5.1 Bandgap and photodetection

a. Determine the maximum value of the energy gap that a semiconductor, used as a photoconductor, can have if it is to be sensitive to yellow light (600 nm).

b. A photodetector whose area is $5 \times 10^{-2}$ cm$^2$ is irradiated with yellow light whose intensity is 2 mW cm$^{-2}$. Assuming that each photon generates one electron-hole pair, calculate the number of pairs generated per second.

c. From the known energy gap of the semiconductor GaAs ($E_g = 1.42$ eV), calculate the primary wavelength of photons emitted from this crystal as a result of electron-hole recombination.

d. Is the above wavelength visible?

e. Will a silicon photodetector be sensitive to the radiation from a GaAs laser? Why?

Solution

a. We are given the wavelength $\lambda = 600$ nm, therefore we need

$$E_g = h \nu = E_{ph}$$

so that,

$$E_g = \frac{hc}{\lambda} = \left(6.626 \times 10^{-34} \text{ J s}\right) \left(3.0 \times 10^8 \text{ m s}^{-1}\right) / (600 \times 10^{-9} \text{ m})$$

∴

$$E_g = 3.31 \times 10^{-19} \text{ J or } 2.07 \text{ eV}$$

b. Area $A = 5 \times 10^{-2}$ cm$^2$ and light intensity $I_{\text{light}} = 2 \times 10^{-3}$ W/cm$^2$. The received power is:

$$P = AI_{\text{light}} = (5 \times 10^{-2} \text{ cm}^2)(2 \times 10^{-3} \text{ W/cm}^2) = 1.0 \times 10^{-4} \text{ W}$$

$$N_{ph} = \text{number of photons arriving per second} = \frac{P}{E_{ph}}$$

∴

$$N_{ph} = \frac{(1.0 \times 10^{-4} \text{ W})}{(3.31 \times 10^{-19} \text{ J})} = 3.02 \times 10^{14} \text{ Photons s}^{-1}$$

Since each photon contributes one electron-hole pair (EHP), the number of EHPs is then:

$$N_{EHP} = 3.02 \times 10^{14} \text{ EHP s}^{-1}$$

c. For GaAs, $E_g = 1.42$ eV and the corresponding wavelength is

$$\lambda = \frac{hc}{E_g} = \left(6.626 \times 10^{-34} \text{ J s}\right) \left(3.0 \times 10^8 \text{ m s}^{-1}\right) / (1.42 \text{ eV} \times 1.602 \times 10^{-19} \text{ J/eV})$$

∴

$$\lambda = 8.74 \times 10^{-7} \text{ m or } 874 \text{ nm}$$

The wavelength of emitted radiation due to electron-hole pair (EHP) recombination is therefore 874 nm.

d. It is not in the visible region (it is in the infrared).

e. From Table 5.1, for Si, $E_g = 1.10$ eV and the corresponding cut-off wavelength is,

$$\lambda_g = \frac{hc}{E_g} = \left(6.626 \times 10^{-34} \text{ J s}\right) \left(3.0 \times 10^8 \text{ m s}^{-1}\right) / (1.1 \text{ eV} \times 1.602 \times 10^{-19} \text{ J/eV})$$

∴

$$\lambda_g = 1.13 \times 10^{-6} \text{ m or } 1130 \text{ nm}$$
Since the 874 nm wavelength of the GaAs laser is shorter than the cut-off wavelength of 1130 nm, the Si photodetector can detect the 874 nm radiation (Put differently, the photon energy corresponding to 874 nm, 1.42 eV, is larger than the $E_g$, 1.10 eV, of Si which means that the Si photodetector can indeed detect the 874 nm radiation).

**5.2 Intrinsic Ge** Using the values of the density of states effective masses $m_e^*$ and $m_h^*$ in Table 5.1, calculate the intrinsic concentration in Ge. What is $n_i$ if you use $N_c$ and $N_v$ from Table 5.1? Calculate the intrinsic resistivity of Ge at 300 K.

**Solution**

From Table 5.1, we get

$$m_e^* = 0.56m_e \quad \text{and} \quad m_h^* = 0.4m_e$$

Now,

$$N_c = 2 \left( \frac{2\pi m_e^* kT}{\hbar^2} \right)^{3/2} = 2 \left[ \frac{2\pi (0.56 \times 9.1 \times 10^{-31} \text{kg})(1.38 \times 10^{-23} \text{J K}^{-1})(300 \text{K})}{(6.626 \times 10^{-34} \text{J s})^2} \right]^{3/2}$$

$$= 1.05 \times 10^{25} \text{ m}^3 \text{ or } 1.05 \times 10^{19} \text{ cm}^3$$

$$N_v = 2 \left( \frac{2\pi m_h^* kT}{\hbar^2} \right)^{3/2} = 2 \left[ \frac{2\pi (0.4 \times 9.1 \times 10^{-31} \text{kg})(1.38 \times 10^{-23} \text{J K}^{-1})(300 \text{K})}{(6.626 \times 10^{-34} \text{J s})^2} \right]^{3/2}$$

$$= 6.35 \times 10^{24} \text{ m}^3 \text{ or } 6.35 \times 10^{18} \text{ cm}^3$$

The intrinsic concentration is

$$n_i = (N_c N_v)^{1/2} \exp \left( -\frac{E_g}{2kT} \right)$$

$$= \left(1.049 \times 10^{19} \text{ cm}^{-3}(6.33 \times 10^{18} \text{ cm}^{-3}) \right)^{1/2} \exp \left( -\frac{0.66 \text{ eV}}{2(8.62 \times 10^{-5} \text{ eV K}^{-1})(300 \text{ K})} \right)$$

$$= 2.33 \times 10^{13} \text{ cm}^{-3} \approx 2.3 \times 10^{13} \text{ cm}^{-3}$$

From Table 5.1, we get $N_c = 1.04 \times 10^{19} \text{ cm}^{-3}$ and $N_v = 6.0 \times 10^{18} \text{ cm}^{-3}$

$$n_i = \left(1.04 \times 10^{19} \text{ cm}^{-3}(6.0 \times 10^{18} \text{ cm}^{-3}) \right)^{1/2} \exp \left( -\frac{0.66 \text{ eV}}{2(8.62 \times 10^{-5} \text{ eV K}^{-1})(300 \text{ K})} \right)$$

$$= 2.26 \times 10^{13} \text{ cm}^{-3} \approx 2.3 \times 10^{13} \text{ cm}^{-3}$$

The intrinsic conductivity is $\sigma = e n_e \mu_e + e p \mu_p = e n_i(\mu_e + \mu_p)$
\[ \sigma = (1.6 \times 10^{-19} \text{ C})(2.34 \times 10^{13} \text{ cm}^{-3})(3900 + 1900) \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} = 0.022 \ \Omega^{-1} \]

And the intrinsic resistivity is \( \rho = 1/\sigma = 45.45 \ \Omega \text{ cm} \)

### 5.3 Fermi level in intrinsic semiconductors

Using the values of the density of states effective masses \( m_e^* \) and \( m_h^* \) in Table 5.1, find the position of the Fermi energy in intrinsic Si, Ge and GaAs with respect to the middle of the bandgap \( (E_g/2) \).

**Solution**

\[ E_{Fi} = E_v + \frac{1}{2} E_g - \frac{3}{4} kT \ln \left( \frac{m_e^*}{m_h^*} \right) \]

For Si, \( m_e^* = 1.08 m_e \) and \( m_h^* = 0.6 m_e \)

\[ E_{Fi} = E_v + \frac{1}{2} E_g - \frac{3}{4} (8.62 \times 10^{-5} \text{ eV K}^{-1})(300 \text{ K}) \ln \left( \frac{1.08 m_e}{0.6 m_e} \right) \]

\[ E_{Fi} = E_v + \frac{1}{2} E_g - 0.011 \text{ eV} \]

So intrinsic Fermi level of Si is 0.011 eV below the middle of the bandgap \( (E_g/2) \).

For Ge, \( m_e^* = 0.56 m_e \) and \( m_h^* = 0.4 m_e \)

\[ E_{Fi} = E_v + \frac{1}{2} E_g - \frac{3}{4} (8.62 \times 10^{-5} \text{ eVK}^{-1})(300 \text{ K}) \ln \left( \frac{0.56 m_e}{0.4 m_e} \right) \]

\[ E_{Fi} = E_v + \frac{1}{2} E_g - 0.0065 \text{ eV} \]

So intrinsic Fermi level of Ge is 0.0065 eV below the middle of the bandgap \( (E_g/2) \).

For GaAs, \( m_e^* = 0.067 m_e \) and \( m_h^* = 0.5 m_e \)

\[ E_{Fi} = E_v + \frac{1}{2} E_g - \frac{3}{4} (8.62 \times 10^{-5} \text{ eV K}^{-1})(300 \text{ K}) \ln \left( \frac{0.067 m_e}{0.5 m_e} \right) \]

\[ E_{Fi} = E_v + \frac{1}{2} E_g + 0.039 \text{ eV} \]

So intrinsic Fermi level of GaAs is 0.039 eV above the middle of the bandgap \( (E_g/2) \).
5.4 Extrinsic Si  A Si crystal has been doped with P. The donor concentration is $10^{15}$ cm$^{-3}$. Find the conductivity, and resistivity of the crystal.

Solution

$N_d = 10^{15}$ cm$^{-3}$

Therefore the conductivity is

$$\sigma = eN_d\mu_e = (1.6 \times 10^{-19} \text{ C})(10^{15} \text{ cm}^{-3})(1350 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}) = 0.216 \text{ } \Omega^{-1}\text{cm}^{-1}$$

And the resistivity is $\rho = 1/\sigma = 4.63 \text{ } \Omega^{-1}\text{cm}^{-1}$

5.5 Extrinsic Si  Find the concentration of acceptors required for a $p$-type Si crystal to have a resistivity of 1 $\Omega$ cm.

Solution

The resistivity, $\rho = \frac{1}{eN_a\mu_h}$

$\therefore \quad N_a = \frac{1}{e\mu_h\rho} = \frac{1}{(1.6 \times 10^{-19} \text{ C})(450 \text{ cm}^2\text{V}^{-1}\text{s}^{-1})(1\Omega \text{ cm})} = 1.38 \times 10^{16} \text{ cm}^{-3}$

It is assumed that the doping concentration does not affect drift mobility. At this concentration, from Figure 5.19, the hole drift mobility is approximately 350 cm$^2$ V$^{-1}$ s$^{-1}$ rather than 450 cm$^2$ V$^{-1}$ s$^{-1}$. Using 350 cm$^2$ V$^{-1}$ s$^{-1}$ we find,

$$N_a = \frac{1}{e\mu_h\rho} = \frac{1}{(1.6 \times 10^{-19} \text{ C})(350 \text{ cm}^2\text{V}^{-1}\text{s}^{-1})(1\Omega \text{ cm})} = 1.78 \times 10^{16} \text{ cm}^{-3}$$

Author's Note: Question 5.7 provides a better example for calculating the required dopant concentration for a given resistivity.

5.6 Minimum conductivity

a. Consider the conductivity of a semiconductor, $\sigma = en\mu_e + ep\mu_h$. Will doping always increase the conductivity?

b. Show that the minimum conductivity for Si is obtained when it is $p$-type doped such that the hole concentration is

$$p_m = n_i \sqrt{\frac{\mu_e}{\mu_h}}$$
and the corresponding minimum conductivity (maximum resistivity) is

\[ \sigma_{\text{min}} = 2en_i \sqrt{\mu_e \mu_h} \]

c. Calculate \( p_m \) and \( \sigma_{\text{min}} \) for Si and compare with intrinsic values.

**Solution**

**a.** Doping does not always increase the conductivity. Suppose that we have an intrinsic sample with \( n = p \) but the hole drift mobility is smaller. If we dope the material very slightly with \( p \)-type then \( p > n \). However, this would decrease the conductivity because it would create more holes with lower mobility at the expense of electrons with higher mobility. Obviously with further doping \( p \) increases sufficiently to result in the conductivity increasing with the extent of doping.

**b.** To find the minimum conductivity, first consider the mass action law:

\[ np = n_i^2 \]

isolate \( n \):

\[ n = n_i^2/p \]

Now substitute for \( n \) in the equation for conductivity:

\[ \sigma = en_1 \mu_e + ep \mu_h \]

\[ \therefore \quad \sigma = \frac{en_i^2 \mu_e}{p} + \mu_h ep \]

To find the value of \( p \) that gives minimum conductivity (\( p_m \)), differentiate the above equation with respect to \( p \) and set it equal to zero:

\[ \frac{d\sigma}{dp} = -\frac{en_i^2 \mu_e}{p^2} + \mu_h e \]

\[ \therefore \quad \frac{en_i^2 \mu_e}{p_m^2} + \mu_h e = 0 \]

Isolate \( p_m \) and simplify,

\[ p_m = n_i \sqrt{\mu_e / \mu_h} \]

Substituting this expression back into the equation for conductivity will give the minimum conductivity:

\[ \sigma_{\text{min}} = \frac{en_i^2 \mu_e}{p_m} + \mu_h ep_m = \frac{en_i^2 \mu_e}{n_i \sqrt{\mu_e / \mu_h}} + \mu_h n_i \sqrt{\mu_e / \mu_h} \]

\[ \therefore \quad \sigma_{\text{min}} = en_i \mu_e \sqrt{\mu_e / \mu_e} + en_i \sqrt{\mu_e \mu_h} = en_i \sqrt{\mu_e \mu_h} + en_i \sqrt{\mu_e \mu_h} \]
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\[ \sigma_{\text{min}} = 2en_i \sqrt{\mu_e \mu_h} \]

c. From Table 5.1, for Si: \( \mu_e = 1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} \), \( \mu_h = 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} \) and \( n_i = 1.0 \times 10^{10} \text{ cm}^{-3} \). Substituting into the equations for \( p_m \) and \( \sigma_{\text{min}} \):

\[
p_m = n_i \sqrt{\frac{\mu_e}{\mu_h}} = (1.0 \times 10^{10} \text{ cm}^{-3}) \sqrt{\frac{1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}}{450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}}} = 1.73 \times 10^{10} \text{ cm}^{-3}
\]

\[
\sigma_{\text{min}} = 2en_i \sqrt{\mu_e \mu_h}
\]

\[
\therefore \sigma_{\text{min}} = 2(1.602 \times 10^{-19} \text{ C})(1.0 \times 10^{10} \text{ cm}^{-3})(1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})(450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})
\]

\[
\therefore \sigma_{\text{min}} = 2.5 \times 10^{-6} \Omega^{-1} \text{ cm}^{-1}
\]

The corresponding maximum resistivity is:

\[
\rho_{\text{max}} = 1 / \sigma_{\text{min}} = 4 \times 10^5 \Omega \text{ cm}
\]

The intrinsic value corresponding to \( p_m \) is simply \( n_i \) (= 1.0 \times 10^{10} \text{ cm}^{-3} \). Comparing it to \( p_m \):

\[
\frac{p_m}{n_i} = \frac{1.73 \times 10^{10} \text{ cm}^{-3}}{1.0 \times 10^{10} \text{ cm}^{-3}} = 1.73
\]

The intrinsic conductivity is:

\[
\sigma_{\text{int}} = en_i(\mu_e + \mu_h)
\]

\[
\therefore \sigma_{\text{int}} = (1.602 \times 10^{-19} \text{ C})(1.0 \times 10^{10} \text{ cm}^{-3})(1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} + 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})
\]

\[
\therefore \sigma_{\text{int}} = 2.88 \times 10^{-6} \Omega^{-1} \text{ cm}^{-1}
\]

Comparing this value to the minimum conductivity:

\[
\frac{\sigma_{\text{min}}}{\sigma_{\text{int}}} = \frac{2.5 \times 10^{-6} \Omega^{-1} \text{ cm}^{-1}}{2.88 \times 10^{-6} \Omega^{-1} \text{ cm}^{-1}} = 0.87
\]

Sufficient \( p \)-type doping that increases the hole concentration by 73% decreases the conductivity by 15% to its minimum value.

5.9 Compensation doping in Si

a. A Si wafer has been doped \( n \)-type with \( 10^{17} \) As atoms \text{ cm}^{-3} \).

1. Calculate the conductivity of the sample at 27 °C.

2. Where is the Fermi level in this sample at 27 °C with respect to the Fermi level \( (E_{Fi}) \) in intrinsic Si?

3. Calculate the conductivity of the sample at 127 °C.
The above *n*-type Si sample is further doped with $9 \times 10^{16}$ boron atoms (*p*-type dopant) per centimeter cubed.

1. Calculate the conductivity of the sample at 27 °C.

2. Where is the Fermi level in this sample with respect to the Fermi level in the sample in (a) at 27 °C? Is this an *n*-type or *p*-type Si?

**Solution**

*a.* Given temperature $T = 27$ °C $= 300$ K, concentration of donors $N_d = 10^{17}$ cm$^{-3}$, and drift mobility $\mu_e \approx 800$ cm$^2$ V$^{-1}$ s$^{-1}$ (from Figure 5.19). At room temperature the electron concentration $n = N_d >> p$ (hole concentration).

![Figure 5.19: The variation of the drift mobility with dopant concentration in Si](image)

For electrons and holes at 300 K.

(1) The conductivity of the sample is:

$$\sigma = eN_d\mu_e \approx (1.602 \times 10^{-19} \text{ C})(10^{17} \text{ cm}^{-3})(800 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 12.8 \Omega^{-1} \text{ cm}^{-1}$$

(2) In intrinsic Si, $E_F = E_{Fi}$,

$$n_i = N_c \exp\left[-(E_c - E_{Fi})/kT\right]$$

In doped Si, $n = N_d$, $E_F = E_{Fn}$,

$$n = N_d = N_c \exp\left[-(E_c - E_{Fn})/kT\right]$$

Eqn. (2) divided by Eqn. (1) gives,

$$\frac{N_d}{n_i} = \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

(3)
\[ \ln \left( \frac{N_d}{n_i} \right) = \frac{E_{Fn} - E_{Fi}}{kT} \]

\[ \Delta E_F = E_{Fn} - E_{Fi} = kT \ln \left( \frac{N_d}{n_i} \right) \] (4)

Substituting we find \((n_i = 1.0 \times 10^{10} \text{ cm}^{-3} \text{ from Table 5.1})\),

\[ \Delta E_F = (8.617 \times 10^{-5} \text{ eV/K})(300 \text{ K})\ln \left( \frac{10^{17} \text{ cm}^{-3}}{1.0 \times 10^{10} \text{ cm}^{-3}} \right) \]

\[ \therefore \Delta E_F = 0.416 \text{ eV above } E_{Fi} \]

**Figure 5.18:** Log-log plot for drift mobility versus temperature for \(n\)-type Ge and \(n\)-type Si samples. Various donor concentrations for Si are shown, \(N_d\) are in \(\text{cm}^{-3}\). The upper right insert is the simple theory for lattice limited mobility whereas the lower left inset is the simple theory for impurity scattering limited mobility.

(3) At \(T_i = 127 \, ^\circ\text{C} = 400 \text{ K}\), \(\mu_e \approx 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}\) (from Figure 5.18). The semiconductor is still \(n\)-type (check that \(N_d \gg n_i\) at 400 K), then

\[ \sigma = eN_d\mu_e \approx (1.602 \times 10^{-19} \text{ C})(10^{17} \text{ cm}^{-3})(450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 7.21 \Omega^{-1} \text{ cm}^{-1} \]

**b.** The sample is further doped with \(N_a = 9 \times 10^{16} \text{ cm}^{-3} = 0.9 \times 10^{17} \text{ cm}^{-3} \) acceptors. Due to compensation, the net effect is still an \(n\)-type semiconductor but with an electron concentration given by,

\[ n = N_d - N_a = 10^{17} \text{ cm}^{-3} 0.9 \times 10^{17} \text{ cm}^{-3} = 1 \times 10^{16} \text{ cm}^{-3} (\gg n_i) \]

We note that the electron scattering now occurs from \(N_d + N_a (1.9 \times 10^{17} \text{ cm}^{-3}) \) number of ionized centers so that \(\mu_e \approx 700 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}\) (Figure 5.18).

(1) \[ \sigma = eN_d\mu_e \approx (1.602 \times 10^{-19} \text{ C})(10^{16} \text{ cm}^{-3})(700 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 1.12 \Omega^{-1} \text{ cm}^{-1} \]
(2) Using Eqn. (3) with \( n = N_d - N_a \) we have,

\[
\frac{n}{n_i} = \frac{N_d - N_a}{n_i} = \exp \left( \frac{E'_{F_n} - E_{F_i}}{kT} \right)
\]

so that

\[
\Delta E'_{F} = E'_{F_n} - E_{F_i} = (0.02586 \text{ eV})\ln\left[\frac{(10^{16} \text{ cm}^{-3})}{(1.0 \times 10^{10} \text{ cm}^{-3})}\right]
\]

\[
\therefore \quad \Delta E'_{F} = 0.357 \text{ eV above } E_{F_i}
\]

The Fermi level from (a) and (b) has shifted “down” by an amount 0.059 eV. Since the energy is still above the Fermi level, this is an \( n \)-type Si.

---

5.10 Temperature dependence of conductivity

An \( n \)-type Si sample has been doped with \( 10^{15} \) phosphorus atoms cm\(^{-3} \). The donor energy level for P in Si is 0.045 eV below the conduction band edge energy.

a. Calculate the room temperature conductivity of the sample.

b. Estimate the temperature above which the sample behaves as if intrinsic.

c. Estimate to within 20 percent the lowest temperature above which all the donors are ionized.

d. Sketch schematically the dependence of the electron concentration in the conduction band on the temperature as \( \log(n) \) versus \( 1/T \), and mark the various important regions and critical temperatures. For each region draw an energy band diagram that clearly shows from where the electrons are excited into the conduction band.

e. Sketch schematically the dependence of the conductivity on the temperature as \( \log(\sigma) \) versus \( 1/T \) and mark the various critical temperatures and other relevant information.

Solution
Figure 5.16: The temperature dependence of the intrinsic concentration.

**a.** The conductivity at room temperature \( T = 300 \text{ K} \) is \( (\mu_e = 1350 \times 10^{-4} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1} \) can be found in Table 5.1):

\[
\sigma = eN_d\mu_e
\]

\[
\therefore \sigma = (1.602 \times 10^{-19} \text{ C})(1 \times 10^{21} \text{ m}^{-3})(1350 \times 10^{-4} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}) = 21.6 \Omega^{-1} \text{ m}^{-1}
\]

**b.** At \( T = T_i \), the intrinsic concentration \( n_i = N_d = 1 \times 10^{15} \text{ cm}^{-3} \). From Figure 5.16, the graph of \( n_i(T) \) versus \( 1/T \), we have:

\[
1000 / T_i = 1.85 \text{ K}^{-1}
\]

\[
\therefore T_i = 1000 / (1.85 \text{ K}^{-1}) = 540 \text{ K} \text{ or } 267 \text{ °C}
\]

**c.** The ionization region ends at \( T = T_s \) when all donors have been ionized, i.e. when \( n = N_d \). From Example 5.7, at \( T = T_s \):

\[
n = N_d = \left( \frac{1}{2} \frac{N_cN_d}{N_cN_d} \right)^\frac{1}{2} \exp \left( \frac{-\Delta E}{2kT_s} \right)
\]

\[
\therefore T_s = \frac{-\Delta E}{2k \ln \left( \frac{N_d}{\frac{1}{2} \frac{N_cN_d}{N_c}} \right)} = \frac{-\Delta E}{2k \ln \left( \sqrt{\frac{2N_d}{N_c}} \right)}
\]
Take $N_c = 2.8 \times 10^{19}$ cm$^{-3}$ at 300 K from Table 5.1, and the difference between the donor energy level and the conduction band energy is $\Delta E = 0.045$ eV. Therefore our first approximation to $T_s$ is:

$$
T_s = \frac{\Delta E}{k \ln \left( \frac{N_c}{2N_d} \right)} = \frac{(0.045 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{(1.381 \times 10^{-23} \text{ J/K}) \ln \left( \frac{2.8 \times 10^{19} \text{ cm}^{-3}}{2 \times 10^{15} \text{ cm}^{-3}} \right)} = 54.68 \text{ K}
$$

Find the new $N_c$ at this temperature, $N_c'$:

$$
N_c' = N_c \left( \frac{T_s}{300} \right)^{\frac{3}{2}} = \left(2.8 \times 10^{19} \text{ cm}^{-3}\right) \left(\frac{54.68 \text{ K}}{300 \text{ K}}\right)^{\frac{3}{2}} = 2.179 \times 10^{18} \text{ cm}^{-3}
$$

Find a better approximation for $T_s$ by using this new $N_c'$:

$$
T_s' = \frac{\Delta E}{k \ln \left( \frac{N_c'}{2N_d} \right)} = \frac{(0.045 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{(1.381 \times 10^{-23} \text{ J/K}) \ln \left( \frac{2.179 \times 10^{18} \text{ cm}^{-3}}{2 \times 10^{15} \text{ cm}^{-3}} \right)} = 74.64 \text{ K}
$$

A better approximation to $T_s$ is:

$$
T_s'' = \frac{\Delta E}{k \ln \left( \frac{N_c''}{2N_d} \right)} = \frac{(0.045 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{(1.381 \times 10^{-23} \text{ J/K}) \ln \left( \frac{3.475 \times 10^{18} \text{ cm}^{-3}}{2 \times 10^{15} \text{ cm}^{-3}} \right)} = 69.97 \text{ K}
$$

We can see that the change in $T_s$ is very small, and for all practical purposes we can consider the calculation as converged. Therefore $T_s = 70.9 \text{ K} = -202.1 \degree \text{C}$.

d. and e.

See Figures 5.15 and 5.20
Figure 5.15: The temperature dependence of the electron concentration in an $n$-type semiconductor.

Figure 5.20: Schematic illustration of the temperature dependence of electrical conductivity for a doped ($n$-type) semiconductor.