Impact of Energy Storage Systems on Electricity Market Equilibrium

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Abstract—Integration of large-scale energy storage systems (ESSs) is desirable nowadays to achieve higher reliability and efficiency for smart grids. Controlling ESS operation usually depends on electricity market prices so as to charge when the price is low and discharge when the price is high. On the other hand, the market-clearing price itself is determined based on the net demand, i.e., including energy storage output, at every hour. Therefore, it is crucial to develop a mathematical model to determine the optimal ESS operation as well as the market-clearing prices. The problem is formulated as a mixed complementarity problem (MCP) that allows the representation of special (incentive) prices, which cannot be represented in a single optimization model. The proposed model is useful for power system operators to determine the optimal storage dispatch simultaneously with the market-clearing price in addition to the conventional generation dispatch. The impact of energy storage size and location on market price, total generation cost, energy storage arbitrage benefit, and total consumer payment is further investigated in this paper. The latter analysis provides some guidelines for power system planners to identify the optimal size and location for installing large-scale ESSs.

Index Terms—Energy storage systems; smart grids; mixed complementarity problem

I. NOMENCLATURE

Sets and indices

\( \mathcal{G} \) Set of system buses equipped with generators

\( \mathcal{E} \) Set of system buses equipped with energy storage

\( i, j \) System bus indices

\( hr \) Hour index

Variables

\( P_{G, i, hr} \) Generated active power at bus \( i \) during hour \( hr \)

\( Q_{G, i, hr} \) Generated reactive power at bus \( i \) during hour \( hr \)

\( P^{ch}_{ESS, i, hr} \) Energy storage charging active power at bus \( i \) during hour \( hr \)

\( P^{dis}_{ESS, i, hr} \) Energy storage discharging active power at bus \( i \) during hour \( hr \)

\( E_{ESS, i, hr} \) Total energy stored at bus \( i \) at hour \( hr \)

\( P_{flow, i, j, hr} \) Power flow of the line between buses \( i \) and \( j \) during hour \( hr \)

\( V_{i, hr} \) Voltage magnitude of bus \( i \) at hour \( hr \)

\( \delta_{i, hr} \) Phase angle of bus \( i \) at hour \( hr \)

\( \delta_{ij, hr} \) Difference in phase angles of buses \( i \) and \( j \) at hour \( hr \), i.e., \( \delta_{i, hr} - \delta_{j, hr} \)

Parameters

\( N \) Total number of system buses

\( \alpha_i, \beta_i \) Generation cost function parameters at bus \( i \) and \( \gamma_i \)

\( C_M \) Energy storage operation and maintenance cost

\( P_D, i, hr \) Demand active power at bus \( i \) during hour \( hr \)

\( Q_D, i, hr \) Demand reactive power at bus \( i \) during hour \( hr \)

\( P^{\text{max}}_{\text{flow}, i, j} \) Rated power flow of the line between buses \( i \) and \( j \)

\( \eta^{ch}_{ESS} \) Energy storage charging efficiency

\( \eta^{dis}_{ESS} \) Energy storage discharging efficiency

\( P_{ESS, i, \text{max}} \) Rated energy storage active power at bus \( i \)

\( E_{ESS, i, \text{max}} \) Rated energy storage capacity at bus \( i \)

\( P_{G, i, \text{min}} \) Minimum generated active power at bus \( i \)

\( P_{G, i, \text{max}} \) Maximum generated active power at bus \( i \)

\( Q_{G, i, \text{min}} \) Minimum generated reactive power at bus \( i \)

\( Q_{G, i, \text{max}} \) Maximum generated reactive power at bus \( i \)

\( V_{\text{min}} \) Minimum voltage at each bus

\( V_{\text{max}} \) Maximum voltage at each bus

\( G_{ij} \) Real part of bus admittance matrix element \((i,j)\)

\( B_{ij} \) Imaginary part of bus admittance matrix element \((i,j)\)

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II. INTRODUCTION

CLASSICAL power systems imply unidirectional flow of energy from generating stations to distribution systems through transmission and sub-transmission networks. This scenario has been changed after allowing for distributed generation to be connected at distribution systems, and thus leading to bidirectional power flow within distribution networks. As a result, distribution systems become active and hence they are referred to as "active distribution networks". Further, renewable portfolio standards place obligations to have certain penetration percentage from renewable energy sources (RESs), e.g., state of California requires that 33% of the supply mix should be provided from RESs by 2020 [1]. From another point of view, customers become interested to participate in electricity market operations to reduce their energy bills. This is referred to as "load management or demand response". Load management can be achieved by implementing two-way communication infrastructure between utilities and customers [2]. Consequently, it is crucial that the distribution system be operated in a fundamentally different manner. One of the initiatives that have been proposed to manage this new environment is the "smart grid" concept. "Smart grid" can be defined from different perspectives; it is the two-way flow of electricity and information between supply and demand. Another definition is the application of intelligent devices and communication technologies in power systems [2]. Generally, smart grids intend to facilitate integration of RESs and to satisfy the needs of higher system reliability and efficiency.

An energy storage system (ESS) is one of the promising technologies that will enable smart grid concepts. ESSs can support RESs integration by reducing the fluctuations associated with their power production. ESSs can further provide some ancillary services, e.g., load following, spinning reserve, black start capability, etc., and they can also be deployed to improve system reliability. Moreover, ESSs can be used to shave peak loads to avoid the high cost associated with peak pick-up generators. Peak load shaving implies power exchange between base generation units and storage devices in order to store some power during off-peak periods and discharge them during peak load periods. This practice increases the capacity factor of base generation units. This is in addition to the fact that off-peak power can be stored or bought with a low price, and then discharged or sold with a high price. This price difference, usually called "arbitrage benefit", is beneficial for an ESS owner, e.g., a utility or a customer driven ESS [3]. In Canada, the first battery-energy storage projects have been recently implemented by British Columbia (BC) Hydro and Toronto Hydro with ratings of 2 MW and 500 kW-250 kWh, respectively [4, 5].

Several research works have addressed the problem of determining the optimal operation strategy of ESSs, i.e., when and how much power is to be charged or discharged, as in [6] [7] [8]. In these works, the storage devices were modeled as "price-taker" firms knowing the electricity price at each hour for some period ahead. Such modeling is based on the assumption that storage capacities are too small to affect the market price. However, the increased potential of adopting large central storage facilities in power systems would contradict that assumption.

Mathematical models were further developed in [9] to find the optimal capacity and the optimal dispatch of a hybrid system comprised of generation and storage facilities. This was achieved by minimizing the annual capital and operation costs of the system. The impact of adopting large-scale energy storage on system price has been also investigated, but since the developed models did not represent the power network, the impact of storage location was not considered. In [10], the authors evaluated the arbitrage benefit of small-size energy storage by optimizing its operation during two weeks period and assuming perfect market price foresight during that period. The impact of large-scale energy storage was further discussed by representing the price as an econometrically estimated non-decreasing function of the net power demand including storage output, aggregated over the whole network. That representation does not actually optimize the market price at every hour nor is the storage location represented. Nevertheless, the results showed lower and higher prices during peak and off-peak periods, respectively. As a result, the arbitrage value was found to be 10-20% less than that calculated considering constant prices at every hour. The results further revealed an increase in consumer surplus which was larger than the decrease in producer surplus, and thus resulting in a net increase in the social welfare. Similarly, the authors in [11] investigated the impact of large-scale energy storage operation on the wholesale electricity price. However, no mathematical models were described for determining either the optimal operation strategy of the storage units or the market-clearing price. The impact on electricity price was instead approximated by observing the effect of storage units on the generation cost of marginal conventional plants. Like [9] [10], [11] did not model the power network, so locational effects were not considered.

From the aforementioned literature survey, it can be concluded that sufficient work has addressed the problem of determining the optimal operation strategy of ESSs assuming forecasted market prices for some period ahead. The impact of large-scale energy storage operation on market prices and generation costs has been further investigated assuming either a historical ESS operation as in [11] or a price function of the net power demand, without modeling the network. As in [10]. These two problems of determining the optimal energy storage dispatch and the market-clearing price were de-coupled to mitigate the complexity of the analysis. Therefore, this paper aims to develop a mathematical model for determining not only the optimal charging/discharging strategy of ESSs, but the optimal market price as well in a perfectly competitive environment, with a physical model for the power network. To the best of the author’s knowledge, this problem has not been tackled yet in the literature; however, it is very useful for

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1 In this paper, the term dispatch is used for determining the optimal ESS operation in day-ahead markets, in which unit commitment is usually performed, contrary to the usual meaning of dispatch that is typically conducted in short-time periods ahead of the actual operation.
power system operators dealing with large-scale ESSs in their networks. The impact of energy storage size and location on market price, total generation cost and benefit, arbitrage benefit, and total consumer payment is further investigated in this paper. The latter analysis provides some guidelines for power system planners to allocate large-scale ESSs.

ESSs are very expensive in capital, operation, and maintenance costs. Therefore, governments may consider special incentives for ESSs in order to encourage private investment. For some forms of special incentives, it is impossible to represent market equilibrium as a single optimization model, e.g., if an ESS is to receive a fixed multiple (greater than one) of the ordinary market price for its discharged output [12]. In such cases, however, a mixed complementarity problem (MCP) formulation can be used. In this paper, we consider one such special incentive whereby the ESS would receive or pay the highest locational marginal price (LMP) among all the buses in the system, and we formulate market equilibrium as an MCP.

The main contributions of this paper can be summarized as follows:

- This paper presents a novel mathematical model for determining the optimal operation strategy of an ESS and the market-clearing price simultaneously in a perfectly competitive environment, even if special incentives make it impossible to formulate a single optimization model.
- The model proposed is used to address the impact of the ESS location and size on electricity market equilibrium.

The rest of this paper is organized in four sections. The problem formulation is derived in Section III. Section IV presents the system under study. Several case studies and simulation results are then discussed in Section V. Finally, Section VI summarizes the conclusions of this research work.

### III. PROBLEM FORMULATION

The rationale behind this research work is to solve two dependent problems, i.e., social welfare maximization, given the energy storage operational decisions, and energy storage arbitrage benefit maximization, given the highest LMP, during charging and discharging, as a special incentive for the ESS. Therefore, the main focus of this paper is to propose a joint optimization strategy that simultaneously solves the two dependent problems, together with a third optimization model that determines the highest LMP. Before discussing how to solve the whole model as an MCP, the three optimization models are defined in the following paragraphs. The MCP is outlined at the end of this section, and it appears in detail in Appendix A.

The single auction market structure is considered in this paper in which the electricity price is settled based on the net demands at buses, i.e., after accounting for ESS charging and discharging, and the bids offered by the generators. Social welfare maximization under such a structure is achieved by minimizing the total generation cost function as follows [13] [14]

\[
\minimize F_1 = \sum_{hr=1}^{24} \sum_{i \in G} \alpha_i P_{G_{i,hr}}^2 + \beta_i P_{G_{i,hr}} + \gamma_i
\]

It is worth mentioning that the above cost function neglects startup costs since the representation of these costs requires binary variables, which cannot be considered in the MCP formulation used in this paper. Therefore, we assume that unit commitment decisions have already been made in an earlier stage, and the committed generators are only considered in \( F_1 \).

When deciding on unit commitments, the system operator would require an estimate of the charging and discharging schedule of the ESS, as the ESS operation could affect the need for some generation units. An estimate of the ESS schedule could be based on experience, to start, and the estimate could be updated after running the MCP model described in this paper, in an iterative process. In this paper, we focus on the MCP model only, and leave the extension to the unit commitment process for future research.

Moreover, if the power rating of the ESS is less than or equal to the capacity of the smallest generator (i.e., minimum spinning reserve), which is usually the case since the capacities of ESS are small compared to those of bulky generators, the ESS would be charged without the need for starting up another offline generator. Consequently, considering the startup costs would not affect the storage scheduling under the above condition. The amount of spinning reserve would be further unchanged, if we neglected the losses in the ESS, since a part of the reserve would be stored in the ESS in this case [15].

Nevertheless, \( F_1 \) is minimized under system constraints such as power flow constraints, generation limits, and voltage limits, as given in (2-9). Note that the variables for ESS charging and discharging are treated as parameters in this optimization model; dual variables are in parentheses.

\[
P_{G_{i,hr}} - P_{D_{i,hr}} = P_{G_{i,hr}}^{ch} + P_{G_{i,hr}}^{ch} = \sum_{j=1}^{N} V_{i,hr} \times V_{j,hr} \]

\[
(G_{ij} \sin \delta_{ij,hr} + B_{ij} \cos \delta_{ij,hr}) \times (\lambda_{i,hr}) \quad \forall i, hr
\]

\[
Q_{G_{i,hr}} - Q_{D_{i,hr}} = \sum_{j=1}^{N} V_{i,hr} \times V_{j,hr} \]

\[
(G_{ij} \sin \delta_{ij,hr} - B_{ij} \cos \delta_{ij,hr}) \times (\mu_{i,hr}) \quad \forall i, hr
\]

\[
P_{flow_{i,j,hr}} = V_{i,hr} \times (G_{ij} \sin \delta_{ij,hr} + B_{ij} \cos \delta_{ij,hr}) \]

\[
+ B_{ij} \times V_{j,hr} \sin \delta_{ij,hr} \leq P_{max} \times V_{j,hr} \]

\[
P_{G_{i,hr}} \leq P_{G_{i,hr}} \leq P_{G_{i,hr}}^{max} \quad \forall i \in G, hr
\]

\[
Q_{G_{i,hr}} \leq Q_{G_{i,hr}} \leq Q_{G_{i,hr}}^{max} \quad \forall i \in G, hr
\]

\[
V_{min} \leq V_{hr} \leq V_{max} \quad \forall i \in G, hr
\]

\[
V_{i,hr} = \text{constant} \quad \forall i \in G, hr
\]

\[
\pi \leq \delta_{ij,hr} \leq \pi
\]
In this work, all generators and customers receive and pay the LMPs at every hour, i.e., the bus dual variables \( \lambda_{i,hr} \), respectively. However, the hourly electricity price \( \rho_{hr} \) that ESSs pay or receive for the energy stored or delivered, respectively, is set to be the highest bus dual variable as in (10), as motivated by [13] [14]

\[
\rho_{hr} = \max_i \left( \lambda_{i,hr} \right) \quad \forall hr
\] (10)

Taking the highest bus dual variable is justified as an incentive for promoting energy storage implementation. There tends to be much smaller differences among LMPs during off-peak, low-price, times when the ESS would charge, and so the “highest price” is close to all LMPs. The special incentive occurs when the “highest price” is very high during peak periods, and the ESS sells its discharged output. Applying this proposed incentive requires that the ESS pays the highest LMP during off-peak times, while the power authority compensates the ESS with the difference between its LMP and the highest LMP in the system during peak periods. This proposed incentive is basically one of many other possible incentives that may be offered to the storage owners. Other incentives could be, for example, to use a weighted average of the LMP at the ESS connection, and the highest LMP, or to use the LMP, but the government pays a constant subsidy as an added amount on the selling price during discharging.

It is worthwhile to mention that if the LMP corresponding to the ESS location were used instead, then in the case study of Sections IV and V, the energy storage would not be dispatched unless the ESS is located at the bus corresponding to the highest market-clearing price. This phenomenon is attributed to the fact that the difference between off-peak and peak price should be large enough in order to justify the ESS operation and maintenance cost, and thus to economically operate the ESS. This condition is not satisfied in the case when using the LMP. However, this conclusion depends on the system data and parameters. In other words, utilizing the LMP might lead to an economic dispatch with other systems of different data.

Equation (10) is implemented via a mathematical device that fits into the framework for developing an MCP: the system operator has a second objective (in addition to social welfare maximization) to minimize the electricity price \( \rho_{hr} \) at every hour, subject to the constraint that \( \rho_{hr} \) exceeds or equals the dual variable at every bus as follows (note that \( \lambda_{i,hr} \) are treated as parameters in this optimization model):

\[
\text{minimize } F_2 = \sum_{hr=1}^{24} \rho_{hr}
\] (11)

Subject to

\[
\rho_{hr} \geq \lambda_{i,hr} \quad \left( m_{i,hr} \right) \quad \forall i, hr
\] (12)

The arbitrage benefit maximization problem associated with the energy storage is mathematically represented as minimizing the following objective function, where the electricity prices at every hour are treated as parameters in this model:

\[
\text{minimize } F_3 = \sum_{hr=1}^{24} \left( \sum_{i \in \mathcal{E}} \left( P^{ch}_{ESS,i,hr} + P^{dis}_{ESS,i,hr} \right) \times C_M - \rho_{hr} \right)
\] (13)

In the above objective function, the first term represents the direct operation and maintenance cost associated with the ESS due to hourly charging and discharging, while the second term represents the arbitrage value. Indirect costs of losses are represented implicitly in (14) below, through the charging and discharging efficiency parameters. This formulation thus prevents dispatching the ESS for a zero arbitrage benefit since the arbitrage benefit should be at least equal to the cost of operating and maintaining the ESS. \( F_3 \) is to be minimized under the energy storage constraints as given in (14-17)

\[
E_{ESS,i,hr} = E_{ESS,i,hr-1} + \eta_{i,hr}^{ch} \times P^{ch}_{ESS,i,hr} - \eta_{i,hr}^{dis} \times P^{dis}_{ESS,i,hr}
\] (14)

\[
P^{dis}_{ESS,i,hr} / \eta_{i,hr}^{dis} \leq P^{dis}_{ESS,i,hr} \quad \forall i \in \mathcal{E}, hr
\] (15)

\[
P^{dis}_{ESS,i,hr} \leq P^{dis}_{i,hr,\text{rated}} \quad \forall i \in \mathcal{E}, hr
\] (16)

\[
P^{dis}_{ESS,i,hr} \leq P^{dis}_{i,hr,\text{rated}} \quad \forall i \in \mathcal{E}, hr
\] (17)

If the ESS was to pay and receive the LMP at its bus \( \lambda_{i,hr} \) instead of \( \rho_{hr} \), then the optimal power flow (OPF) problem (1-9) could be combined with (14-17) in a single OPF that minimizes generation costs in (1) plus ESS costs in the first term of (13). However, such a formulation cannot be used to determine the optimal dispatch of the ESS with the special (incentive) price in (10). In other words, conventional market models cannot take into consideration the formulations introduced in (11-17) while solving for the 24-hour OPF problem (1-9). Therefore, we propose a novel method that determines the optimal operation strategy of ESS and the special price that ESS will pay or receive at each hour, in addition to the conventional dispatch variables, i.e., the generation power levels at each hour. As a result, the main challenge of this work is to solve the aforementioned three dependent problems together.

In order to solve the three dependent problems simultaneously, for equilibrium, they are formulated as an MCP. In such a formulation, the KKT conditions of all optimization problems are combined together to form a square set of equations, inequalities, and complementarity conditions, as given in Appendix A.

However, any solution to those KKT conditions is not guaranteed to give the optimal solutions to the three problems unless certain sufficiency conditions are met, e.g., the objective function is a continuously differentiable convex function, and the constraints constitute a convex set [12]. By examining the three problems under study, we can conclude that the objective functions are convex functions since the first one is a quadratic function with non-negative parameters, while the second and third objectives are linear. However, all constraints are linear functions, i.e., convex, except the power
flow equations which are non-convex functions in most cases [16]. Consequently, the obtained solutions are only guaranteed to be local optima.

IV. SYSTEM UNDER STUDY

In all case studies, the system under study consists of a 6-bus transmission system and a 3-bus distribution system, as shown in Fig. 1 [13]. The distribution system is interconnected to the transmission system through a 100-MVA transformer. Two generating stations (G1 and G2) are connected at buses 1 and 3; their cost function parameters, generation limits, and daily peak demand are given in [13]. This system is simple enough to be solved in a short time, i.e., nearly one minute on a personal computer. Therefore, the model can be run several times, in a good timely manner, in order to provide a sensitivity analysis for the impact of the ESS location and size on market equilibrium. However, the IEEE 14 and 30 bus systems are also used in the first case study in order to demonstrate the performance of the model developed in solving larger real systems. The data of the IEEE 14 and 30 bus systems can be found in [17].

The demand profile at each node is assumed to follow the IEEE-RTS summer load profile in [18] which provides the hourly load magnitude as a percentage of the daily peak demand. However in real case studies, one day ahead forecasted load demand profile as in [19] can be utilized in the developed model to set the market price and determine the optimal operation of allocated ESSs. On the other hand, the size and the location of the energy storage will be varied as will be discussed later on in section V. The ESS charging and discharging efficiencies are assumed to be 90% (i.e., 81% round trip efficiency) in all case studies [9]. The operation and maintenance cost of ESS ($C_{OM}$) is assumed to be 0.6 cents/kWh [20].

V. SIMULATION RESULTS

The aforementioned MCP is solved by means of the PATH solver in the General Algebraic Modeling System (GAMS) environment. Several case studies are presented in this section; the first one shows the significance of the proposed MCP model to power system operators, i.e., to determine the optimal ESS dispatch and the highest market-clearing price in a perfectly competitive market. Afterwards, other case studies are discussed to show the impact of ESS size and location on the highest market-clearing price, total generation cost and benefit, energy storage arbitrage benefit, and total consumer payments. The simulation results are discussed in the next subsections. For all case studies, we report only the highest market-clearing price ($\rho_0$), but not all nine LMPs, to save space.

Case study 1: impact of energy storage operation

Basically, any energy storage consists of an energy reservoir and a power conversion system (PCS). Therefore, the ESS size is defined by two parameters: rated charging/discharging rate (in MW) and rated storage capacity (in number of discharge hours at rated discharging rate), which correspond to PCS and energy reservoir sizes, respectively.

In this case study, the 9 bus system is utilized first, and it is assumed that one ESS is allocated at the distribution system (bus #7) with a rating of 100 MW and 2 hours discharge, i.e., $P_{ESS_{new}} = 100$ MW, and $E_{ESS_{new}} = 200$ MWh. The proposed MCP model is utilized to determine the optimal ESS dispatch and the highest market-clearing price for one day ahead.

The highest market-clearing price is presented in Fig. 2 with and without the ESS integration to the system. With the presence of the ESS, the electricity prices become almost leveled during off-peak and peak periods. Furthermore, it is clear that the prices are increased during off-peak times, while they are reduced during peak periods. This is because of the optimal ESS charging and discharging during off-peak and peak times, respectively, as shown in Fig. 3, which shows the ESS output power as a percentage of the rated charging/discharging rate (i.e., 100 MW in this case study).

Moreover, the model proposed is used to solve the IEEE 14 and 30 bus systems in order to assess the increase in computational time as the model grows in size. The run times corresponding to the systems used in this paper are summarized in Table I. Although it seems that the model developed could scale up to solve larger systems, we would like to mention that for larger real-world systems, faster computer equipment and possibly specialized algorithms such as decomposition, as discussed at chapter 9 in [12], might be needed, to be solvable within a reasonable amount of time for day-ahead markets.

Fig. 1. System under study

Fig. 2. Highest market-clearing price with and without ESS integration
Case study 2: impact of energy storage size

This case study investigates the impact of energy storage size on the highest market-clearing price, total generation cost and benefit, arbitrage benefit, and total consumer payments. Like the previous case study, one ESS is assumed connected to bus #7. Firstly, the PCS size is fixed at 40 MW, while the reservoir size is varied between 2-6 discharge hours in steps. Afterwards, the PCS size is changed in steps between 40-80 MW, while the reservoir size is kept constant at 2 discharge hours.

Fig. 4 shows the highest market-clearing prices for the different energy reservoir sizes compared to the case of no energy storage in the system. With the increase of energy storage capacity, the ESS can exchange more power with the generating stations during the day. Therefore, the market prices are settled at higher prices during low demand periods and at lower prices, on the other hand, when the demand is high. Similar results are obtained when the PCS size is increased from 40 MW to 80 MW, as shown in Fig. 5.

The total generation cost profiles are presented in Fig. 6 and Fig. 7 for the different ESS sizes adopted in this case study. Fig. 6 reveals that up to 20% higher generation costs are incurred during off-peak periods, while generation costs can be reduced by 10% during peak times.

With respect to the energy storage arbitrage benefit (in Fig. 8), it is clear that the arbitrage benefit increases with an increment of the energy storage size up to a certain size. The arbitrage benefit then becomes steady at the maximum value since the optimal ESS operation does not change beyond this point. The maximum arbitrage benefit depicted is almost $3,100 per day, which can be achieved with different combinations between the PCS and reservoir sizes. For example, 80MW - 4 hours discharge and 40 MW - 6 hours discharge ESSs result in the same (maximum) arbitrage benefit. Therefore, the ESS owner needs to calculate the economic size to be installed that maximizes the present value of the arbitrage benefit over the lifetime of the ESS, minus the installation cost.

Moreover, Fig. 9 and Fig. 10 show total consumer payments and total generation benefit for the different ESS sizes, respectively, where total generation benefit is calculated by subtracting total generation cost from total generation revenue. It can be noticed that consumer payments and generation benefits are reduced with the increase of ESS size.
Case study 3: impact of energy storage location

The impact of energy storage installation location is analyzed in this case study. Two possible locations are considered as follows: bus #5 (in the transmission system) and bus #8 (in the distribution system). In this case study, the size of ESS installed is fixed at 40 MW - 4 hours discharge.

As shown in Fig. 11, the energy storage location has an impact on the highest market-clearing price. When the ESS is installed in the transmission system (bus #5), the differences between off-peak and peak market prices are higher than those depicted when installing the same ESS in the distribution system (bus #8). This observation is attributed to the fact that the market prices are higher when more power losses are incurred in the system and vice versa. Therefore, when ESS units are installed in the distribution system, the system power losses are increased during charging periods of ESS, while they are reduced during discharging periods of ESS, thus increasing and decreasing the market prices during off-peak and peak periods, respectively.

Moreover, the arbitrage value associated with transmission system installation equals $2,947 (per day), which is almost two times the value of the distribution system installation. As a result, it is more profitable for the ESS owner to install the ESS in the transmission system in order to achieve higher arbitrage benefit. On the other hand, Fig. 12 reveals that total generation costs are larger with transmission system installation during off-peak times, while they are smaller when the load is high, compared to installing the same ESS in the distribution system. This observation can be explained as generation costs depend on the net system demand shown in Fig. 13. The net system demand is calculated taking into consideration the optimal ESS operation in each case.
since it is charged during off-peak periods, when the lines are lightly loaded, and discharged during peak times, thus relieving the congestion in the system. As shown in Table II, the lines 3-4 and 3-5, for example, were congested before the ESS was installed in the system; however, these congestions are alleviated after connecting the ESS at either bus #5 or bus #8.

Table II

<table>
<thead>
<tr>
<th>Line between buses</th>
<th>Rated power flow (pu)*</th>
<th>Maximum power flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>No ESS</td>
</tr>
<tr>
<td>1-2</td>
<td>2</td>
<td>1.83</td>
</tr>
<tr>
<td>1-6</td>
<td>1.7</td>
<td>1.70</td>
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<td>2-5</td>
<td>1.0</td>
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<td>1.8</td>
<td>0.58</td>
</tr>
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<td>6-7</td>
<td>1</td>
<td>0.76</td>
</tr>
<tr>
<td>7-8</td>
<td>0.9</td>
<td>0.77</td>
</tr>
<tr>
<td>8-9</td>
<td>0.5</td>
<td>0.31</td>
</tr>
</tbody>
</table>

*Base power = 100 MVA

Case study 4: central vs. distributed storage

This case study investigates the difference between central and distributed implementation of energy storage. The installation of one ESS (60 MW - 2 hours discharge) at bus #8 is compared to the installation of three ESSs (each is 20 MW – 2 hours discharge) at buses #7, #8, and #9.

It is shown in Fig. 16 that the highest market-clearing prices are exactly the same with both implementations since the power losses in the transmission network, constituting the major part of total system losses, are independent of the storage location in the distribution system. However, the arbitrage benefit associated with the distributed storage is found to be slightly higher than that with the central storage case. Therefore, it may be more desirable for the ESS owner to install distributed storage facility than installing central energy storage. Similarly, total generation costs are found to be slightly influenced by central/distributed storage implementation, as shown in Fig. 17, since the ESS operation is almost the same in both cases.

VI. CONCLUSIONS

This paper develops a mathematical model for determining the optimal operation strategy of ESSs simultaneously with the optimal market prices in a perfectly competitive environment, modified with special incentive pricing for ESSs based on the highest LMP in the system. The problem formulation solves two dependent problems, social welfare and energy storage arbitrage benefit maximization. Moreover, this paper investigates the impact of energy storage size and location on market price, total generation cost and benefit, arbitrage benefit, and total consumer payments, through several case studies.

Due to ESS operation, the highest market-clearing price becomes almost leveled during off-peak and peak intervals. The larger the energy storage size, the less difference between off-peak and peak prices. The total generation costs are also affected by the energy storage size; for larger sizes, higher costs are observed during low demand, while lower costs are incurred when the demand is high. With respect to the arbitrage benefit, it increases up to a certain maximum value when it is plotted versus the storage capacity size.
Furthermore, consumer payments and generation benefits are reduced with the increase of arbitrage benefit.

Regarding the impact of energy storage location, a higher difference between off-peak and peak prices is noticed in the case of connecting the energy storage to the transmission system compared to the case of installing the same ESS in the distribution system. Thus, the arbitrage benefit is higher in the case of transmission system installation. Therefore, the ESS owner is economically encouraged to install the ESS in the transmission system. Moreover, central and distributed energy storage implementations are compared in this paper. The results show that the highest market-clearing price and the generation costs are almost not influenced by central/distributed implementation; however, the arbitrage benefit associated with distributed storage is slightly higher than that found in the case of central storage. As a result, distributed storage may be desirable to be implemented in power systems, from the ESS owner’s perspective.

In conclusion, energy storage size and location have a direct impact on electricity market prices and arbitrage benefit. The conclusions are very useful for power system planners in allocating large-scale ESSs. Moreover, the proposed mathematical model is helpful for power system operators to determine the optimal charging/discharging strategy of ESSs and the optimal market prices in a perfect competition.

VII. APPENDIX A

The KKT conditions can be derived by taking derivatives of Lagrangian functions \( \mathcal{L}_1, \mathcal{L}_2, \) and \( \mathcal{L}_3 \) that correspond to \( F_1, F_2, \) and \( F_3, \) respectively, with respect to the primal variables and the Lagrangian multipliers (dual variables) associated with equality constraints, and setting those derivatives equal to zero. KKT conditions further include the complementarity conditions associated with inequality constraints and their dual variables. The complete MCP model is then given as follows:

\[
\frac{\partial \mathcal{L}_1}{\partial P_{G_{i,hr}}} = 2\alpha_i \times P_{G_{i,hr}} + \beta_i - \lambda_{i,hr} - a_{i,hr} = 0 \quad \forall i \in \mathcal{G}, hr
\]

\[
\frac{\partial \mathcal{L}_2}{\partial Q_{G_{i,hr}}} = -\mu_{i,hr} - c_{i,hr} + d_{i,hr} = 0 \quad \forall i \in \mathcal{G}, hr
\]
\[
\frac{\partial L}{\partial \lambda_{i,hr}} = \sum_{j=1}^{N} \left[ V_{i,hr} \times V_{j,hr} \times \left( G_{ij} \cos \delta_{ij,hr} + B_{ij} \sin \delta_{ij,hr} \right) \right] - (A.6)
\]
\[
P_{G,1,hr} + P_{D,1,hr} + P_{E_{\text{ess},1,hr}} - P_{\text{dis}_{\text{ess},1,hr}} = 0 \quad \forall i, hr
\]
\[
\frac{\partial L}{\partial \Omega_{i,hr}} = V_{i,hr} \text{ is constant} \quad \forall i \in G_{,hr}
\]
\[
\frac{\partial L}{\partial \hat{\lambda}_{i,hr}} = \sum_{i=1}^{N} m_{i,hr} = 1 \quad \forall hr
\]
\[
\frac{\partial L}{\partial P_{r_{\text{ch}}}} = C_M + \rho_{hr} - n_{i,hr} \times \eta_{\text{dis}_{\text{ch}}} - (A.10)
\]
\[
\frac{\partial L}{\partial P_{r_{\text{dis}}}} = C_M - \rho_{hr} + n_{i,hr} \times \eta_{\text{dis}} - (A.11)
\]
\[
q_{i,hr} + r_{i,hr} = 0 \quad \forall i \in \mathcal{E}, hr
\]
\[
\frac{\partial L}{\partial \eta_{\text{ess}_{\text{ess}}}} = n_{i,hr} - n_{i,hr+1} - (A.12)
\]
\[
s_{i,hr} + 1_{i,hr} = 0 \quad \forall i \in \mathcal{E}, hr
\]
\[
\frac{\partial L}{\partial \hat{\lambda}_{i,hr}} = E_{\text{ess}_{i,hr}} - E_{\text{ess}_{i,hr-1}} - \eta_{\text{ess}} \times P_{r_{\text{ch}}_{i,hr}} + (A.13)
\]
\[
P_{\text{dis}_{\text{ess}_{i,hr}}} \times \eta_{\text{dis}_{\text{ess}_{i,hr}}} = 0 \quad \forall i \in \mathcal{E}, hr
\]
\[
0 \leq P_{\text{flow}_{i,hr}} - P_{\text{flow}_{j,hr}} \perp w_{i,j,hr} \geq 0 \quad \forall i \neq j, hr
\]
\[
0 \leq P_{G_{i,hr}} - P_{G_{i,hr}} \perp d_{i,hr} \geq 0 \quad \forall i \in G_{,hr}
\]
\[
0 \leq P_{G_{i,hr}} - P_{G_{i,hr}} \perp b_{i,hr} \geq 0 \quad \forall i \in G_{,hr}
\]
\[
0 \leq Q_{G_{i,hr}} - Q_{G_{i,hr}} \perp c_{i,hr} \geq 0 \quad \forall i \in G_{,hr}
\]
\[
0 \leq Q_{G_{i,hr}} - Q_{G_{i,hr}} \perp d_{i,hr} \geq 0 \quad \forall i \in G_{,hr}
\]
\[
0 \leq V_{i,hr} \perp V_{i,hr} \geq 0 \quad \forall i \in G_{,hr}
\]
\[
0 \leq V_{i,hr} \perp V_{i,hr} \geq 0 \quad \forall i \in G_{,hr}
\]
\[
0 \leq \delta_{i,hr} + \pi \perp k_{i,hr} \geq 0 \quad \forall i, hr
\]
\[
0 \leq \rho_{hr} - \lambda_{i,hr} \perp l_{i,hr} \geq 0 \quad \forall i, hr
\]
\[
0 \leq P_{\text{ch}_{i,hr}} \perp \alpha_{i,hr} \geq 0 \quad \forall i \in \mathcal{E}, hr
\]
\[
0 \leq P_{\text{ess}_{i,hr}} - P_{\text{ch}_{i,hr}} \perp p_{1_{i,hr}} \geq 0 \quad \forall i \in \mathcal{E}, hr
\]
\[
0 \leq P_{\text{dis}_{i,hr}} \perp q_{1_{i,hr}} \geq 0 \quad \forall i \in \mathcal{E}, hr
\]
\[
0 \leq P_{\text{ess}_{i,hr}} - P_{\text{dis}_{i,hr}} \perp r_{1_{i,hr}} \geq 0 \quad \forall i \in \mathcal{E}, hr
\]

\[
0 \leq E_{\text{ess}_{,hr}} \perp s_{i,hr} \geq 0 \quad \forall i \in \mathcal{E}, hr
\]
\[
0 \leq E_{\text{ess}_{,hr}} - E_{\text{ess}_{,hr}} \perp t_{1_{i,hr}} \geq 0 \quad \forall i \in \mathcal{E}, hr
\]

**VIII. References**


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