Tutorial - 09
Section6: Dielectric Materials
ECE:209 (Electronic and Electrical Properties of Materials)
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Solutions:

7.3 Electronic polarization in liquid xenon Liquid xenon has been used in radiation detectors. The density of the liquid is 3.0 g cm^{-3}. What is the relative permittivity of liquid xenon given its electronic polarizability in Table 7.1? (The experimental ε_r is 1.96.)

Solution

Given $d = 3.0\text{g/cm}^{-3}$ we can calculate the number of Xe atoms per unit volume,

$$N = \frac{N_A \times d}{M_{at}} = \frac{6.023\times10^{23}\times3}{131.3} = 1.3762\times10^{22}\,\text{cm}^{-3}$$

With $N = 1.3762\times10^{28}\,\text{m}^{-3}$ and the given values from Table 7.1, $\alpha_e = 4.4\times10^{-40}\,\text{Fm}^2$ and $\varepsilon_o = 8.854\times10^{-12}\,\text{CV}^{-1}\text{m}^{-1}$, from Eq 7.14, we have

$$\varepsilon_r = 1 + \frac{N\alpha_e}{\varepsilon_0} = 1.6839$$

But, if we use Clausius-Mossotti Equation,

$$\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_o}$$

$$\therefore \qquad \varepsilon_r = \frac{1 + \dfrac{2N\alpha_e}{3\varepsilon_0}}{1 - \dfrac{N\alpha_e}{3\varepsilon_0}} = 1.8858 \text{ or } 1.89$$

$$\therefore \qquad \varepsilon_r(\text{Xe}) = 1.8858$$

Which is closer to the experimental value.

7.4 Relative permittivity, bond strength, bandgap and refractive index Diamond, silicon, and germanium are covalent solids with the same crystal structure. Their relative permittivities are shown in Table 7.10.

a. Explain why ε_r increases from diamond to germanium.

b. Calculate the polarizability per atom in each crystal and then plot polarizability against the elastic modulus *Y* (Young's modulus). Should there be a correlation?

c. Plot the polarizability from part (*b*) against the bandgap energy, E_g. Is there a relationship?

d. Show that the refractive index n is $\sqrt{\varepsilon_r}$. When does this relationship hold and when does it fail?

e. Would your conclusions apply to ionic crystals such as NaCl?

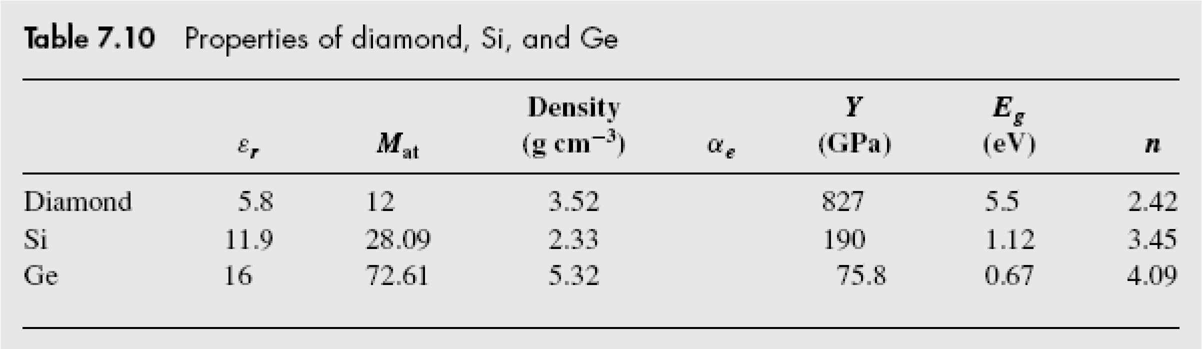

Table 7.10 Properties of diamond, Si, and Ge

	ε_r	M_{at}	Density (g cm^{-3})	α_e	Y (GPa)	E_g (eV)	n
Diamond	5.8	12	3.52		827	5.5	2.42
Si	11.9	28.09	2.33		190	1.12	3.45
Ge	16	72.61	5.32		75.8	0.67	4.09

Solution

a. In diamond, Si, and Ge, the polarization mechanism is electronic (bond). There are two factors that increase the polarization. First is the number of electrons available for displacement and the ease with which the field can displace the electrons. The number of electrons in the core shells increases from diamond to Ge. Secondly, and most importantly, the bond strength per atom decreases from diamond to Ge, making it easier for valence electrons in the bonds to be displaced.

b. For diamond, atomic concentration *N* is:

$$N = \frac{DN_A}{M_{\text{at}}} = \frac{(3.52\times10^3\ \text{kg/m}^3)(6.022\times10^{23}\ \text{mol}^{-1})}{(12\times10^{-3}\ \text{kg/mol})} = 1.766 \times 10^{29}\ \text{m}^{-3}$$

The polarizability can then be found from the Clausius-Mossotti equation:

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N}{3\varepsilon_o}\alpha_e$$

$$\therefore \qquad \alpha_e = \frac{3\varepsilon_o(\varepsilon_r - 1)}{N(\varepsilon_r + 2)} = \frac{3(8.854\times10^{-12}\ \text{F/m})(5.8-1)}{(1.766\times10^{29}\ \text{m}^{-3})(5.8+2)}$$

$$\therefore \qquad \boldsymbol{\alpha_e = 9.256 \times 10^{-41}\ \textbf{F m}^2}$$

The polarizability for Si and Ge can be found similarly, and are summarized in Table 7Q2-2:

	N (m^{-3})	**α_e (F m^2)**
Diamond	1.766×10^{29} m^{-3}	**9.256×10^{-41} F m^2**
Si	4.995×10^{28} m^{-3}	**4.170×10^{-40} F m^2**

Ge	4.412×10^{28} m^{-3}	**5.017×10^{-40} F m^{2}**

Table 7Q4-1: Polarizability values for diamond, Si and Ge

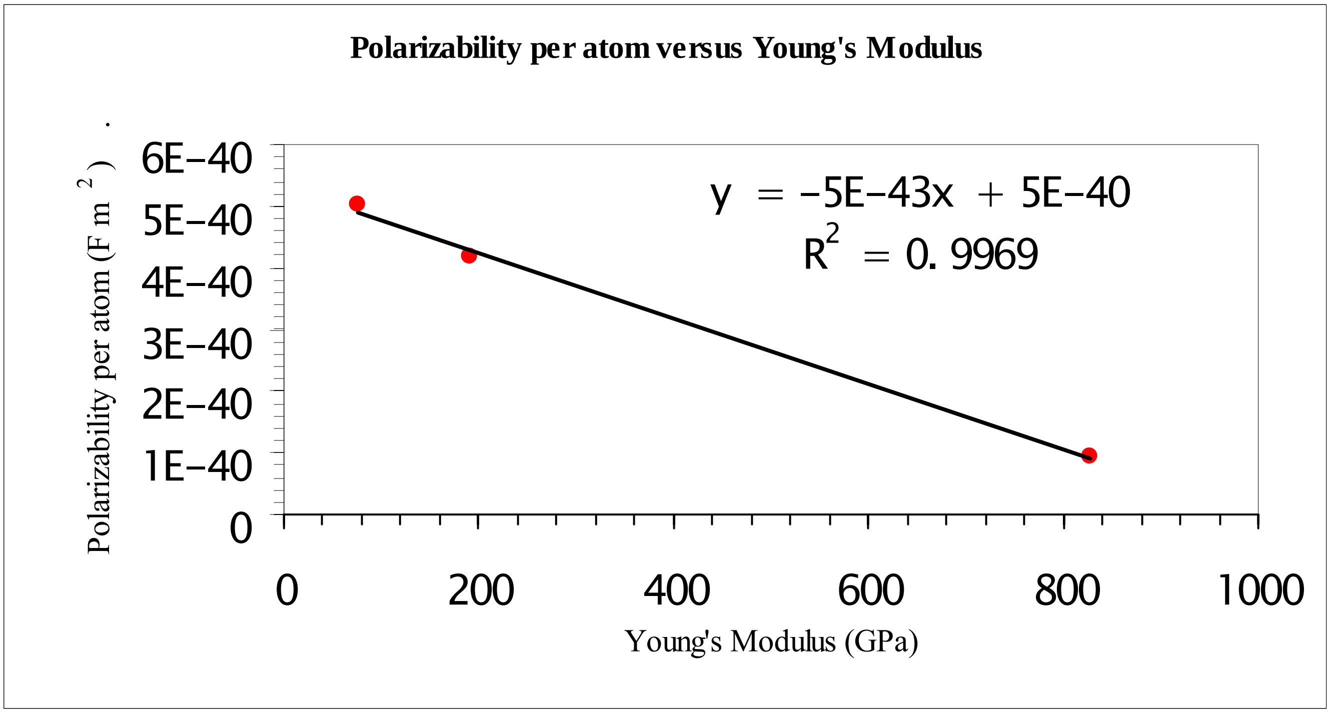

Figure 7Q4-1: Plot of polarizability per atom versus Young's modulus.

As the polarization mechanism in these crystals is due to electronic bond polarization, the displacement of electrons in the covalent bonds depends on the flexibility or elasticity of these bonds and hence also depends on the elastic modulus.

c.

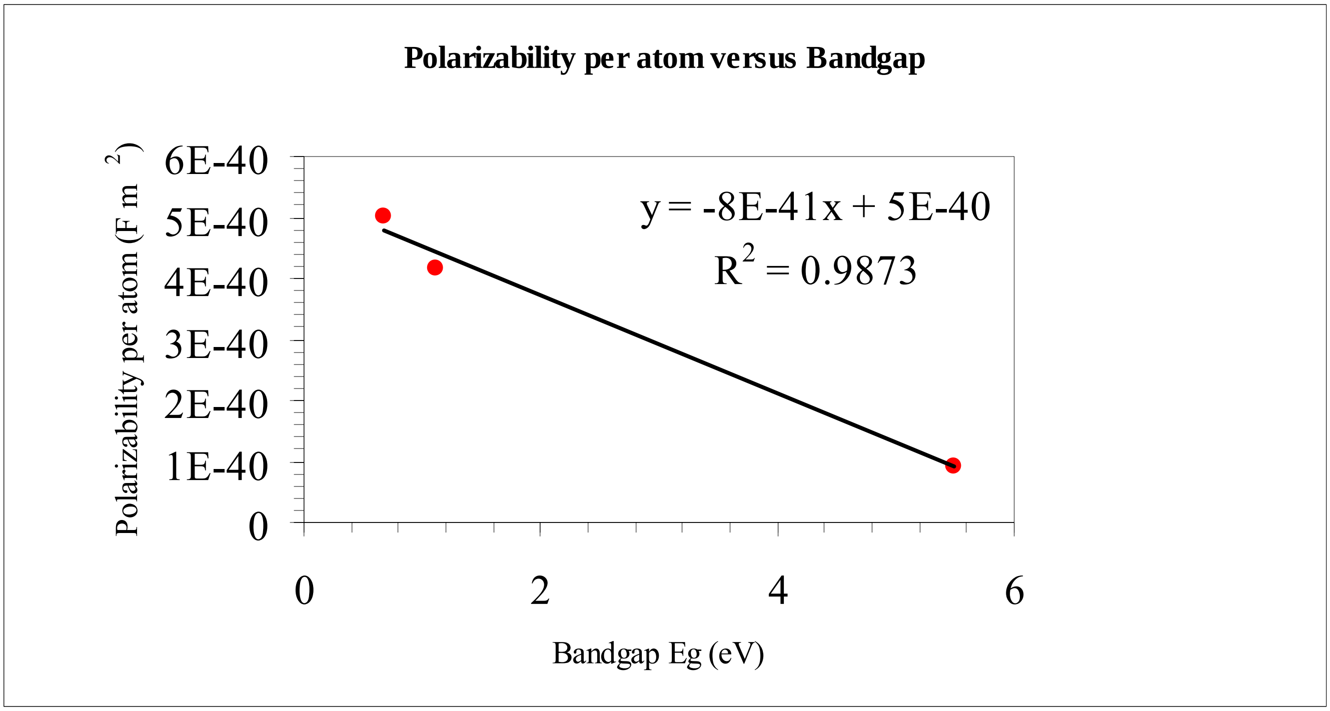

Figure 7Q4-2: Plot of polarizability versus bandgap energy.

There indeed seems to be a linear relationship between polarizability and bandgap energy.

d. To facilitate this proof, we can plot a graph of refractive index, *n*, versus relative permittivity, ε_r.

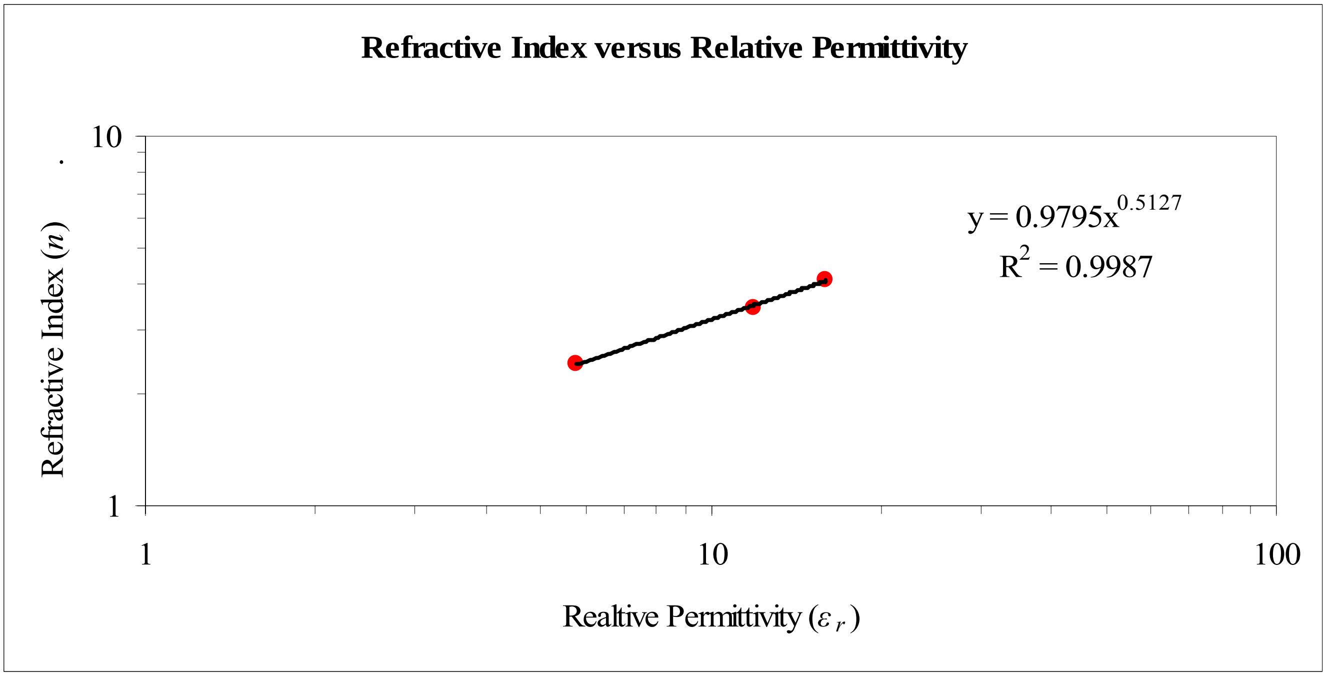

Figure 7Q4-3: Logarithmic plot of refractive index versus relative permittivity.

The log-log plot exhibits a straight line through the three points. The best fit line is $n = A\varepsilon_r^x$ (Correlation coefficient is 0.9987) where $x = 0.513 \approx 1/2$ and $A = \exp(0.02070) \approx 1$. Thus n is $\sqrt{\varepsilon_r}$.

The refractive index n is an optical property that represents the speed of a light wave, or an electromagnetic wave, through the material ($v = c/n$). The light wave is a high frequency electromagnetic wave where the frequency is of the order of 10^{14} to 10^{15} Hz (f_{optical}). n and polarizability (or ε_r) will be related if the polarization can follow the field oscillations at this frequency (f_{optical}). This will be the case in electronic polarization because electrons are light and rapidly respond to the fast oscillations of the field. The relationship between n and ε_r will not hold if we take ε_r at a low frequency ($<< f_{\text{optical}}$) where other slow polarization contributions (such as ionic polarization, dipolar polarization, interfacial polarization) also contribute to ε_r.

e. $n = \sqrt{\varepsilon_r}$ would apply to ionic crystals if ε_r is taken at the corresponding optical frequency rather than at frequencies below f_{optical}. Tabulated data for ionic crystals typically quote ε_r that includes ionic polarization and hence this data does NOT conform to $n = \sqrt{\varepsilon_r}$.

7.6 Dielectric constant of water vapor or steam The isolated water molecule has a permanent dipole p_o of 6.1×10^{-30} C m. The electronic polarizability α_e of the water molecule under dc conditions is about 4×10^{-40} C m. What is the dielectric constant of steam at a pressure of 10 atm (10×10^5 Pa) and at a temperature of 400 °C? [Note: The number of water molecules per unit volume N can be found from the simple gas law, $P = (N/N_A)RT$. The Clausius–Mossotti equation does not apply to orientational polarization. Since N is small, use Equation 7.14.]

Solution

Given the gas law,

$$P = \frac{NRT}{N_A}$$

where the gas constant, $R = 8.314\ \text{J.mol}^{-1}.\text{K}^{-1}$ and $N_A = 6.023 \times 10^{23}\ \text{mol}^{-1}$,

At $T = 673$ K and $P = 10\,\text{atm} = \dfrac{10\,\text{N}}{9.869 \times 10^{-6}\,\text{m}^2}$ we will have,

$$N = \frac{PN_A}{RT} = 1.0907 \times 10^{26}\,\text{m}^{-3}\ \text{of H}_2\text{O molecules.}$$

Since $p_0 = 6.10 \times 10^{-30}$ Cm and $k = 1.38 \times 10^{-23}$ J K^{-1}, the dipolar polarizability will be,

$$\alpha_d = \frac{p_o^2}{3kT} = 1.3345 \times 10^{-39}\ \text{Fm}^2$$

Therefore, given $\alpha_e = 4 \times 10^{-40}\,\text{Fm}^2$, from Eq 7.14

$$\varepsilon_r = 1 + \frac{N(\alpha_e + \alpha_d)}{\varepsilon_0} = 1.0214$$

∴ $\varepsilon_r(\text{steam}) = 1.0214$ at 10atm and $T = 673$ K.

7.8 Ionic and electronic polarization Consider a CsBr crystal that has the CsCl unit cell crystal structure (one Cs^+-Br^- pair per unit cell) with a lattice parameter (a) of 0.430 nm. The electronic polarizability of Cs^+ and Br^- ions are 3.35×10^{-40} F m^2 and 4.5×10^{-40} F m^2, respectively, and the mean ionic polarizability per ion pair is 5.8×10^{-40} F m^2. What is the low frequency dielectric constant and that at optical frequencies?

Solution

The CsBr structure has a lattice parameter given by a = 0.430 nm, and there is one CsBr ion pair per unit cell. If n is the number of ion pairs in the unit cell, the number of ion pairs, or individual ions, per unit volume (N) is

$$N = \frac{n}{a^3} = \frac{1}{\left(0.430 \times 10^{-9}\ \text{m}\right)^3} = 1.258 \times 10^{28}\ \text{m}^{-3}$$

At low frequencies both ionic and electronic polarizability contribute to the relative permittivity. Thus, from Equation 7.21, (where α_i is the mean ionic polarizability per ion pair, $\alpha_{e\text{Cs}}$ is the electronic polarizability of Cs^+ and $\alpha_{e\text{Br}}$ is the electronic polarizability of Br^-):

$$\frac{\varepsilon_{r(low)} - 1}{\varepsilon_{r(low)} + 2} = \frac{1}{3\varepsilon_o}\left(N\alpha_i + N\alpha_{e\text{Cs}} + N\alpha_{e\text{Br}}\right)$$

Remember that ($N\alpha_i + N\alpha_{e\text{Cs}} + N\alpha_{e\text{Br}}$) should be written as ($N_i\alpha_i + N_{\text{Cs}}\alpha_{e\text{Cs}} + N_{\text{Br}}\alpha_{e\text{Cl}}$), but since there is a one-to-one ratio between the number of molecules and ions in CsBr, we can take all the N's to be the same.

∴ $$\varepsilon_{r(low)} = \frac{1}{3\varepsilon_o}\left(N\alpha_i + N\alpha_{eCs} + N\alpha_{eBr}\right)\left(\varepsilon_{r(low)} + 2\right) + 1$$

∴ $$\varepsilon_{r(low)} - \frac{1}{3\varepsilon_o}\left(N\alpha_i + N\alpha_{eCs} + N\alpha_{eBr}\right)\varepsilon_{r(low)} = \frac{2}{3\varepsilon_o}\left(N\alpha_i + N\alpha_{eCs} + N\alpha_{eBr}\right) + 1$$

Isolate and simplify:

$$\varepsilon_{r(low)} = \frac{2N\left(\alpha_i + \alpha_{eCs} + \alpha_{eBr}\right) + 3\varepsilon_o}{3\varepsilon_o - N\left(\alpha_i + \alpha_{eCs} + \alpha_{eBr}\right)}$$

$$\varepsilon_{r(low)} = \frac{2\left(1.258 \times 10^{28}\ \text{m}^{-3}\right)\left(5.8 \times 10^{-40}\ \text{F m}^2 + 3.35 \times 10^{-40}\ \text{F m}^2 + 4.50 \times 10^{-40}\ \text{F m}^2\right) + 3\left(8.854 \times 10^{-12}\ \text{F/m}\right)}{3\left(8.854 \times 10^{-12}\ \text{F/m}\right) - \left(1.258 \times 10^{28}\ \text{m}^{-3}\right)\left(5.8 \times 10^{-40}\ \text{F m}^2 + 3.35 \times 10^{-40}\ \text{F m}^2 + 4.50 \times 10^{-40}\ \text{F m}^2\right)}$$

$\therefore \qquad \boldsymbol{\varepsilon_{r(low)} = 6.48}$

At optical frequencies there is no contribution from ionic polarization. We only consider electronic polarization of individual ions and therefore the relative permittivity at optical frequencies, $\varepsilon_{r(op)}$, is:

$$\varepsilon_{r(op)} = \frac{2(1.258\times10^{28}\text{ m}^{-3})(3.35\times10^{-40}\text{ F m}^2 + 4.50\times10^{-40}\text{ F m}^2) + 3(8.854\times10^{-12}\text{ F/m})}{3(8.854\times10^{-12}\text{ F/m}) - (1.258\times10^{28}\text{ m}^{-3})(3.35\times10^{-40}\text{ F m}^2 + 4.50\times10^{-40}\text{ F m}^2)}$$

$\therefore \qquad \boldsymbol{\varepsilon_{r(op)} = 2.77}$

7.9 Electronic and ionic polarization in KCl KCl has the same crystal structure as NaCl. KCL's lattice parameter is 0.629 nm. The ionic polarizability per ion pair (per K^+-Cl^- ion) is 4.58× 10^{-40} F m^2. The electronic polarizability of K^+ is 1.264 × 10^{-40} F m^2 and that of Cl^- is 3.408 × 10^{-40} F m^2. Calculate the dielectric constant under dc operation and at optical frequencies. Experimental values are 4.84 and 2.19.

Solution

The KCl structure has a lattice parameter given by a = 0.629 nm, and there are 4 KCl ion pairs per unit cell (see Table 1.3). The number of ion pairs, or individual ions, per unit volume (N) is therefore:

$$N = \frac{4}{a^3} = \frac{4}{(0.629\times10^{-9}\text{ m})^3} = 1.607\times10^{28}\text{ m}^{-3}$$

The electronic polarizability of the K^+ ion is given as $\alpha_{eK} = 1.264\times10^{-40}$ F m^2, and polarizability of the Cl^- ion is given as $\alpha_{eCl} = 3.408\times10^{-40}$ F m^2. From Equation 7.21, the relative permittivity at optical frequencies, $\varepsilon_{r(op)}$, can be found (see solution for question 7.5 for derivation):

$$\varepsilon_{r(op)} = \frac{2N(\alpha_{eK} + \alpha_{eCl}) + 3\varepsilon_o}{3\varepsilon_o - N(\alpha_{eK} + \alpha_{eCl})}$$

$$\therefore \qquad \varepsilon_{r(op)} = \frac{2(1.607\times10^{28}\text{ m}^{-3})(1.264\times10^{-40}\text{ F m}^2 + 3.408\times10^{-40}\text{ F m}^2) + 3(8.854\times10^{-12}\text{ F/m})}{3(8.854\times10^{-12}\text{ F/m}) - (1.607\times10^{28}\text{ m}^{-3})(1.264\times10^{-40}\text{ F m}^2 + 3.408\times10^{-40}\text{ F m}^2)}$$

$\therefore \qquad \boldsymbol{\varepsilon_{r(op)} = 2.18}$

This value is very close to the experimental value of 2.19.

7.13 **Student microwaves mashed potatoes** A microwave oven uses electromagnetic waves at 2.48 GHz to heat food by dielectric loss, that is, making use of ε_r'' of the food material, which normally has substantial water content. An undergraduate student microwaves 10 cm^3 of mashed potatoes in 60 seconds. The microwave generates an rms field of E_{rms} of 200 V cm^{-1} in mashed potatoes. At 2.48 GHz, mashed potatoes have ε_r'' = 21. Calculate the average power dissipated per cm^3, and also the total energy dissipated heating the food (Note: You can use E_{rms} instead of E in Equation 7.32.)

Solution

Using Eq7.32 and substituting Eq7.31 into it we have,

$$W_{\text{vol}} = \omega E^2 \varepsilon_0 \varepsilon_r^{"}$$

Substituting the relevant parameters into this equation we get

Average power,

$$W_{\text{vol}} = 2\pi \times 2.48 \times 10^9 \times 200^2 \times 8.854 \times 10^{-12} \times 21 = \mathbf{1.16 \times 10^3\ \ W\ cm^{-3}}$$

$$= \mathbf{1.16\ k\ W\ cm^{-3}}$$

Since, $\text{Energy} = \text{Power} \times \text{Time} \times \text{Area}$

Average $\text{Energy} = 1.1589 \times 10^3 \times 60 \times 10 = 6.9539 \times 10^5\,\text{J}$ or 700 kJ

7.14 Dielectric loss per unit capacitance Consider the three dielectric materials listed in Table 7Q8-1 with the real and imaginary dielectric constants, ε_r' and ε_r''. At a given voltage, which dielectric will have the lowest power dissipation per unit capacitance at 1 kHz and at an operating temperature of 50 °C? Is this also true at 120 °C?

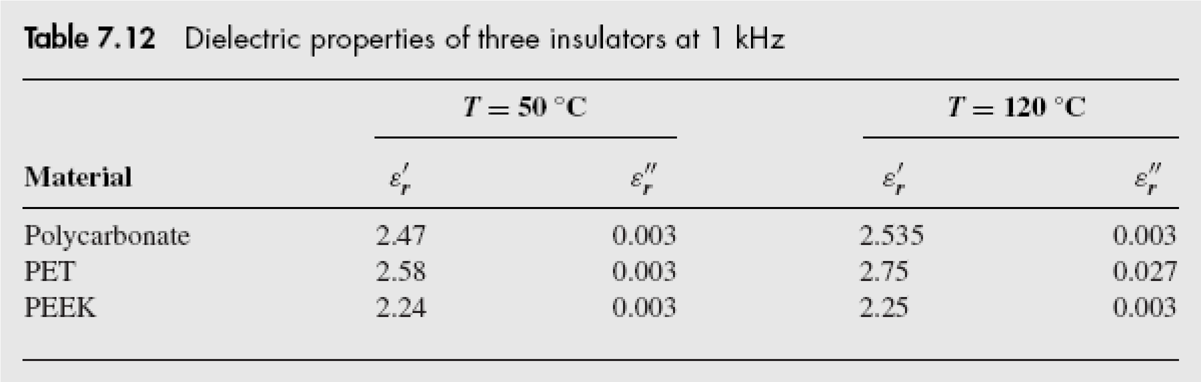

Table 7.12 Dielectric properties of three insulators at 1 kHz

	$T = 50\ ^\circ\text{C}$		$T = 120\ ^\circ\text{C}$	
Material	ε_r'	ε_r''	ε_r'	ε_r''
Polycarbonate	2.47	0.003	2.535	0.003
PET	2.58	0.003	2.75	0.027
PEEK	2.24	0.003	2.25	0.003

Solution

Since we are merely comparing values, assume voltage $V = 1$ V for calculation purposes. From example 7.5, the power dissipated per unit capacitance (W_{cap}) is given by:

$$W_{cap} = V^2 \omega \frac{\varepsilon_r''}{\varepsilon_r'}$$

where ω is the angular frequency ($2\pi f$) and ε_r' and ε_r'' represent the real and imaginary components of the relative permittivity ε_r, respectively. As a sample calculation, the power dissipated in polycarbonate is:

$$W_{cap} = (1\,\text{V})^2 [2\pi(1000\,\text{Hz})] \frac{(0.003)}{(2.47)} = \mathbf{7.63\ W/F}$$

Therefore, 7.63 W/F is dissipated at 50 °C at 1 V. The values for the other materials at both 50 °C and 120 °C are listed below in Table 7Q14-2:

	50 °C	120 °C
Material	**Power Dissipated (W per F)**	**Power Dissipated (W per F)**
Polycarbonate	**7.63**	**7.44**
PET	**7.31**	**61.7**
PEEK	**8.41**	**8.38**

Table 7Q14-2: Power dissipated at different temperatures for the given materials.

At 50 °C, all three are comparable in magnitude, but PET has the lowest power dissipation.

At 120 °C, polycarbonate has the lowest dissipation, while PET is almost ten times worse.

7.19 Dielectric breakdown of gases and Paschen curves Dielectric breakdown in gases typically involves the avalanche ionization of the gas molecules by energetic electrons accelerated by the applied field. The mean free path between collisions must be sufficiently long to allow the electrons to gain sufficient energy from the field to impact-ionize the gas molecules. The breakdown voltage, V_{br}, between two electrodes depends on the distance, d, between the electrodes as well as the gas pressure,

P, as shown in Figure 7.63. V_{br} versus Pd plots are called Paschen curves. We consider gaseous insulation, air and SF_6, in an HV switch.

a. What is the breakdown voltage between two electrodes of a switch separated by a 5 mm gap with air at 1 atm when the gaseous insulation is air and when it is SF_6?

b. What are the breakdown voltages in the two cases when the pressure is 10 times greater? What is your conclusion?

c. At what pressure is the breakdown voltage a minimum?

d. What air gap spacing d at 1 atm gives the minimum breakdown voltage?

e. What would be the reasons for preferring gaseous insulation over liquid or solid insulation?

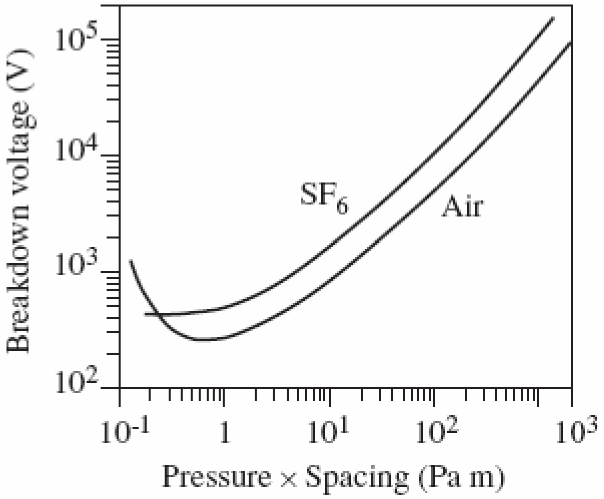

Figure 7.63 Breakdown voltage versus (pressure × electrode spacing) (Paschen curves).

Solution

a. At pressure P = 1 atm = 1.013×10^5 Pa and air gap d = 5 mm, $P \times d = (1.013 \times 10^5$ Pa$)(0.005$ m$) = 506.5$ Pa m. From Figure 7Q19-1, the corresponding values of breakdown voltage for air (V_{air}) and for SF_6 (V_{SF6}) are:

$\boldsymbol{V_{\text{air}}}$ **= 21000 V** or **21.0 kV**

$\boldsymbol{V_{\text{SF6}}}$ **= 50000 V** or **50.0 kV**

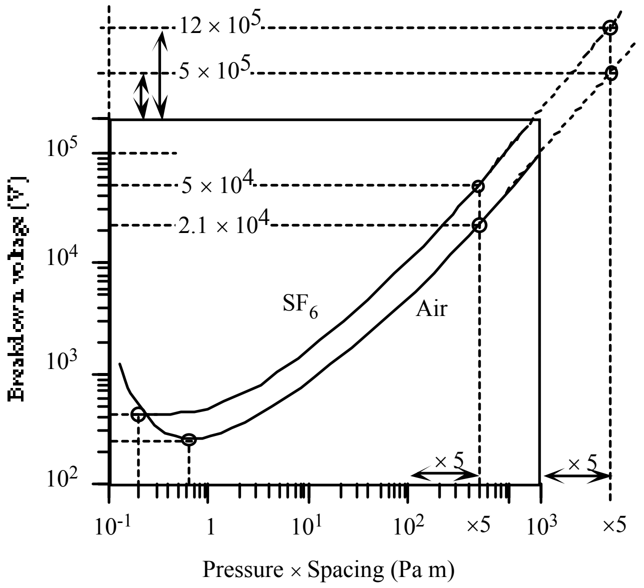

Figure 7Q19-1: Breakdown voltage versus (pressure × electrode spacing) (Paschen curves)

b. At $P = 10$ atm $= 1.013 \times 10^6$ Pa and $d = 5$ mm, $P \times d = (1.013 \times 10^6 \text{ Pa})(0.005 \text{ m}) = 5065$ Pa m. Using linear extrapolation on Figure 7Q19-1:

$\boldsymbol{V_{\text{air}} = 500000}$ **V** or **500 kV**

$\boldsymbol{V_{\text{SF6}} = 1200000}$ **V** or **1200 kV**

Pressure increases by 10 times, but the breakdown voltages increase by a factor of about 25 times - this is a good improvement.

c. With a gap length of 5 mm, we need to know the pressure at which the breakdown voltage is a minimum. From the graph, the minimum breakdown voltage of air is about $V_{\text{air}} = 250$ V, and the minimum for SF$_6$ is about $V_{\text{SF6}} = 420$ V. The corresponding values for $P \times d$ are $(P \times d)_{\text{air}} = 0.62$ Pa m and $(P \times d)_{\text{SF6}} = 0.2$ Pa m. From these we can determine the values of pressure needed for minimum breakdown voltage:

$$P_{\text{air}} d = 0.62 \text{ Pa m} \qquad P_{\text{SF6}} d = 0.2 \text{ Pa m}$$

$$\therefore \quad P_{\text{air}} = \frac{0.62 \text{ Pa m}}{0.005 \text{ m}} \qquad P_{\text{SF6}} = \frac{0.2 \text{ Pa m}}{0.005 \text{ m}}$$

$$\therefore \quad \boldsymbol{P_{\text{air}} = 124 \textbf{ Pa}} \text{ or } \boldsymbol{0.00122 \textbf{ atm}} \qquad \boldsymbol{P_{\text{SF6}} = 40.0 \textbf{ Pa}} \text{ or } \boldsymbol{0.000395 \textbf{ atm}}$$

A low pressure is needed for minimum breakdown which explains why discharge tubes operate at a low pressure.

d. At a set pressure $P = 1$ atm $= 1.013 \times 10^5$ Pa, the air gap spacing d for minimum breakdown voltage can be found in a similar manner to the one above, using the same values for $P \times d$:

$$Pd_{\text{air}} = 0.62\ \text{Pa m}$$

$$\therefore \qquad d_{\text{air}} = \frac{0.62\ \text{Pa m}}{1.013 \times 10^{5}\ \text{Pa}}$$

$$\therefore \qquad \boldsymbol{d_{air} = 6.12 \times 10^{-6}\ \text{m}}$$

This value corresponds to a breakdown voltage of 250 V. Therefore a gap of about 6 μm will only need 250 V for breakdown.

e. HV and high current switches or relays that have moving parts cannot be practically insulated using solid dielectrics. Liquid dielectrics are not as efficient as gaseous dielectrics because some undergo chemical changes under partial discharges. Further, they have a higher viscosity than gases that may affect the efficiency of the moving parts. Gas naturally permeates all the necessary space or locations where insulation is critical.

***7.20 Capacitor design** Consider a nonpolarized 100 nF capacitor design at 60 Hz operation. Note that there are three candidate dielectrics, as listed in Table 7.13.

a. Calculate the volume of the 100 nF capacitor for each dielectric, given that they are to be used under low voltages and each dielectric has its minimum fabrication thickness. Which one has the smallest volume?

b. How is the volume affected if the capacitor is to be used at a 500 V application and the maximum field in the dielectric must be a factor of 2 less than the dielectric strength? Which one has the smallest volume?

c. At a 500 V application, what is the power dissipated in each capacitor at 60 Hz operation? Which one has the lowest dissipation?

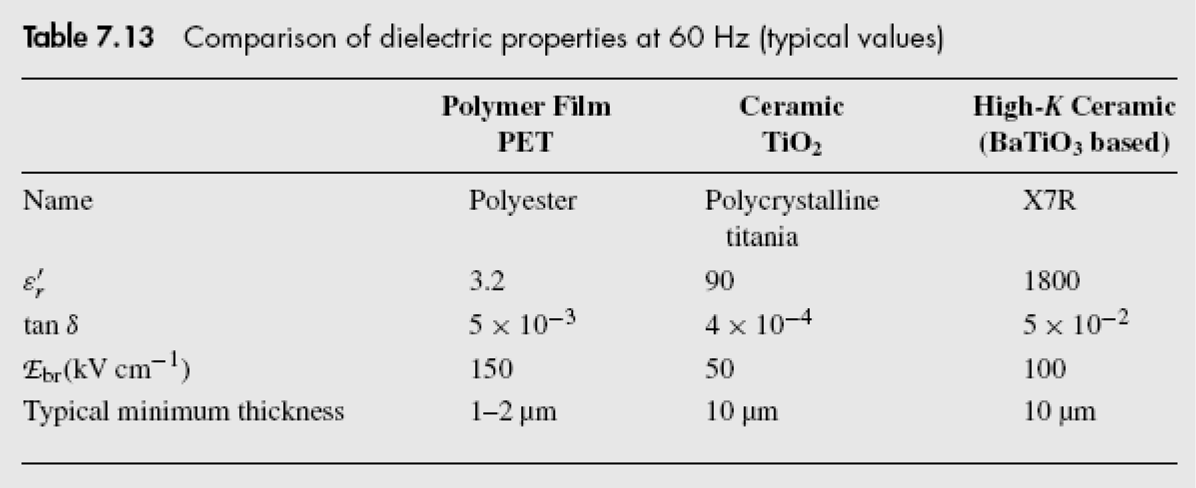

Table 7.13 Comparison of dielectric properties at 60 Hz (typical values)

	Polymer Film PET	Ceramic TiO_2	High-K Ceramic ($BaTiO_3$ based)
Name	Polyester	Polycrystalline titania	X7R
ε_r'	3.2	90	1800
tan δ	5×10^{-3}	4×10^{-4}	5×10^{-2}
$\mathcal{E}_{\text{br}}$(kV cm^{-1})	150	50	100
Typical minimum thickness	1–2 μm	10 μm	10 μm

Solution

Note: All sample calculations are for Polymer film (PET). All methods of calculation for the other materials are identical, and the obtained values are summarized in Table 7Q20-1.

a. To find the volume needed for C = 100 nF given that the dielectric has the minimum practical thickness, d (Table 7.13), find the capacitance per unit volume (C_{vol}):

$$C_{\text{vol}} = \frac{\varepsilon_o \varepsilon_r}{d^2} = \frac{(8.854\times10^{-12}\ \text{F/m})(3.2)}{(1\times10^{-6}\ \text{m})^2} = 28.33\ \text{F/m}^3$$

The volume V can now be found as follows:

$$\boldsymbol{V} = C / C_{\text{vol}} = (100\times10^{-9}\ \text{F}) / (28.33\ \text{F/m}^3) = \mathbf{3.53\times10^{-9}\ m^3}$$

This is the volume at low voltage operation based on the minimum practical thickness.

b. Suppose that d is the minimum thickness (in m) which gives a maximum field of half of E_{br} at 500 V. Then:

∴ $$d = 2\frac{V_{\max}}{E_{br}} = 2\left(\frac{500\ \text{V}}{1.50\times10^{7}\ \text{V/m}}\right) = 6.667\times10^{-5}\ \text{m}$$

Now the capacitance per unit volume can be found:

$$C_{\text{vol}} = \frac{\varepsilon_o \varepsilon_r}{d^2} = \frac{(8.854\times10^{-12}\ \text{F/m})(3.2)}{(6.667\times10^{-5}\ \text{m})^2} = 0.006374\ \text{F/m}^3$$

∴ $$\boldsymbol{V} = C / C_{\text{vol}} = (100\times10^{-9}\ \text{F}) / (0.006374\ \text{F/m}^3) = \mathbf{1.57\times10^{-5}\ m^3}$$

This is the dielectric volume at 500 V.

c. The power dissipation in the capacitor at 500 V (60 Hz operation) can be found by first obtaining the power lost per unit volume W_{vol}. It is given by:

$$W_{\text{vol}} = \frac{E_{br}^{\ 2}}{\eta^2}\omega\varepsilon_o\varepsilon_r' \tan\delta$$

where η is the safety factor (assumed to be equal to 2) and $\omega = 2\pi f$ is the angular frequency. Evaluating:

$$W_{\text{vol}} = \frac{(1.5\times10^{7}\ \text{V/m})^2}{(2)^2}(2\pi(60\ \text{Hz}))(8.854\times10^{-12}\ \text{F/m})(3.2)(5\times10^{-3})$$

∴ $$W_{\text{vol}} = 3004\ \text{W/m}^3$$

The power dissipated (W) is therefore:

$$W = W_{\text{vol}}V = (3004\ \text{W/m}^3)(1.57 \times 10^{-5}\ \text{m}^3)$$

$\therefore$ $\boldsymbol{W = 0.0472\ \text{W}}$

		Polymer Film PET	Ceramic TiO_2	High-*K* Ceramic ($BaTiO_3$ based)
A	Low voltage volume (m^3)	**3.53×10^{-9}**	**1.25×10^{-8}**	**6.27×10^{-10}**
B	High voltage volume (m^3)	**1.57×10^{-5}**	**5.02×10^{-6}**	**6.27×10^{-8}**
C	Power dissipated (W)	**0.0472**	**0.00377**	**0.471**

Table 7Q20-1: Summarized values for volume and power of given capacitors.

Upon inspection we see that for part ***a*** and part ***b***, high-*K* ceramic has the smallest volumes, and for part ***c***, ceramic has the lowest power dissipation.