}
\text{‘true’} \\
\text{‘false’} \\
\text{‘not’ } b \\
e_1 \text{ rop } e_2 \quad \text{for } e_1, e_2 \in \text{Aexp} \\
e_1 \text{ bop } b_2 \quad \text{for } e_1, e_2 \in \text{Bexp} \\
\text{‘(‘ } b \text{ ‘)}
\end{array}
\]

rop ::= ‘<’ | ‘<=‘ | ‘=‘ | ‘>=‘ | ‘>’

bop ::= ‘and’ | ‘or’
Syntax of Statements

Statements
s ::= skip
    | x ::= e
    | if b then s [ else s ]
    | while b do s
    | '{' slist '}'
    | print_state
    | assert b | assume b | havoc v1, ..., vN
slist ::= s ( ';' s )*
prog ::= slist

Notes:
• Semi-colon ';' is a statement composition, not statement terminator!!!
• Statements contain all the side-effects in the language
• Many usual features of a PL are missing: references, function calls, …
  – the language is very very simple yet hard to analyze
Abstract Syntax Tree (AST)

AST is an abstract tree representation of the source code

- each node represents a syntactic construct occurring in the code
  - statement, variable, operator, statement list
- called "abstract" because some details of concrete syntax are omitted
  - AST normalizes (provides common representation) of irrelevant differences in syntax (e.g., white space, comments, order of operations)
- example AST: \((x + 3) \times (y - 5)\)
Language Parsing in a Nutshell

Parser generator
- input: BNF grammar; output: parser (program)

Parser
- input: program source code; output: AST or error
WHILE AST in Python

One class per syntactic entity

One field per child

Class hierarchy corresponds to the semantic one
Behavior Pattern: Visitor

Applicability
- Object hierarchy with many classes
- Operations depend on classes
- Set of classes is stable
- Want to define new operations

Consequences
- Simplifies adding new operations
- Groups related behavior in one class
- Extending class hierarchy is difficult
- Visitor can maintain state
- **Element** must expose interface

In Python
- Method name is used instead of polymorphism, e.g., `visit_stmt()`
- Visitor’s `visit()` method dispatches calls based on reflection. No need for `accept()`
Example Visitor in Python

class AstVisitor(object):
    """Base class for AST visitor"""
    def __init__(self):
        pass

    def visit(self, node, *args, **kwargs):
        """Visit a node."""
        method = 'visit_' + node.__class__.__name__
        visitor = getattr(self, method)
        return visitor(node, *args, **kwargs)

    def visit_BoolConst(self, node, *args, **kwargs):
        visitor = getattr(self, 'visit_' + Const.__name__)
        return visitor(node, *args, **kwargs)

class PrintVisitor(AstVisitor):
    """A printing visitor"""
    def visit_IntVar(self, node, *args, **kwargs):
        self._write(node.name)

    def visit_Const(self, node, *args, **kwargs):
        self._write(node.val)

    def visit_Exp(self, node, *args, **kwargs):
        if node.is_unary():
            self._write(node.op)
            self.visit(node.arg(0))
        else:
            self._open_brkt(**kwargs)
            self.visit(node.arg(0))
            for a in node.args[1:]:
                self._write(' ')
                self._write(node.op)
                self._write(' ')
                self.visit(a)
            self._close_brkt(**kwargs)
Exercise: Implement a state counting visitor

Write a visitor that counts the number of statements in a program

• (a) Implementation 1:
  – the visitor should be stateless and return the number of statements

• (b) Implementation 2:
  – uses an internal state (field) to keep track of the number of statements
Stateless Visitor

class StmtCounterStateless (wlang.ast.AstVisitor):
    def __init__ (self):
        super (StmtCounterStateless, self).__init__ ()

    def visit_StmtList (self, node, *args, **kwargs):
        if node.stmts is None:
            return 0
        res = 0
        for s in node.stmts:
            res = res + self.visit (s)
        return res

    def visit_IfStmt (self, node, *args, **kwargs):
        res = 1 + self.visit (node.then_stmt)
        if node.has_else ():
            res = res + self.visit (node.else_stmt)
        return res

    def visit_WhileStmt (self, node, *args, **kwargs):
        return 1 + self.visit (node.body)

    def visit_Stmt (self, node, *args, **kwargs):
        return 1
class StmtCounterStatefull (wlang.ast.AstVisitor):
    def __init__ (self):
        super (StmtCounterStatefull, self).__init__()  
        self._count = 0

    def get_num_stmts (self):
        return self._count

    def count (self, node, *args, **kwargs):
        self._count = 0
        self.visit (node, *args, **kwargs)

    def visit_StmtList (self, node, *args, **kwargs):
        if node.stmts is None:
            return
        for s in node.stmts:
            self.visit (s)

    def visit_Stmt (self, node, *args, **kwargs):
        self._count = self._count + 1
    def visit_IfStmt (self, node, *args, **kwargs):
        self.visit_Stmt (node)
        self.visit (node.then_stmt)
        if node.has_else ():
            self.visit (node.else_stmt)

    def visit_WhileStmt (self, node, *args, **kwargs):
        self.visit_Stmt (node)
        self.visit (node.body)
Tutorial Friday @ 5:30pm in E7 5353

Friday at 5:30pm in E7 5353

Topics

• Docker
• Python
• AST
• Visitors
Non-determinism vs. Randomness

A **deterministic** function always returns the same result on the same input
- e.g., $F(5) = 10$

A **non-deterministic** function may return different values on the same input
- e.g., $G(5)$ in $[0, 10]$ “$G(5)$ returns a non-deterministic value between 0 and 10”

A **random** function may choose a different value with a probability distribution
- e.g., $H(5) = (3$ with prob. $0.3$, $4$ with prob. $0.2$, and $5$ with prob. $0.5$)

Non-deterministic choice cannot be implemented!
- used to model the worst possible adversary/environment
SEMANTICS
Reference for Semantics

Nielson². Semantics with Applications: An Appetizer

Available FREE at SpringerLink

Chapter 1 and Chapter 2

Link on course web page
Syntax and Semantics

Syntax

• MW: the way in which linguistic elements (such as words) are put together to form constituents (such as phrases or clauses)

• Determines and restricts how things are written

Semantics

• MW: the study of meanings

• Determines how syntax is interpreted to give meaning
Meaning of WHILE Programs

Questions to answer:

• What is the “meaning” of a given WHILE expression/statement?

• How would we evaluate WHILE expressions and statements?

• How are the evaluator and the meaning related?

• How can we reason about the effect of a command?
Semantics of Programming Languages

Denotational Semantics
• Meaning of a program is defined as the mathematical object it computes (e.g., partial functions).
• example: Abstract Interpretation

Axiomatic Semantics
• Meaning of a program is defined in terms of its effect on the truth of logical assertions.
• example: Hoare Logic

Operational Semantics
• Meaning of a program is defined by formalizing the individual computation steps of the program.
• example: Natural (Big-Step) Semantics, Structural (Small-Step) Semantics
Semantics of WHILE

The meaning of WHILE expressions depends on the values of variables, i.e. the current state.

A state $s$ is a function from $L$ to $Z$

- assigns a value for every location/variable
- notation: $s(x)$ is the value of variable $x$ in state $s$

The set of all states is $Q = L \rightarrow Z$

We use $q$ to range over $Q$
Judgement: Natural Semantics

Expression $e$ in state $q$ has a value $n$
Natural Semantics in the Book

The book uses a slightly different notation

\[ <e, q> \Rightarrow n \]

Versus notation in the slides

\[ <e, q> \downarrow n \]
Judgments

We write $<e, q> \Downarrow n$ to mean that expression $e$ evaluates to $n$ in state $q$.

- The formula $<e, q> \Downarrow n$ is called a judgment (a judgement is a relation between an expression $e$, a state $q$ and a number $n$)
- We can view $\Downarrow$ as a function of two arguments $e$ and $q$

This formulation is called natural operational semantics

- also known as big-step operational semantics
- the judgment relates the expression and its “meaning”

How to define $<e_1 + e_2, q> \Downarrow \ldots$ ?
Notation: Inference Rule

\[ F_1 \ldots F_n \quad \underline{\quad G \quad} \quad H \]

Premise

Side-condition

Conclusion
Notation: Axiom

An axiom states that the conclusion $G$ is true independently of any premises or side-conditions.
Inference Rules

We express the evaluation rules as inference rules for our judgments.
The rules are also called evaluation rules.

An inference rule

\[
\begin{array}{c}
F_1 \ldots F_n \\
\hline
G
\end{array}
\]

where \( H \)

defines a relation between judgments \( F_1, \ldots, F_n \) and \( G \).

- The judgments \( F_1, \ldots, F_n \) are the premises of the rule;
- The judgments \( G \) is the conclusion of the rule;
- The formula \( H \) is called the side condition of the rule.

If \( n=0 \) the rule is called an axiom. In this case, the line separating premises and conclusion may be omitted.
Inference Rules for Aexp

In general, we have one rule per language construct:

\[ \langle n, q \rangle \downarrow n \]
\[ \langle x, q \rangle \downarrow q(x) \]

\[ \langle e_1, q \rangle \downarrow n_1 \quad \langle e_2, q \rangle \downarrow n_2 \]
\[ \langle e_1 + e_2, q \rangle \downarrow (n_1 + n_2) \]

\[ \langle e_1 - e_2, q \rangle \downarrow (n_1 - n_2) \]

\[ \langle e_1, q \rangle \downarrow n_1 \quad \langle e_2, q \rangle \downarrow n_2 \]
\[ \langle e_1 \times e_2, q \rangle \downarrow (n_1 \times n_2) \]

This is called structural operational semantics of expressions.

- rules are defined based on the structure of the expressions.
Inference Rules for $Bexp$

\[
<true, q> \Downarrow true
\]

\[
\begin{align*}
&<e_1, q> \Downarrow n_1 \quad <e_2, q> \Downarrow n_2 \\
&\quad \hline
&<e_1 = e_2, q> \Downarrow (n_1 = n_2)
\end{align*}
\]

\[
\begin{align*}
&<e_1, q> \Downarrow n_1 \quad <e_2, q> \Downarrow n_2 \\
&\quad \hline
&<e_1 \leq e_2, q> \Downarrow (n_1 \leq n_2)
\end{align*}
\]

\[
\begin{align*}
&<b_1, q> \Downarrow t_1 \quad <e_2, q> \Downarrow t_2 \\
&\quad \hline
&<b_1 \land b_2, q> \Downarrow (t_1 \land t_2)
\end{align*}
\]

\[
<false, q> \Downarrow false
\]
Derivation

A well-formed application of inference rules is called a \textit{derivation}
Derivation \textit{infers} new facts from existing ones

\[<5 + (7*2), q> \Downarrow 19 \]
\[<5, q> \Downarrow 5 \quad <7*2, q> \Downarrow 14 \]
\[<5 + (7*2), q> \Downarrow 19 \]
Semantics of Statements

\[ <s, q> \Downarrow q' \]

Statement \( s \) executed in state \( q \) results in state \( q' \)
Notation: state and state change

A *state* $s$ is an assignment of values to *memory locations* (often called *variables*)

**Notation:**

- **empty state** $[\ ]$
- **state** $[x := 10, y:=15, z:=5]$
- **substitution** $s[x:=10]$
  - a state like $s$, BUT, the value of $x$ is 10
Aside: Notation for Substitution

We need a concise notation for “Find-Replace”

- Replace every occurrence of variable (string, value, …) \( X \) with variable (string, value, …) \( Y \) in a formula (string, object, state) \( F \), and return the new formula (string, object, state) without changing \( F \) in place.

In Python, this looks like

- `string.replace(old, new [, count])`
- `F.replace(X, Y)`

In books, logic, slides, this course

- \( F[X := Y] \) --- implies that \( F \) is a state, \( X \) a state variable, \( Y \) a value
- \( F[X / Y] \) – “\( F \) with \( X \) replaced by \( Y \)”
- \( F[X \downarrow Y] \) --- ”\( F \) with \( X \) replacing \( Y \)”
- \( F[X \rightarrow Y] \) – “\( F \) with \( X \) replacing \( Y \)”
- \( F[X \leftarrow Y] \) – “\( F \) with \( X \) replaced by \( Y \)”
- \( s/X/Y/g \) – sed syntax
Evaluation of Statements

Evaluation of a statement produces a side-effect

- The *result* of evaluation of a statement is a **new state**

We write \(<s, q> \downarrow q'\) to mean that evaluation of statement \(s\) in state \(q\) results in a new state \(q'\)

\[
\begin{align*}
<s_1, q> & \downarrow q' \\
<s_2, q> & \downarrow q'
\end{align*}
\]

\[
\begin{align*}
<s_1; s_2, q> & \downarrow q' \\
<e, q> & \downarrow n
\end{align*}
\]

\[
\begin{align*}
<b, q> & \downarrow true \\
<s_1, q> & \downarrow q'
\end{align*}
\]

\[
\begin{align*}
<b, q> & \downarrow false \\
<s_2, q> & \downarrow q'
\end{align*}
\]
Derivation and Execution

Derivation of statement facts corresponds to execution / interpretation

For example

- Show that \(<p:=0; x:=1; n:=2, [ ]> \downarrow [p:=0,x:=1,n:=2]\)

\[
\begin{align*}
<0, []> & \downarrow 0 \\
<1, [p:=0]> & \downarrow 1 \\
\hline
<p:=0, []> & \downarrow [p:=0] \\
<x:=1, [p:=0]> & \downarrow [p:=0,x:=1] \\
\hline
<p:=0,x:=1, [ ]> & \downarrow [p:=0,x:=1] \\
<n:=2, [p:=0,x:=1]> & \downarrow [p:=0,x:=1,n:=2] \\
\hline
<p:=0; x:=1; n:=2, [ ]> & \downarrow [p:=0,x:=1,n:=2]
\end{align*}
\]
Semantics of Loops

What about infinite execution?
• Can introduce a special state \( T \), called \( top \), that represents divergence
• Infinite loop enters divergent state

\[
\begin{align*}
<\text{while } b \text{ do } s, q> & \downarrow q & <b, q> & \downarrow false \\
<\text{while } b \text{ do } s, q> & \downarrow q & <b, q> & \downarrow true & <s; \text{ while } b \text{ do } s, q> & \downarrow q' \\
<\text{while true do } s, q> & \downarrow T
\end{align*}
\]

• Any statement in divergent state is treated like ‘skip’

\[
<\text{s, } T > \downarrow T
\]

Need structural (or \textit{small step}) semantics to deal with reactive execution
• execution that does not terminate, but produces useful result
Properties of Semantics

A semantics is *deterministic* if every program statement has exactly one possible derivation in any state

- If \(<s, q> \Downarrow q_1\) and \(<s, q> \Downarrow q_2\) then \(q_1 = q_2\)

Two statements are *semantically equivalent* if for every input state they derive the same output state

- \(s_1\) and \(s_2\) are sem. equiv. if \(<s_1, q> \Downarrow q_1\) and \(<s_2, q> \Downarrow q_2\) imply \(q_1 = q_2\)
- e.g., \((\text{while } b \text{ do } s)\) and \(\text{if } b \text{ then } (s ; \text{while } b \text{ do } s) \text{ else skip}\) are sem. equiv.

**Structural induction**: To prove a property \(P\) on a derivation tree

- **Base case**: prove \(P\) for all of the axioms
- **Inductive Hypothesis**: assume \(P\) holds before every rule
- **Induction**: prove that \(P\) holds at the end of every rule

Use structural induction to prove that our semantics are deterministic
Structural Operational Semantics (Small-Step)

The meaning of executing ONE statement of a program

\[ <s, q> \Rightarrow <t, q'> \]

- Program to execute
- Program state
- The REST of the program
- Output state

For the final statement, the output is only a state

\[ <s, q> \Rightarrow q' \]
Small-step semantics for WHILE

- `<skip, q> ⇒ q`

- `<s_1, q> ⇒ q’`

- `<s_1 ; s_2, q> ⇒ <s_2, q’>`

- `<b, q> ⇓ true`
  - `<if b then s_1 else s_2, q> ⇒ <s_1, q>`

- `<b, q> ⇓ false`
  - `<if b then s_1 else s_2, q> ⇒ <s_2, q>`

- `<while b do s, q> ⇒ <if b then (s ; while b do s) else skip, q>`
Properties of Small Step Semantics

Small step semantics can be viewed as a transition system $TS=(S, R)$

- $S$ is a set of states; Each configuration $<s, q>$ is a state.
- $R$ is a transition relation on pair of states
  - $(x, y)$ in $R$ iff $(x \Rightarrow y)$ is a true judgement in small-step semantics

A path $x_1, x_2, x_3, \ldots$ in this TS is called a *derivation sequence*

A derivation sequence in TS corresponds to a program execution

Properties of small-step semantics are established by induction on the length of the derivation

Small step semantics is *deterministic* if there is only one derivation for every configuration
From Programming to Modeling

Extend a programming language with 3 modeling features

**Assertions**
- `assert e` – aborts an execution when `e` is false, no-op otherwise

```c
void assert (bool b) { if (!b) error(); }
```

**Non-determinism**
- `havoc x` – assigns variable `x` a non-deterministic value

```c
void havoc(int &x) { int y; x = y; }
```

**Assumptions**
- `assume e` – blocks execution if `e` is false, noop otherwise

```c
void assume (bool e) { while (!e); }
```
Safety Specifications as Assertions

A program is **correct** if all executions that satisfy all assumptions also satisfy all assertions.

A program is **incorrect** if there exists an execution that satisfies all of the assumptions AND violates at least one an assertion.

**Assumptions** express **pre-conditions** on which the program behavior relies.

**Assertions** express desired **properties** that the program must maintain.
Writing Specifications with Assert and Assume

```c
int x, y;

void main (void)
{
    havoc (x);
    assume (x > 10);
    assume (x <= 100);

    y = x + 1;

    assert (y > x);
    assert (y < 200);
}
```
Order of Assumptions is IMPORTANT!!!

```c
int x, y;

void main (void)
{
    havoc (x);
    y = x + 1;
    assume (x > 10);
    assume (x <= 100);
    assert (y > x);
    assert (y < 200);
}
```

```c
int x, y;

void main (void)
{
    havoc (x);
    y = x + 1;
    assume (x > 10);
    assume (x <= 100);
    assert (y > x);
    assert (y < 200);
}
```
GRAPHS
Graphs

A graph, \( G = (N, E) \), is an ordered pair consisting of
- a node set, \( N \), and
- an edge set, \( E = \{(n_i, n_j)\} \)

If the pairs in \( E \) are ordered, then \( G \) is called a directed graph and is depicted with arrowheads on its edges

If not, the graph is called an undirected graph

Graphs are suggestive devices that help in the visualization of relations
- The set of edges in the graph are visual representations of the ordered pairs that compose relations

Graphs provide a mathematical basis for reasoning about programs
Paths

A path, \( P \), through a directed graph \( G = (N, E) \) is a sequence of edges, 
\( (u_1, v_1), (u_2, v_2), \ldots, (u_t, v_t) \) such that

- \( v_{k-1} = u_k \) for all \( 1 < k \leq t \)
- \( u_1 \) is called the start node and \( v_t \) is called the end node

The length of a path is the number of edges (or nodes-1 😊) in the path

Paths are also frequently represented by a sequence of nodes
- \( (u_1, u_2, u_3, \ldots, u_t) \)
Cycles

A cycle in a graph $G$ is a path whose start node and end node are the same.

A simple cycle in a graph $G$ is a cycle such that all of its nodes are different (except for the start and end nodes).

If a graph $G$ has no path through it that is a cycle, then the graph is called acyclic.
Example of Cycles

Cycle: 1, 3, 2, 4, 3, 1
Simple cycle: 3, 2, 4, 3
Trees

An acyclic, undirected graph is called a tree.

If the undirected version of a directed graph is acyclic, then the graph is called a directed tree.

If the undirected version of a directed graph has cycles, but the directed graph itself has no cycles, then the graph is called a Directed Acyclic Graph (DAG).

Every tree is isomorphic to a prefix-closed subset of $\mathbb{N}^*$ for some natural number $N$. 
Examples

- **Tree**: A directed tree with nodes labeled as 0, 1, 1.0, and 1.1.
- **Directed Tree**: A directed acyclic graph (DAG) with nodes labeled as 0, 1, 1.0, and 1.1.
- **Cyclic Undirected Graph**: A cyclic undirected graph with nodes labeled as 0, 1, 1.0, and 1.1.
- **Directed Acyclic Graph (DAG)**: A directed acyclic graph (DAG) with nodes labeled as 0, 1, 1.0, and 1.1.
GRAPHS AS MODELS OF COMPUTATION
Computation tree

A tree model of all the possible executions of a system

At each node represents a state of the system
  • valuation of all variables

Can have infinite number of paths

Can have infinite paths
Example Computation Tree

```
total := 0;
count := 1;
max := input();
while (count <= max)
  do {
    val := input();
    total := total + val;
    count := count + 1};
print (total)
```

Is this tree infinite?
Disadvantages of Computation Trees

Represent the space that we want to reason about

For anything interesting, they are too large to create or reason about

Other models of executable behavior are providing abstractions of the computation tree model

• Abstract values
• Abstract flow of control
• Specialize abstraction depending on focus of analysis
Control Flow Graph (CFG)

Represents the flow of execution in the program

\[ G = (N, E, S, T) \]

where

- the nodes \( N \) represent executable instructions (statement, statement fragments, or basic blocks);
- the edges \( E \) represent the potential transfer of control;
- \( S \) is a designated start node;
- \( T \) is a designated final node
- \[ E = \{ (n_i, n_j) \mid \text{syntactically, the execution of } n_j \text{ follows the execution of } n_i \} \]

Nodes may correspond to single statements, parts of statements, or several statements (i.e., basic blocks)

Execution of a node means that the instructions associated with a node are executed in order from the first instruction to the last
Example of a Control Flow Graph

total := 0;
count := 1;
max := input();
while (count <= max) do {
    val := input();
    total := total + val;
    count := count + 1;
} print (total)

1. total := 0;
count := 1
max := input()

2. count <= max
   T
   print (total)
   F

3. val := read()
total := total + val
count := count + 1

4. print (total)
Deriving a Control Flow Graph

```java
public static String collapseNewlines(String argStr) {
    char last = argStr.charAt(0);
    StringBuffer argBuf = new StringBuffer();
    for (int cIdx = 0; cIdx < argStr.length(); cIdx++) {
        char ch = argStr.charAt(cIdx);
        if (ch != '\n' || last != '\n') {
            argBuf.append(ch);
            last = ch;
        }
    }
    return argBuf.toString();
}
```
Control Flow Graph

A CFG is a graph of basic blocks
- edges represent different control flow

A CFG corresponds to a program syntax
- where statements are restricted to the form $L_i; S$; goto $L_j$
  and $S$ is control-free (i.e., assignments and procedure calls)

1. initial setup:
   
   ```
   total := 0;
   count := 1
   max := input()
   ```

2. condition:
   
   ```
   count <= max
   ```

3. reading value:
   
   ```
   val := read()
   total := total + val
   count := count + 1
   ```

4. printing total:
   
   ```
   print (total)
   ```
Control Flow Graph

1: total := 0; count := 1;
   max := input(); goto 2

2: if count <= max
   then goto 3 else goto 4

3: val := read();
   total := total + val;
   count := count + 1; goto 2

4: print(total)
a **sub-path** through a CFG is a sequence of nodes \((n_i, n_{i+1}, \ldots, n_t)\), \(i \geq 1\) where for each \(n_k, i \leq k < t, (n_k, n_{k+1})\) is an edge in the graph

- e.g., 2, 3, 2, 3, 2, 4

a **complete path** starts at the start node and ends at the final node

- e.g., 1, 2, 3, 2, 4

```python
val := read()
total := total + val
count := count + 1
```

```python
total := 0;
count := 1
max := input()
```

```python
print (total)
```

F

T

\( \text{T} \) and \( \text{F} \) accept different paths.
Infeasible Paths

Every executable sequence in the represented component corresponds to a path in G

Not all paths correspond to executable sequences

- requires additional semantic information
- “infeasible paths” are not an indication of a fault

CFG usually overestimates the executable behavior
Example with an infeasible path

1. $X > 0$

2. $Y := X \div 2$

3. $Y := 5$

4. $X \cdot Y > 0$

5. $Z := 10$

6. $Z := 20$

7. $X := Y + Z$

8. $X <= 0$

9. $X <= 0, Y = 5$
Example Paths

Feasible path: 1, 2, 4, 5, 7

Infeasible path: 1, 3, 4, 5, 7

Determining if a path is feasible or not requires additional semantic information

• In general, undecidable
• In practice, intractable
  – Some exceptions are studied in this course
Benefits of CFG

Probably the most commonly used representation
- Numerous variants

Basis for inter-component analysis
- Collections of CFGs

Basis for various transformations
- Compiler optimizations
- S/W analysis

Basis for automated analysis
- Graphical representations of interesting programs are too complex for direct human understanding
Paths

- Paths:
  - 1, 2, 4, 5, 7
  - 1, 2, 4, 6, 7
  - 1, 3, 4, 5, 7
  - 1, 3, 4, 6, 7
Paths can be identified by predicate outcomes

- outomes
  - t, t
  - t, f
  - f, t
  - f, f
Paths can be identified by domains

- $\{ X, Y \mid X > 0 \text{ and } X \times Y > 0 \}$
- $\{ X, Y \mid X > 0 \text{ and } X \times Y \leq 0 \}$
- $\{ X, Y \mid X \leq 0 \text{ and } X \times Y > 0 \}$
- $\{ X, Y \mid X \leq 0 \text{ and } X \times Y \leq 0 \}$

1. $X > 0$
2. $Z := 1$
3. $Z := 5$
4. $X \times Y > 0$
5. $Z := Z + 10$
6. $Z := Z + 20$
7. $X := Y + Z$
CFG Abstraction Level?

Loop conditions? (yes)
Individual statements? (no)
Exception handling? (no)

*What’s best depends on type of analysis to be conducted*
Draw a control flow graph with 7 nodes.

```c
int binary_search(int a[], int low, int high, int target) { /* binary search for target in
the sorted a[low, high] */
    while (low <= high) {
        int middle = low + (high - low)/2;
        if (target < a[middle])
            high = middle - 1;
        else if (target > a[middle])
            low = middle + 1;
        else
            return middle;
    }
    return -1; /* return -1 if target is not
found in a[low, high] */
}
```
int binary_search(int a[], int low, int high, int target) { /* binary search for target in
the sorted a[low, high] */
    while (low <= high) {
        int middle = low + (high - low)/2;
        if (target < a[middle])
            high = middle - 1;
        else if (target > a[middle])
            low = middle + 1;
        else
            return middle;
    }
    return -1; /* return -1 if target is not
found in a[low, high] */
}
int binary_search(int a[], int low, int high, int target) { /* binary search for target in the sorted a[low, high] */
    while (low <= high) {
        int middle = low + (high - low)/2;
        if (target < a[middle])
            high = middle - 1;
        else if (target > a[middle])
            low = middle + 1;
        else
            return middle;
    }
    return -1; /* return -1 if target is not found in a[low, high] */
}
Draw a control flow graph with 8 nodes.

```c
int binary_search(int a[], int low, int high, int target) { /* binary search for target in the sorted a[low, high] */
    while (low <= high) {
        int middle = low + (high - low)/2;
        if (target < a[middle])
            high = middle - 1;
        else if (target > a[middle])
            low = middle + 1;
        else
            return middle;
    }
    return -1; /* return -1 if target is not found in a[low, high] */
}
```
public static double ReturnAverage(int value[], int AS, int MIN, int MAX) {
    int i, ti, tv, sum;
    double av;
    i = 0; ti = 0; tv = 0; sum = 0;
    while (ti < AS && value[i] != -999) {
        ti++;
        if (value[i] >= MIN && value[i] <= MAX) {
            tv++;
            sum = sum + value[i];
        }
        i++;
    }
    if (tv > 0) av = (double)sum/tv;
    else av = (double) -999;
    return (av);
CFG of ReturnAverage

\[ i = 0, \; ti = 0, \]
\[ tv = 0, \; sum = 0 \]

\[ \text{F} \quad ti < AS \]
\[ \text{T} \quad \text{value}[i] \neq -999 \]

\[ \text{F} \quad tv > 0 \]
\[ \text{T} \quad \] 
\[ \text{av} = (\text{double})-999 \]
\[ \text{av} = (\text{double})\text{sum/tv} \]

\[ \text{return(}\text{av}) \]

\[ \text{ti}++ \]

\[ \text{F} \quad \] 
\[ \text{X} \quad \text{value}[i] \geq \text{MIN} \]

\[ \text{F} \quad \] 
\[ \text{value}[i] \leq \text{MAX} \]
\[ \text{TV} \]
\[ \] 
\[ \text{TV} \]
\[ \text{tv}++ \]
\[ \text{sum} = \text{sum} + \text{value}[i] \]

\[ \text{i}++ \]