Energy Storage and Regulation: An Analysis

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***Abstract*—Electric system operators rely on regulation services to match the total system supply to the total system load in quasi real-time. The regulation contractual framework requires that a regulation unit declares its regulation parameters at the beginning of the contract; the operator guarantees that the regulation signals will be within the range of these parameters; and the regulation unit is rewarded proportionally to what it declares and what it supplies. We study how this service can be provided by a unit with a non-ideal storage. We consider two broad classes of storage technologies characterized by different state of charge evolution equations, namely batteries and flywheels. We first focus on a single contract, and obtain formulas for the upward and downward regulation parameters that a unit with either a battery or a flywheel should declare to the operator to maximize its reward. We then focus on a multiple contract setting, and show how to analytically quantify the reward that such a unit could obtain in successive contracts. We quantify this reward using bounds and expectation, and compare our analytical results with those obtained from a dataset of real-world regulation signals. Finally, we provide engineering insights by comparing different storage technologies in terms of potential rewards for different contract durations and parameters.**

***Index Terms*—Frequency regulation, energy storage.**

I. Nomenclature

EST	Energy storage technology
(S)RU	(Storage) regulation unit
D, δ	Contract and time-slot (*ts*) duration
η_c, η_d	Charging and discharging efficiencies of the storage
ρ	Depth of discharge of the storage
B', B	Capacity of the regulating storage, $B = \rho B'$ (Wh)
Δ_c, Δ_d	Max charging and discharging power limits (W)
Γ	Self-discharge efficiency ($\Gamma = e^{-\delta/T_{loss}}$)
α, γ	Charge/discharge time ($\alpha = \frac{B'}{\Delta_c}, \gamma = \frac{B'}{\Delta_d}$) (hour)
β	Discharge to charge ratio ($= \frac{\Delta_d}{\Delta_c} = \frac{\alpha}{\gamma}$)
C_n	Contract n
$\mathcal{K}$	Set of K time-slots ($D = K\delta$)
R, r	Upward and downward regulation parameters (W)
a_n, b_n	Prices for each unit of upward and downward regulation per unit of time in C_n
$f_n(\cdot)$	Fixed part of regulation reward in C_n ($)
F_T	Fixed part of regulation reward over a period of length T hours ($)
s_k	Received regulation signal at *ts* k (W)
U_n	State of charge (SoC) at the beginning of C_n (Wh)
$b(k)$	SoC of the storage at the end of *ts* k (Wh)
ASC	Annual storage cost ($)
ACC	Annual capital cost ($)
ARC	Annual replacement cost ($)
FRP	Future replacement price ($/KWh)
AOMC	Annual operation and maintenance cost ($)
OMC	Fixed operation and maintenance cost ($)
CC	Capital cost ($)
CRF	Capital recovery factor
T, IR	Planning horizon (year) and interest rate (%)
L_s	Lifetime of storage technology s (year)

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II. Introduction

ELECTRICAL grid operators have the responsibility to maintain the target grid frequency. To achieve this goal, operators need to balance demand and supply at all times using services that operate at different time-scales [1]. Typically, operators predict the demand a day ahead, and schedule some slow-ramping generators with different characteristics (e.g., ramp rates) for supplying electricity a day in advance. Then, using generators or flexible loads, quasi real-time adjustments are made to maintain the grid's reliability [1]-[2].

In this study, we focus on a regulation service which balances demand and supply in quasi real-time. Typically, this service is offered by conventional fossil-fuel generators (e.g., natural-gas-fired steam turbines, etc.). These regulation units (RU) can vary their supply rate in response to regulation signals sent from every ten seconds to every few minutes. With high penetration of intermittent resources in the near future, conventional regulation services that use slow ramping RUs, such as generators, might have difficulties coping with high levels of fluctuations in supply and demand. Hence, the participation of fast-response storage devices in regulation services might be necessary to maintain the balance between demand and supply [2]. Energy storage technologies (EST), such as flywheels and batteries, can provide fast and accurate frequency regulation services [3]-[6], and also enable energy-recycling [7]. However, there are also challenges linked to their limited capacity, charging and discharging power limits, and self-discharge. We will refer to an RU that uses an EST to provide regulation services as an storage regulation unit (SRU).

The context of this study is the engineering and operation of SRUs. We will consider two broad classes of ESTs, namely flywheels and batteries. These two classes of ESTs have different characteristics not only in terms of their parameters [21]-[22], but also in terms of their state of charge (SoC) evolution equations. In our previous study in [18], we used a single simple SoC evolution model for energy storage technologies (EST). While this SoC model is appropriate for batteries, it does not accurately model the SoC evolution of flywheels. Flywheels which are considered as a promising potential storage technology for regulation services, are composed of an induction motor-generator and an active power controller. In [27], we obtain formulas for the flywheel SoC evolution equations, and show that they significantly differ from the SoC evolution model used in [18]. More precisely, the formulas for the SoC evolution equations for flywheels account (a) for frictional windage and magnetic losses from the bearings and motor-generator components (i.e., the self-discharge phenomenon), and (b) for the inertia of the active power controller inside the flywheel energy storage system. Clearly, the significant differences on the SoC evolution equations of the two classes of ESTs might yield significant differences in their performances in regulation services. In this paper, we use the accurate SoC evolution model for flywheels to provide completely new analytical results for frequency regulation for this class of ESTs.

We begin by describing the regulation service more precisely. Every D units of time, a new regulation contract, negotiated a few units of time ahead, starts. At the beginning of each contract, the operator and the RU agree on the regulation parameters r and R in Watts ($r, R \in \mathbb{R},\ r < R$) for the contract duration $D = K\delta$ (where δ is the duration of a time-slot). The operator will send regulation signals to the RU in every time-slot and commits that the signal s_k in time-slot $k \in \{1, \ldots, K\}$ will obey the following constraint:

$$r \le s_k \le R \ .$$

There is no other constraint on the sequence of regulation signals, sent by the operator. The RU commits to provide any power in the range $[r, R]$ in response to the regulation signals sent by the operator. The RU must supply constant power s_k during time-slot k if s_k is positive, and draw constant power $(-s_k)$ from the grid if it is negative. Typically, the RU is rewarded for its flexibility in terms of R and r, and for the amount of energy that it effectively supplies/draws during the contract. There is also a penalty to pay if the RU cannot fulfill its commitments in terms of R and r. During the negotiation phase, the RU needs to select its parameters r and R so as to maximize its reward while keeping the risk of a regulation failure close to zero.

The purpose of this paper is threefold, and covers both the planning and operation of an SRU. First, from an operational perspective, we aim to provide formulas on what the SRU should declare in terms of its regulation parameters R and r for both classes of storage technologies. The challenge here stems from the fact that the regulation signals are not known in advance. Second, from a planning perspective, our goal is to help an SRU to determine, ahead of time, the storage technology, sizing, and the contract duration that would maximize its operational rewards and minimize its costs. Finally, in order to answer the planning question above, our goal is to enable an SRU to estimate, beforehand, the reward that can be obtained in successive contracts. Note that in consecutive contracts, the SoC at the beginning of a contract depends on the regulation signals received during the previous contract.

The following contributions have been obtained for *two broad classes of non-ideal ESTs characterized by different SoC evolution equations*:

1) For a single contract, assuming the initial SoC is known, and that the SRU wants to keep the risk of a regulation failure equal to zero, we obtain simple formulas for the values of the regulation parameters R and r that the SRU should declare to the operator to maximize its reward.
2) We then provide means to analytically quantify, ahead of time, the reward that an SRU could obtain in successive contracts using analytical upper and lower bounds, and an approximation method for the average reward.
3) We validate our approximations for the expected reward, and then provide engineering insights by comparing the potential rewards the EST can receive for different contract durations and parameters.

The paper is organized as follows: Section III presents the related work. The system model and the EST models are introduced in Section IV. In Section V, we present our analytical results first for a single contract and then for N consecutive contracts. Numerical results are provided in Section VI. In Section VII, we compare the two classes of ESTs in terms of their minimum achievable reward. All the proofs are presented in the Appendix.

III. Literature Background

In conventional power grids, grid frequency is maintained by generators and flexible loads that can vary their supply rate quickly. A comprehensive overview of the conventional frequency regulation services is provided in [8]. Practical installations and simulations have shown that there are several benefits in using ESTs in regulation service [1]-[2], [9]-[13].

One of the key challenges in the context of frequency regulation is the risk of regulation failure. In the context of an SRU, this would happen if it is unable to meet its contractual obligation of supplying or drawing a certain amount of power if it is, for instance, fully charged or depleted. In [14], Lu *et al.* show, via simulation, that there is indeed a risk of a regulation failure when a flywheel provides regulation services by itself *without any control over its charging/discharging rate*.

Several approaches have been proposed to address regulation failures including: (a) combining SRUs with hydro-power plants [1], (b) allowing SRUs to purchase (or sell) electricity [15], (c) sending "energy neutral" regulation signals [16], and (d) over-dimensioning [17]. While combining an SRU with a hydro-power plant has been shown to reduce the risk of regulation failure significantly [1], [14], it is not always practical or feasible. Some independent system operators (ISO) (e.g., the New York and Midwest ISOs) allow SRUs to purchase (or sell) electricity from the market to charge (or discharge) their storage devices when needed [15]. Nonetheless, the risk of regulation failure still exists because it is difficult to perfectly estimate when the storage will be fully charged or discharged. The California Independent System Operator (CAISO) has adopted a different approach, called *ACE smoothing*, in which the slow moving component of the regulation signal is fulfilled by conventional RUs, while SRUs fulfill the remaining fast-changing component [7], [16]. This fast-changing component tends to be *energy-neutral* since it does not contain any long term trends [16]. Energy-neutral regulation signals are optimal for ideal storage devices with no charging or discharging losses, since they help maintain their SoC close to or at the preferred level in consecutive contracts. However, in the presence of losses, the risk of regulation failure in a sequence of contracts is not zero. Finally, conservatively over-dimensioning an SRU [17] can also minimize the risk of regulation failure, but at the expense of increased installation cost.

In [33], Ghiassi-Farrokhfal *et al.* propose an analytical model for a class of ESTs that includes all battery technologies and compressed air energy storage systems. Using teletraffic techniques, the authors propose power performance bounds for energy systems with non-ideal ESTs in firming-like scenarios.

In our preliminary work [18], we computed the regulation parameters R and r that an SRU with a non-ideal EST governed by a simple SoC evolution equation should declare to the operator at the beginning of a contract so as to maximize its reward while ensuring that there is *no* regulation failure. However, the simple SoC evolution equation used in [18] did not account for self discharge, and as we will discuss, is not suited for ESTs like the flywheel. This paper extends the results in [18] by considering: (a) a more accurate SoC evolution equation for batteries that takes self-discharge into account, and (b) detailed SoC evolution equations for flywheels that model the inertia of the active power controller and core losses inside the flywheel energy storage system. In addition, we have validated our analytically derived (but approximate) expressions for average reward using actual regulation signals from the Bonneville Power Administration (BPA) dataset [19]. Finally, we include a cost analysis of different ESTs in Section VII that was not part of [18].

We believe that this *analytical* study on the operational and planning aspects of SRUs is the first of its kind. Extensive work has been done on energy storage and frequency regulation, but none of them propose formulas for the regulation parameters R and r that an SRU with non-ideal storage should declare to the operator, nor do they provide expressions for the minimum, maximum, and average reward that an SRU can expect over successive contracts as we do in this paper.

IV. System Model

We consider an SRU that offers regulation services in a region whose power system is controlled by an independent system operator. We assume that the time is slotted in time-slot (*ts*) of size δ, and that the duration of each contract is K time-slots (i.e., $D = K\delta$). At the beginning of a contract, the SRU selects the regulation parameters R and r so that it can always respond to the regulation signals without any failure. We assume that the operator always accepts the parameters that the SRU proposes at the beginning of a contract.

A. EST Models

The SRU is using a non-ideal energy storage of size B' (Watt-hour), with charging efficiency η_c, discharging efficiency η_d, maximum charging and discharging power limits Δ_c and Δ_d (Watt), respectively, and depth of discharge (DoD) ρ. Therefore, the available capacity for the regulation service is equal to $B = \rho B'$.

We define the charge time α (resp. discharge time γ) as the ratio of the storage size to its maximum charging (resp. discharging) power limit, and the discharge to charge ratio β as the ratio of the discharging power limit of the storage to its charging power limit. Note that, in this study, we do not model the state of health of ESTs. We assume that the values of the parameters α, β, and γ are constant for a given technology, i.e., they do not change over the lifetime of the storage. Hence, Δ_c and Δ_d are proportional to the storage capacity, i.e., $\Delta_c = B'/\alpha$ and $\Delta_d = B'/\gamma$.

It is assumed that the energy stored in a EST decays exponentially with a time constant T_{loss}, which can vary significantly depending on the EST. This time constant is only 50 hours for flywheels while for batteries, T_{loss} is more than one year. Let Γ denote the self-discharge efficiency of the storage, and it is equal to $e^{-\delta/T_{loss}}$.

Different ESTs have different SoC evolution models [27]. To present the SoC evolution models for batteries and flywheels, let us focus on one contract, and let $b(k)$ denote the SoC of the storage at the end of *ts* k. The SoC evolution equation determines $b(k)$ as a function of $b(k-1)$, the regulation signal s_k (in Watts), and the EST parameters.

Battery: For this technology, $b(k)$ evolves as follows:

$$b(k) = \Gamma\ b(k-1) - \eta_d \delta [s_k]^+ + \eta_c \delta [-s_k]^+ \quad \forall k \in \mathcal{K} \quad (1)$$

where $b(0) = U$ is the initial SoC, $\mathcal{K} = \{1, \ldots, K\}$, and $[x]^+$ is equal to x if $x \geq 0$; otherwise, it is zero. By convention, in this model, we have $\eta_c \leq 1 \leq \eta_d$. The SoC evolution equation shows that the SoC at *ts* k is a function of the SoC at *ts* $(k-1)$, and the input power at *ts* k.

Flywheel: A flywheel energy storage is composed of a flywheel, an induction motor-generator, and an active power[1] controller. In [27], we approximate the impact of windage and lamination losses (i.e., the self-discharge phenomenon) on the SoC of the flywheel with a first-order differential equation with time constant T_{loss}, and model the electrical sub-system (i.e., the combination of the active power controller and induction machine) with a first order system. By doing this, we obtain SoC evolution equations that account for the inertia of the active power controller and core losses inside the flywheel energy storage system.

Let T_{cont} denote the time constant of the electrical sub-system which is the combination of the active power controller and induction machine. The value of this time constant depends on the inductance of the electrical machine and the control gains of the active power controller. Our SoC evolution equations for the flywheel energy storage are given below [27]:

$$b(k) = \Gamma\ b(k-1) + \overline{E_k} \quad \forall k \in \mathcal{K} \quad (2)$$

where $\overline{E_k}$ is given by (3) where z_k, z_{k-1}, and z_{eff} are given by

$$\begin{aligned} z_k &= (\eta_c \mathbf{1}_{s_k \leq 0} + \eta_d \mathbf{1}_{s_k > 0}) \\ z_{k-1} &= (\eta_c \mathbf{1}_{s_{k-1} \leq 0} + \eta_d \mathbf{1}_{s_{k-1} > 0}) \\ z_{\text{eff}} &= \left(z_k \mathbf{1}_{|s_k| > |s_{k-1}|} + z_{k-1} \mathbf{1}_{|s_k| \leq |s_{k-1}|}\right) \end{aligned}$$

and P and Q are defined as follows:

$$P = T_{loss} - \frac{T_{loss} T_{cont}}{T_{cont} - T_{loss}}, \quad Q = \frac{T_{cont} - T_{loss}}{T_{loss} T_{cont}} \ .$$

By convention, in our model, we have $\eta_c \leq 1 \leq \eta_d$.

Our analytical results show the SoC evolution of the flywheel is not only a function of the SoC at *ts* $(k-1)$ and the input power at *ts* k, but also of the input power at *ts* $(k-1)$, and that this dependence is complex and cannot be neglected. In contrast, the stored energy in a battery at *ts* k is a function of the stored energy at *ts* $(k-1)$ and the input power at *ts* k. Clearly, such differences should have a significant impact on the optimal values of the downward and upward parameters that the SRU can declare at the beginning of a contract, and on the reward that the SRU can obtain in successive contracts.

[1] Active power is the power that is dissipated in the resistance of the load.

$$\overline{E_k} = \begin{cases} G(s_k, s_{k-1}), & \textbf{if } s_k s_{k-1} < 0 \textbf{ and } \frac{s_k}{s_k - s_{k-1}} > e^{\frac{-\delta}{T_{cont}}} \\ H(s_k, s_{k-1}), & \textbf{otherwise} \end{cases}$$

$$G(s_k, s_{k-1}) = \Gamma \left[P(z_k - z_{k-1}) s_k \left(\frac{s_k - s_{k-1}}{s_k} \right)^{\frac{T_{cont}}{T_{loss}}} + s_k T_{loss} \left(z_{k-1} - \Gamma^{-1} z_k \right) + \left(\frac{s_k - s_{k-1}}{Q} \right) \left(e^{\delta Q} z_k - z_{k-1} \right) \right]$$

$$H(s_k, s_{k-1}) = z_{\text{eff}} \Gamma \left[s_k T_{loss} \left(1 - \Gamma^{-1} \right) + \left(\frac{s_k - s_{k-1}}{Q} \right) \left(e^{\delta Q} - 1 \right) \right] \tag{3}$$

B. Frequency Regulation Market Mechanism

As mentioned earlier, the SRU is rewarded for its flexibility in terms of R and r, and for what it actually supplies/draws during the contract. To select the values of the regulation parameters, we focus on the fixed part of the reward, and assume that, in contract C_n, the SRU is paid a fixed price $a_n \geq 0$ (resp. $b_n \geq 0$) per Watt of upward regulation (resp. downward regulation) for the contract duration D. Therefore, the fixed part of the regulation reward in a contract C_n in which the regulation parameters are R_n and r_n is

$$f(R_n, |r_n|) = (a_n R_n + b_n |r_n|) D \quad \text{(in dollars)} .$$

V. Analytical Results

A. Single Contract

Let us consider a single contract. We assume that the SRU knows its SoC at the beginning of the contract. The SRU has to choose its parameters R and r so that it can respond to all possible sequences of regulation signals $\{s_k\}$ without any failure. Given regulation parameters R and r, we show what the worst-case regulation sequences are from the SoC standpoint. Then, we show how to choose the regulation parameters to maximize the reward for these worst-case sequences.

Given a pair (R, r) [2], define the polyhedron $F(R, r)$ as follows[3]:

$$F(R, r) = \left\{ \pi \in \mathbb{R}^K | \pi = [s_1, \ldots, s_K]^t, \ r \leq s_k \leq R \right\} .$$

The operator will send regulation signals $\{s_k\}$ to the SRU ensuring that $r \leq s_k \leq R$ for all $k \in \mathcal{K}$, i.e., any sequence $\pi \in F(R, r)$ can be sent to the SRU by the operator. The SRU can provide any power in the range $[r, R]$ in response to the regulation signals $\{s_k\}$ *if and only if* the following constraints are satisfied:

$$0 \leq b(k) \leq B \quad \forall k \in \mathcal{K} \tag{4}$$

$$[-s_k]^+ \leq \Delta_c \quad \forall k \in \mathcal{K} \tag{5}$$

$$[s_k]^+ \leq \Delta_d \quad \forall k \in \mathcal{K} . \tag{6}$$

The only constraint imposed on the regulation signals is that $s_k \in [r, R]$ for all k. Therefore, to satisfy the constraints (5)-(6), the SRU needs to make sure that $R \leq \Delta_d$ and $|r| \leq \Delta_c$.

We say that a pair (R, r) with $0 \leq R \leq \Delta_d$ and $0 \leq |r| \leq \Delta_c$, is **feasible** (i.e., it can be selected by the SRU without any risk of a regulation failure) if (4) is satisfied for all $\pi \in F(R, r)$. Our goal is to find the **feasible** pair $(R^\star, r^\star)$ that maximizes the reward $f(R, |r|) = (aR + b|r|) D$. To do so, we first find the worst-case sequences that can be sent by the operator to the SRU given a pair (R, r).

[2]We assume that $r \leq 0$ and $R \geq 0$.

[3]The superscript "t" denotes the transpose operation.

Let us consider the two following sequences of regulation signals:

$$\pi_1 : \ s_k = r \ \ \forall k \in \mathcal{K}; \ \pi_2 : \ s_k = R \ \ \forall k \in \mathcal{K}$$

Proposition 1 establishes that pair (R, r) is feasible *if and only if* the value of $b(K)$ obtained by the sequence π_1 (resp. π_2) is less than or equal to B (resp. greater than or equal to zero) for the two classes of ESTs that we study. The proof is provided in the appendix.

Proposition 1. *Given δ, K, η_d, η_c, Γ, and B, pair (R, r) is **feasible** if and only if the sequences π_1 and π_2 yield $0 \leq b(K) \leq B$.*

Sequences π_1 and π_2 can be seen as worst-case sequences. Using the SoC equations in (1) and (2), we can show that constraint (4) is satisfied for sequences π_1 and π_2 *iff*

$$0 \leq |r| \leq \frac{B - \Gamma^K U}{\eta_c \overline{D}} \ , \text{ and } \ 0 \leq R \leq \frac{\Gamma^K U}{\eta_d \overline{D}} \tag{7}$$

where

$$\overline{D} = \begin{cases} T_{loss}\left(1 - \Gamma^K\right), & \text{for flywheels} \\ K\delta, & \text{for batteries} \end{cases}$$

Note that $\overline{D} \leq K\delta$ irrespective of the values of Γ, K, and δ.

Pair (R, r) is feasible (i.e., (4) is satisfied for all $\pi \in F(R, r)$) *if and only if* (7) is satisfied, and $0 \leq R \leq \Delta_d$ and $0 \leq (-r) \leq \Delta_c$. Therefore, given B, U, K, δ, Δ_d, Δ_c, Γ, η_c, and η_d, pair (R, r) is **feasible** *if and only if*:

$$0 \leq R \leq \overline{\mathbf{R}} \triangleq \min \left(\Delta_d, \frac{\Gamma^K U}{\eta_d \overline{D}} \right) \tag{8}$$

$$|r| \leq \overline{\mathbf{r}} \triangleq \min \left(\Delta_c, \frac{B - \Gamma^K U}{\eta_c \overline{D}} \right) . \tag{9}$$

As long as f is increasing in its arguments, $\overline{\mathbf{R}}$ and $\overline{\mathbf{r}}$ are the values of R and r that maximize $f(R, |r|)$. Note that they do not depend on the regulation prices a and b.

Engineering Insights: As mentioned earlier, we assume that for a given technology, the parameters α and β are constant. Hence, the only free parameter from the standpoint of the SRU is the storage capacity B'. To understand the impact of the storage capacity B' and the initial SoC on the reward $f(\overline{\mathbf{R}}, |\overline{\mathbf{r}}|)$, let us present our analytical results in terms of the storage parameters α, β, B', η_c, η_d, Γ, and ρ. We assume that $U = xB$ where $x \in [0, 1]$. Given α, β, B', η_c, η_d, Γ, ρ, δ, K, and x, the regulation parameters $\overline{\mathbf{R}}$ and $\overline{\mathbf{r}}$ that the SRU would declare, can be written as follows:

$$\overline{\mathbf{R}} \triangleq B' \min\left(\frac{\beta}{\alpha}, \rho\frac{x\Gamma^K}{\eta_d \overline{D}}\right)$$

$$\overline{\mathbf{r}} \triangleq B' \min\left(\frac{1}{\alpha}, \rho\frac{(1 - x\Gamma^K)}{\eta_c \overline{D}}\right) \ .$$

Recall that $\Gamma = e^{-\delta/T_{loss}}$, and note that $\overline{D} = D$ for batteries and is slightly less than D for flywheels as long as $D < 3$ hours. The results show that the reward $f(\overline{\mathbf{R}}, |\overline{\mathbf{r}}|)$ is linearly proportional to the storage capacity B'. We can also easily compute the best value of x, i.e., the initial SoC, for both classes of EST, given D, the EST parameters, and the per unit upward and downward prices a and b. We will discuss the impact of D in more details in Section VI.

Next, we focus on a multiple contract setting, and characterize the reward that can be obtained in successive contracts.

B. Multiple Contracts

Let us assume that the SRU wants to bid for a set of N consecutive contracts for a total duration of $T = ND$ time units. Our goal is to characterize a priori the reward $F_T = \sum_{n=1}^{N} f_n(\overline{\mathbf{R}}_n, |\overline{\mathbf{r}}_n|)$ that the SRU can obtain over the N contracts. The index n in $f_n(\cdot)$ reflects the fact that the prices for each unit of upward regulation (a_n) and downward regulation (b_n) might be different for each contract.

The regulation parameters $\overline{\mathbf{R}}_n$ and $\overline{\mathbf{r}}_n$ are functions of U_n, the SoC at the beginning of contract C_n. We assume that U_1 is fixed and known a priori. Hence, $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{r}}_1$ can be computed by using (8) and (9). For $n > 1$, U_n is equal to the SoC at the end of contract C_{n-1}, and hence its value depends on the sequence of regulation signals sent by the operator during contracts $C_1, \cdots, C_{n-1}$. This sequence of regulation signals is unknown a priori, and hence we do not know the SoC at the beginning of C_n for $n > 1$. Therefore, the initial SoC U_n (for $n > 1$) as well as the reward F_T are random variables whose values depend on the sequences of regulation signals being produced by the operator during the $(N - 1)$ contracts. We characterize the potential reward from the N contracts by providing upper and lower bounds on F_T, and by computing an approximation of the average reward $E\{F_T\}$.

1) ***Bounds****:* Let us begin with a single contract n, and assume that we do not know the SoC U_n at the beginning of that contract. We only know that U_n can take any value in the range $[0, B]$. Proposition 2 derives an upper bound $\overline{f_n}$ and a lower bound $\underline{f_n}$ for the reward $f_n(\overline{\mathbf{R}}_n, |\overline{\mathbf{r}}_n|)$. To understand the impact of the storage capacity B' and the contract duration D on the lower and upper bounds of the reward, we present our analytical results in terms of the parameters α, β, B', η_c, η_d, ρ, Γ, δ, and K. The proof is provided in the appendix.

Proposition 2. *Let $a_n \geq 0$ and $b_n \geq 0$ denote the prices for each unit of upward and downward regulation in contract n. Given α, β, η_c, η_d, K, δ, B', Γ, and ρ, the SRU's reward $f_n(\overline{\mathbf{R}}_n, |\overline{\mathbf{r}}_n|)$ is bounded as follows:*

$$\underline{f_n} \leq f_n(\overline{\mathbf{R}}_n, |\overline{\mathbf{r}}_n|) \leq \overline{f_n}$$

where:
Case 1) *If $\overline{D} \geq \rho\frac{\alpha}{\eta_c}$, then*

$$\underline{f_n} = \rho\frac{DB'}{\overline{D}} \min\left\{Q_n, \frac{b_n}{\eta_c}\right\}, \ \overline{f_n} = \rho\frac{DB'}{\overline{D}} \max\left\{Q_n, \frac{b_n}{\eta_c}\right\}.$$

Case 2) *If $\frac{\gamma\rho}{\eta_d} \leq \overline{D} < \rho\frac{\alpha}{\eta_c}$, then*

$$\underline{f_n} = DB' \min\left\{\frac{\rho Q_n}{\overline{D}}, \frac{b_n}{\alpha}\right\}, \ \overline{f_n} = DB' \max\left\{\frac{W_n}{\alpha}, \frac{\rho Q_n}{\overline{D}}\right\}.$$

Case 3) *If $\rho\frac{\alpha}{(\eta_c+\eta_d\beta)} \leq \overline{D} < \frac{\gamma\rho}{\eta_d}$, then*

$$\underline{f_n} = \frac{DB'}{\alpha} \min\{b_n, P_n, Z_n\}, \ \overline{f_n} = \frac{DB'}{\alpha} \max\{Z_n, W_n, P_n\}.$$

Case 4) *If $\overline{D} < \rho\frac{\alpha}{(\eta_c+\eta_d\beta)}$, then*

$$\underline{f_n} = \frac{DB'}{\alpha} \min\{b_n, P_n\}, \ \overline{f_n} = \frac{DB'}{\alpha} \max\{(a_n\beta + b_n), P_n\}.$$

where

$$Q_n = \frac{b_n}{\eta_c}(1 - \Gamma^K) + \Gamma^K\frac{a_n}{\eta_d}, \quad W_n = b_n + \frac{a_n}{\eta_d}\left(\frac{\alpha\rho}{\overline{D}} - \eta_c\right)$$

$$P_n = a_n\beta + \frac{b_n\rho\alpha(1 - \Gamma^K)}{\eta_c\overline{D}}, \quad Z_n = a_n\beta + \frac{b_n}{\eta_c}\left(\frac{\rho\alpha}{\overline{D}} - \eta_d\beta\right).$$

The reward F_T over the N contracts is then bounded as follows:

$$\sum_{n=1}^{N} \underline{f_n} \leq F_T \leq \sum_{n=1}^{N} \overline{f_n} \ .$$

Our analytical results show that the upper and lower bounds are proportional to the storage capacity. Therefore, the spread between the upper and lower bounds over the N contracts (i.e., the uncertainty on the reward) is also proportional to B'.

To understand the impact of K and δ, we consider two battery technologies (Lithium-ion and Lead-acid) and a flywheel technology. The ranges of values for α, γ, β, Γ, ρ, η_d, and η_c for these technologies are shown in Table I. Note that we have also computed results for a Sodium-Sulfur (NaS) battery. The values of the parameters η_c, η_d, α, γ, β, ρ, L_s, and W_s for different types of batteries (including NaS) can be obtained from [20]- [26]. We decided not to report these results to not clutter the figures. The results for NaS batteries are in between the results for Lead-acid and Lithium-ion.

- Flywheel: We are in Case 1 since $\rho\frac{\alpha}{\eta_c}$ is of the order of a few minutes (less than ten) and $\overline{D}$ is just slightly less than D which is typically larger than 30 minutes. Hence, the minimum reward during the period of T hours is inversely proportional to D which means that shorter contracts (larger than 10 minutes though) are favorable for SRUs using flywheels.
- Battery: We recall that $\overline{D} = D$, and in view of the values of the battery parameters, we are in Case 2, 3 or 4 and the relationship between the lower bound and D is not straightforward.

2) ***Average Reward****:* The expected reward $E\{F_T\}$ over the N contracts is equal to $\sum_{n=1}^{N} E_n$ where E_n denotes the expected value of the reward in contract n (i.e., $E_n = E\{f_n(\overline{\mathbf{R}}_n, |\overline{\mathbf{r}}_n|)\}$). We can easily compute E_1 since U_1 is known a priori. As mentioned earlier, $\overline{\mathbf{R}}_n$ and $\overline{\mathbf{r}}_n$ are a function of U_n, and U_n is a random variable for $n \geq 2$. Therefore, $\overline{\mathbf{R}}_n$ and $\overline{\mathbf{r}}_n$ are random variables whose potential values depend on the past history. To compute the expected value E_n, let $\overline{\mathbf{R}}_n(x)$ (resp. $\overline{\mathbf{r}}_n(x)$) denote $\overline{\mathbf{R}}_n$ (resp. $\overline{\mathbf{r}}_n$) given $U_n = x$. We can compute E_n by

$$E_n = \int_0^B f_n(\overline{\mathbf{R}}_n(x), |\overline{\mathbf{r}}_n(x)|) \ g_n(x)dx, \quad n \geq 2 \ .$$

TABLE I
STORAGE CHARACTERISTICS [20]-[22]

Storage technology	Lead-acid	Lithium-ion	Flywheel
Charging efficiency η_c	0.75	0.85	0.95
Discharging efficiency η_d	1	1	1.05
Charge time α	8-16 h	2-4 h	1-3 min
Discharge time γ	48-96 min	24-48 min	1-3 min
Discharge to charge ratio β	10	5	1
DoD ρ	0.8	0.8	1
Self-discharge rate	2-4% per month	2-4% per month	2% per hour
Lifetime L_s (years)	4	8	10
Storage price W_s (\$/KWh)	315	500	1000

where $g_n(\cdot)$ denotes the probability distribution function of U_n over the range $[0, B]$.

The distribution of U_n over $[0, B]$ is very complex, and it is highly dependent on the distribution of the regulation signals in contract C_{n-1} over the range $\left[\overline{\mathbf{r}}_{n-1}, \overline{\mathbf{R}}_{n-1}\right]$. It is not easy to estimate the probability distribution function of the regulation signals for a given contract since typically an operator works with multiple RUs, and it distributes the required regulation effort over the available RUs based on their flexibility and characteristics. Even with a specific probability distribution for the regulation signals, it is hard to compute the probability distribution function of U_n when the SRU is using a non-ideal storage. To illustrate this, let us assume that the regulation signals in contract n are uniformly distributed in $\left[\overline{\mathbf{r}}_n, \overline{\mathbf{R}}_n\right]$. We can show that U_n is distributed according to an Irwin-Hall distribution when the SRU is using an ideal battery with $\Gamma = \eta_d = \eta_c = 1$. However, it is hard to analytically determine the probability distribution function of U_n for a non-ideal storage.

We propose a method to approximate the average reward $E\{F_T\}$ over the N contracts. We do this in two steps. In the first step, we compute an estimate for $E\{U_n\}$ assuming that the initial SoC in each contract n is uniformly distributed between its minimum and maximum values. In the second step, we propose an approximation for E_n which is a function of $E\{U_n\}$. Using the proposed approximation for $E\{U_n\}$, we approximately compute E_n for all $n > 1$.

STEP 1 : Let $\widetilde{X}$ represent an approximation of the real-valued variable X. We approximate $E\{U_n\}$ by

$$\widetilde{E\{U_n\}} = \frac{X_n + Y_n}{2}$$

where X_n and Y_n denote the minimum and maximum SoCs at the end of contract n, *averaged* over all initial SoCs at the beginning of contract n. The distribution of the initial SoC is approximated as a uniform distribution between its minimum and maximum values. The values of X_n and Y_n are given in Table II.

STEP 2 : We approximate E_n by

$$\widetilde{E_n} = D\left[a_n \min\left(\Delta_d, \frac{\Gamma^K E\{U_n\}}{\eta_d \overline{D}}\right) + b_n \min\left(\Delta_c, \frac{B - \Gamma^K E\{U_n\}}{\eta_c \overline{D}}\right)\right]. \quad (10)$$

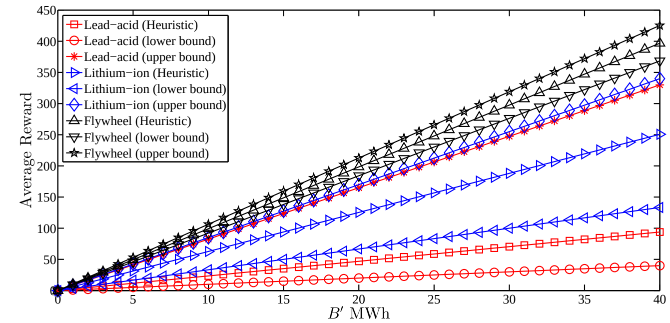

Fig. 1. The expected value of the reward $E\{F_T\}$ (in \$) as well as the lower and upper bounds on F_T, as a function of B' for $D = 1\ h$ when $\alpha = 10\ h$ (Lead-acid), $\alpha = 3\ h$ (Lithium-ion), $\alpha = 2\ min$ (Flywheel), $T = 10\ h$, $K = 12$, and $a_n = b_n = 1$ \$/(h MW).

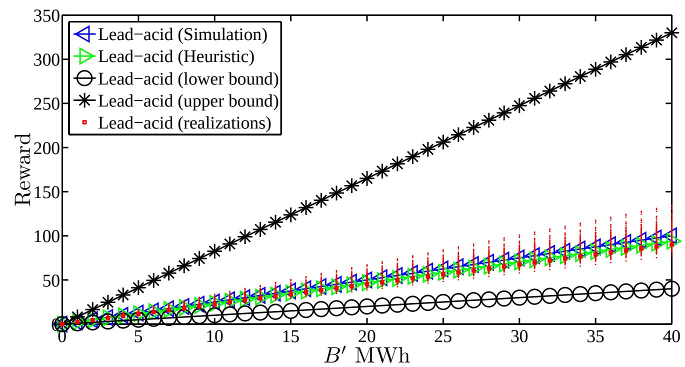

Fig. 2. Lead-acid: Our approximation of the average reward $E\{F_T\}$, the lower and upper bounds on F_T, and the average and realization of reward obtained from simulations, all as a function of B'. The system parameters are indicated in Figure 1.

Using the approximation of $E\{U_n\}$, we can iteratively compute $\widetilde{E\{U_n\}}$ for all n. After computing $\widetilde{E\{U_n\}}$ for all n, we can easily compute $\widetilde{E_n}$ using (10). Finally, we compute our approximation of $E\{F_T\}$ which is equal to $E_1 + \sum_{n=2}^{N} \widetilde{E_n}$.

Next, we validate our approximation on the average reward. Then, using the proposed approximation and the lower and upper bounds, we provide some engineering insights on the selection of a storage technology, its sizing, and the impact of the parameters B', K, and δ on the reward.

VI. NUMERICAL RESULTS

We focus on a period of $T = 10$ hours, and consider the three storage technologies discussed earlier, i.e., Lead-acid, Lithium-ion, and Flywheel. We take $K = 12$, $U_1 = \frac{B}{2}$, and α to be equal to 10 hours, 3 hours, and 2 minutes for Lead-acid, Lithium-ion, and Flywheel, respectively. The other parameters are given in Table I. Note that the proposed formulas can be used to quantify the reward for any other battery technology that obeys the SoC evolution equation given in (1). In this study, we focus on the regulation market in eastern ISOs in the United States (e.g., PJM and NYISO) [28], and assume that $a_n = b_n$ for all the N contracts. More precisely, we take $a_n = b_n = 1$ \$/(h MW), and compute the reward using real-world regulation signals obtained from the BPA dataset [19].

TABLE II
THE HEURISTIC PARAMETERS $X_{(n+1)}$ AND $Y_{(n+1)}$ FOR CONTRACT $(n+1)$

Conditions	$X_{(n+1)}$	$Y_{(n+1)}$
Case 1: $\overline{D} \geq \rho\frac{\alpha}{\eta_c}$	0	B
Case 2: $\frac{\gamma\rho}{\eta_d} \leq \overline{D} < \rho\frac{\alpha}{\eta_c}$	0	$\begin{cases} \Gamma^K E\{U_n\} + \eta_c\overline{D}\Delta_c, & \text{if } Y_n \leq Z_c \\ G_n, & \text{if } Y_n \geq Z_c \end{cases}$
Case 3: $\rho\frac{\alpha}{(\eta_c+\eta_d\beta)} \leq \overline{D} < \frac{\gamma\rho}{\eta_d}$	$\begin{cases} 0, & \text{if } X_n \leq Y_n \leq Z_d \\ H_n, & \text{if } X_n \leq Z_d \leq Y_n \\ \Gamma^K E\{U_n\} - \eta_d\overline{D}\Delta_d, & \text{if } Z_d \leq X_n \leq Y_n \end{cases}$	$\begin{cases} \Gamma^K E\{U_n\} + \eta_c\overline{D}\Delta_c, & \text{if } X_n \leq Y_n \leq Z_c \\ G_n, & \text{if } X_n \leq Z_c \leq Y_n \\ B, & \text{if } Z_c \leq X_n \leq Y_n \end{cases}$
Case 4: $\overline{D} < \rho\frac{\alpha}{(\eta_c+\eta_d\beta)}$	$\begin{cases} 0, & \text{if } X_n \leq Y_n \leq Z_d \\ H_n, & \text{if } X_n \leq Z_d \leq Y_n \\ \Gamma^K E\{U_n\} - \eta_d\overline{D}\Delta_d, & \text{if } Z_d \leq X_n \leq Y_n \end{cases}$	$\begin{cases} \Gamma^K E\{U_n\} + \eta_c\overline{D}\Delta_c, & \text{if } X_n \leq Y_n \leq Z_c \\ G_n, & \text{if } X_n \leq Z_c \leq Y_n \\ B, & \text{if } Z_c \leq X_n \leq Y_n \end{cases}$

$$Z_d = \frac{\eta_d\overline{D}\Delta_d}{\Gamma^K},\ Z_c = \frac{B-\eta_c\overline{D}\Delta_c}{\Gamma^K},\ G_n = \frac{1}{Y_n-X_n}\left[B(Y_n-X_n) - 0.5\Gamma^K(Z_c-X_n)^2\right],\ H_n = \frac{1}{Y_n-X_n}\left[0.5\Gamma^K(Y_n-Z_d)^2\right]$$

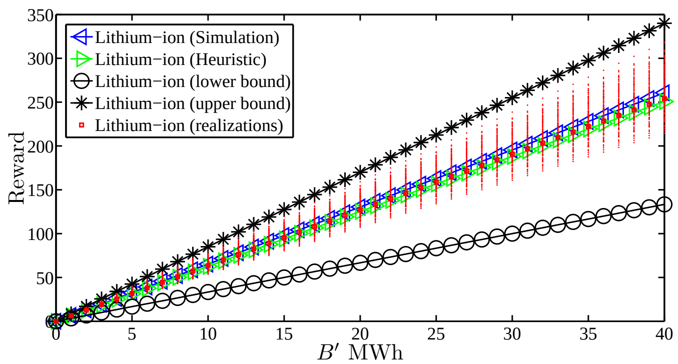

Fig. 3. Lithium-ion: Our approximation of the average reward $E\{F_T\}$, the lower and upper bounds on F_T, and the average and realization of reward obtained from simulations, all as a function of B'. The system parameters are indicated in Figure 1.

To validate our approximation of the average reward, we have used regulation signals from a real-world dataset from the BPA, where imbalance generation is dispatched every 5 minutes and is analogous to an automatic control generation (AGC) signal [19]. We note that in the contractual framework, the regulation signals would actually depend on the regulation parameters declared by SRU. Therefore, we had to translate the raw regulation signals we obtained from the dataset to lie between r and R. For clarity, the exact process we followed is described below.

We use the data available in [19] to generate regulation signals for 2 months. We divide the period (i.e., 2 months) into blocks of length 10 hours. Starting with the first contract, C_1 (and assuming $U_1 = \frac{B}{2}$) we compute $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{r}}_1$. We then translate the first K signals (we call them $\{d_1, \cdots, d_K\}$) using linear shifting and scaling operations such that each regulation signal is in the range $\left[\overline{\mathbf{r}}_1, \overline{\mathbf{R}}_1\right]$[4]. Using these regulation signals and the battery evolution equations, we can determine U_2, and repeat this process till we know the reward over n successive contracts. Using this technique, we have computed the reward for 140 realizations for different values of the storage capacity.

[4]Let $d_{max} = \max\{d_k\}$ and $d_{min} = \min\{d_k\}$. We first shift the signals into the range $[0, d_{max} - d_{min}]$, scale them to lie in the range $[0, \overline{\mathbf{R}}_1 - \overline{\mathbf{r}}_1]$ and shift them back to lie in the range $[\overline{\mathbf{r}}_1, \overline{\mathbf{R}}_1]$.

Figure 1 compares the three ESTs (Lead-acid, Lithium-ion, and Flywheel) in terms of the minimum, maximum and average reward as a function of battery capacity. In addition, Figure 2 and Figure 3 compare the rewards estimated using the proposed analytical techniques to those obtained from simulations using the BPA dataset for Lead-acid and Lithium-ion. A similar plot for Flywheel is excluded for space constraints, but we note that for Flywheel our heuristic approximation of average rewards aligns almost perfectly with simulation data.

We observe that (a) Flywheel provides not only greater rewards than Lead-acid and Lithium-ion, but also provides more predictable rewards, i.e., has a lower spread between the minimum and maximum rewards, and (b) our approximation of the average reward (line marked Heuristic) provides very accurate estimates of the average reward obtained from simulations (line marked Simulation). We have obtained similar results for different values of K and δ.

The SRU can also influence the operator to negotiate a contract duration D which is favorable to its storage technology. To understand the impact of varying D (equivalently, varying δ if K is fixed) on the SRU's reward, we compute our approximation of $E\{F_T\}$ as well as the lower and upper bounds on F_T, as a function of D for $B' = 20\ MWh$ and $K = 12$. Our numerical results in Fig. 4 show that (a) the reward is a non-increasing function of D, which can be explained by the engineering insights provided in Section V, (b) as observed before, the Flywheel provides a greater average reward and a lower spread between the minimum and maximum rewards than the other two technologies, and (c) the reward for Flywheel is particularly sensitive to contract duration and increases by as much as 500% when D is reduced to 10 min from 1 $hour$. We have also studied the impact of varying K on F_T while keeping δ fixed, and have observed that the impact of varying K on reward is similar to that of varying δ. The details are omitted due to space limitations.

VII. Comparison of Energy Storage Technologies

The objective of this section is to show how our framework could be used to perform some (simple) offline cost analysis to compare the usefulness of different storage technologies to deliver regulation services. Capital costs, lifetime, and operation and maintenance (O&M) costs can significantly affect the

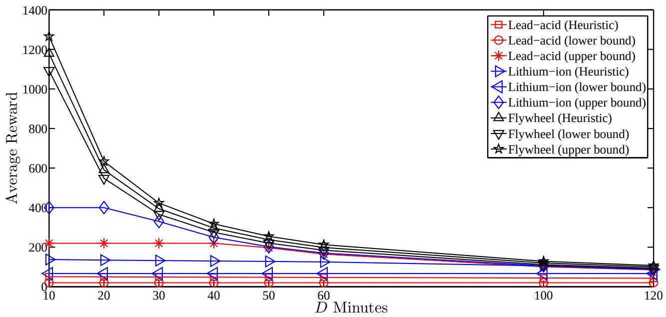

Fig. 4. The expected value of the reward $E\{F_T\}$ (in \$) as well as the lower and upper bounds on F_T, as a function of D for $B' = 20\ MWh$. All other system parameters are the same as in Figure 1.

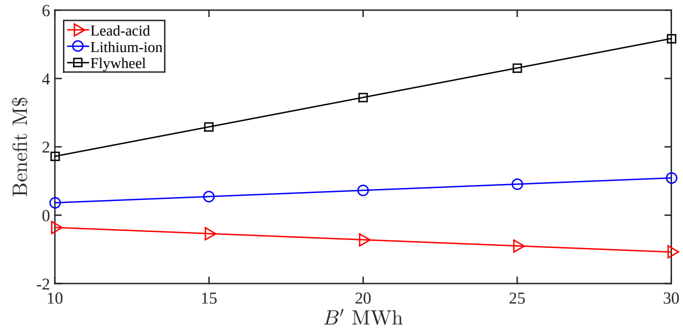

Fig. 5. The annual benefit (in M\$) as a function of the storage capacity when $a_n = b_n = 30$ \$/(hour MW), $\delta = 1$ minute, IR $= 2\%$, and $D = 1$ hour.

operational rewards of using ESTs for ancillary services. To develop guidelines on how to select the appropriate EST for an SRU, we assume that a regulation unit is planning to install an SRU with capacity B' to participate in regulation services over a horizon of length T years, and that the system operator has guaranteed minimum upward and downward regulation prices a and b[5], respectively, over the planning horizon[6]. We use the cost-benefit analysis method proposed in [29]-[30] to compare different ESTs in terms of their costs and rewards.

Given EST s with capacity B', the annual storage cost (which we refer to as ASC_s) can be computed by the following formula given in [29]- [30]

$$\text{ASC}_s = (\text{ACC}_s + \text{ARC}_s + \text{AOMC}_s)$$

where ACC_s, ARC_s, and AOMC_s, are the annual capital cost (i.e., the annual cost of financing the capital required to set up the installation), annual replacement cost, and annual operation and maintenance cost, respectively. The annual capital cost ACC_s is equal to $(\text{CRF} \times \text{CC}_s)$ where CC_s and CRF are the capital cost and capital recovery factor, respectively. CC_s is equal to $(W_s \times B')$ where W_s is the storage price (in dollars per KWh) at the installation time, and CRF can be calculated as

$$\text{CRF} = \frac{IR(1+IR)^T}{(1+IR)^T - 1}\ .$$

where IR is the interest rate.

The storage element may have to be replaced one or more times during the planning horizon of the SRU depending on its lifetime. This cost can be annualized as follows [30]:

$$\text{ARC}_s = \text{CRF}\left[\frac{B' \times \text{FRP}_s(L_s)}{(1+IR)^{L_s}} + \frac{B' \times \text{FRP}_s(2L_s)}{(1+IR)^{2L_s}} + \cdots\right]$$

where L_s and $\text{FRP}_s(t)$ are the lifetime (in years) and the future replacement cost in the t'th year of the planning horizon T, respectively. The number of terms in the factor of the equation above is equal to the number of times storage units are replaced during the planning horizon of the SRU. For an EST with lifetime L_s, there should be $\lfloor T/L_s \rfloor$ terms in the equation above, i.e., the storage should be replaced $\lfloor T/L_s \rfloor$ times during the SRU life. The values of the parameters L_s and W_s are provided in Table I.

In recent years, EST prices have steadily decreased. This trend is expected to continue. We assume that the storage price decays exponentially. More precisely, we assume that the storage price changes by a factor of 5% per year (i.e., $\text{FRP}_s(t) = 0.95 \times \text{FRP}_s(t-1)$ where $\text{FRP}_s(1) = W_s$) [32].

Similar to the study in [31], we only consider fixed maintenance costs, and set the annual O&M cost AOMC_s to 3% of the capital cost, i.e., $\text{AOMC}_s = 0.03 \times \text{CC}_s$. This approximation reflects the fact that flywheels have higher O&M costs since the capital cost is equal to $(W_s \times B')$ and the storage price for flywheels is higher than the storage price for batteries.

For our numerical results, we take a planning horizon of length $T = 20$ years, and $\delta = 1$ minute, $IR = 2\%$, and $D = 1$ hour. Having fixed the storage capacity to B', we compute for each technology s, the minimum reward over one year (call it $\text{Reward}(s)$) assuming that the operator will always accept what the SRU can declare. We then compute the annual benefit $(\text{Reward}(s) - \text{ASC}(s))$. The results are shown in Fig. 5.

Our numerical results show that, flywheels can bring an annual benefit that is significantly greater than the other two technologies. These observations can be explained by the fact that flywheels have higher lifetime and rewards, despite their higher capital and O&M costs. Note that the benefits obtained for Lead-acid are always negative, i.e., Lead-acid batteries should not be used to offer regulation services (at least for the set of parameters that we tried). The proposed framework enables SRUs to perform some offline cost analysis and to compare different technologies under different sets of parameters.

VIII. Conclusion

In this study, we analytically quantify the reward that an SRU could obtain in successive contracts using analytical upper and lower bounds, and an approximation method for the average reward. We consider the following three storage technologies, namely two battery technologies (Lithium-ion and Lead-acid) and a flywheel technology, and study the impact of the storage parameters and the contract duration on the expected reward in multiple contracts. Finally, we compare these technologies in terms of the reward that they can obtain for different values of some critical parameters.

[5]This can be generalized to any known sequence (maybe different values every year).

[6]For the analysis in this section, we will ignore affects such as currency fluctuations and inflation.

APPENDIX A
PROOF OF PROPOSITION 1

Claim 1. *Given $r, R \in \mathbb{R}$ and $k \in \mathcal{K}$, the maximum (resp. minimum) value of $b(k)$ over all $\pi \in F(R,r)$ is obtained by the worst case sequence π_1 (resp. π_2) of regulation signals.*

Proof : The SoC of a battery at time-slot k is a function of the SoC at time-slot $(k-1)$ and the input power at time-slot k [27]. In contrast, the SoC of a flywheel energy storage is not only a function of the SoC at time-slot k and the input power at time-slot k, but also the input power at time-slot $(k-1)$ [27]. Clearly, the maximum (resp. minimum) value of $b(k)$ over all $\pi \in F(R,r)$ is obtained by the sequence π_1 (resp. π_2) for batteries. While the result is trivial for batteries, it is not obvious for flywheels. To prove the claim for flywheels, let us introduce the SoC evolution equation derived in [27].

Consider a flywheel energy storage system with the parameters η_c, η_d, T_{loss}, and T_{cont}. At time instant $t_k = k\delta$, the SoC can be computed by $b(k) = \sum_{i=0}^{k} \overline{E_i}\Gamma^{(k-i)}$ where

$$\overline{E_i} = \int_{t_{i-1}}^{t_i} \left(\widehat{P_{in}^m(\tau)} \times z_{\text{eff}}(\tau)\right) e^{\frac{-(t_i-\tau)}{T_{loss}}} d\tau$$
$$\widehat{P_{in}^m(t)} = \left[s_i - (s_i - s_{i-1})\, e^{\frac{-(t-t_{i-1})}{T_{cont}}}\right] \;,\; t \in [t_{i-1}, t_i]$$

Note that $\overline{E_0} = U$ denotes the initial SoC of the flywheel.

The charging/discharging efficiency $z_{\text{eff}}(t)$ is equal to η_c if $\widehat{P_{in}^m(t)}$ is non-negative; otherwise it is equal to η_d. Therefore, $\overline{E_i}$ is maximized if and only if $\widehat{P_{in}^m(t)}$ is maximized. $\widehat{P_{in}^m(t)}$ is maximized if and only if $s_i = R$ for all $i \in \mathcal{K}$. Hence, the maximum value of $b(k)$ over all $\pi \in F(R,r)$ is obtained by the sequence π_1. Similarly, the minimum value of $b(k)$ over all $\pi \in F(R,r)$ is obtained by the sequence π_2. □

Claim 1 shows that (4) will be satisfied for all $\pi \in F(R,r)$ if (4) is satisfied for the sequences π_1 and π_2. The converse is trivial. Hence, (R,r) is feasible if and only if (4) is satisfied for the sequences π_1 and π_2.

Given $k \in \mathcal{K}$, the SoC at ts k for the sequences π_1 and π_2 can be written as follows:

$$\pi_1: \; b(k) = U + A_k\overline{A}(\eta_c|r|), \quad \forall k \in \mathcal{K}$$
$$\pi_2: \; b(k) = U - A_k\overline{A}(\eta_d R), \quad \forall k \in \mathcal{K}$$

where

$$A_k = \begin{cases} \frac{1-\Gamma^k}{1-\Gamma}, & \text{for } \Gamma < 1 \\ k, & \text{for } \Gamma = 1 \end{cases}.$$

$\overline{A}$ is equal to $T_{loss}(1-\Gamma)$ and δ for flywheels and batteries, respectively. This shows that $b(k)$ is increasing (resp. decreasing) in k for the sequence π_1 (resp. π_2) since A_k is increasing in k. Hence, the constraint (4) is satisfied for the sequences π_1 and π_2 if and only if the value of $b(K)$ obtained by the sequence π_1 (resp. π_2) is less than or equal to B (resp. greater than or equal to zero). This completes the proof. □

APPENDIX B
PROOF OF PROPOSITION 2

Given K, δ, η_c, η_d, B, and Γ, let us define the linear functions $P(U_n) = \frac{\Gamma^K U_n}{\eta_d \overline{D}}$ and $Q(U_n) = \frac{B - \Gamma^K U_n}{\eta_c \overline{D}}$ where $U_n \in [0, B]$. The SRU's reward for contract n depends on the values of the parameters Δ_c, Δ_d, K, δ, η_c, η_d, and B, and on the value of U_n in contract n. We consider the following cases:

Case 1: Let us assume that $\eta_c\Delta_c \geq \frac{B}{D}$ and $\eta_d\Delta_d \geq \frac{B}{D}$ (i.e., $\overline{D} \geq \frac{\rho\alpha}{\eta_c}$). We can verify that the optimal value of $f_n(R_n, r_n)$ is determined by $(a_nP(U_n) + b_nQ(U_n)) \times D$ irrespective of the value of U_n, and the reward is bounded as follows:

$$\frac{BD}{\overline{D}} \min\left\{Q_n, \frac{b_n}{\eta_c}\right\} \leq f_n(R_n, r_n) \leq \frac{BD}{\overline{D}} \max\left\{Q_n, \frac{b_n}{\eta_c}\right\}$$

where $Q_n = \frac{b_n}{\eta_c}(1-\Gamma^K) + \Gamma^K \frac{a_n}{\eta_d}$.

Case 2: Let us assume that $\eta_d\Delta_d \geq \frac{B}{D}$ and $\eta_c\Delta_c < \frac{B}{D}$ (i.e., $\frac{\gamma\rho}{\eta_d} \leq \overline{D} < \frac{\rho\alpha}{\eta_c}$). Let us define $Z_c = (B - \Delta_c\eta_c\overline{D})/\Gamma^K$. Given Δ_c, Δ_d, K, δ, η_c, η_d, and B, depending on the value of U_n in contract n, the SRU's reward for contract n is determined by one of the following cases:

- If $Z_c \leq U_n \leq B$, then $P(U_n) \leq \Delta_d$, $Q(U_n) \leq \Delta_c$, and $f_n(R_n, r_n) = (a_nP(U_n) + b_nQ(U_n)) \times D$.
- If $0 \leq U_n \leq Z_c$, then $P(U_n) \leq \Delta_d$, $Q(U_n) \geq \Delta_c$, and $f_n(R_n, r_n) = (a_nP(U_n) + b_n\Delta_c) \times D$.

Let $W_n = \left(b_n + \frac{a_n}{\eta_d}\left(\frac{\alpha\rho}{\overline{D}} - \eta_c\right)\right)$. The reward in contract n is bounded as follows:

- If $Z_c \leq U_n \leq B$, then $D\Delta_c \min\left\{\frac{\rho\alpha Q_n}{\overline{D}}, W_n\right\} \leq f_n(R_n, r_n) \leq D\Delta_c \max\left\{\frac{\rho\alpha Q_n}{\overline{D}}, W_n\right\}$.
- If $0 \leq U_n \leq Z_c$, then $b_n\Delta_c D \leq f_n(R_n, r_n) \leq W_nD\Delta_c$.

Note that in this case, we have $W_n \geq b_n$.

Case 3: Typically, discharging power limits are greater than the charging power limits $\Delta_d \geq \Delta_c$, and $\eta_c \leq \eta_d$ [20], [21]. Therefore, it is impossible to have $\eta_d\Delta_d < \frac{B}{D}$ and $\eta_c\Delta_c \geq \frac{B}{D}$.

Case 4: Let us assume that $\eta_d\Delta_d < \frac{B}{D}$, $\eta_c\Delta_c < \frac{B}{D}$, and $Z_c \leq Z_d$ where $Z_d = \left(\eta_d\Delta_d\overline{D}/\Gamma^K\right)$ (i.e., $\rho\frac{\alpha}{(\eta_c+\eta_d\beta)} \leq \overline{D} < \frac{\gamma\rho}{\eta_d}$). Given Δ_d, Δ_c, K, δ, η_c, η_d, and B, depending on the value of U_n in contract n, the SRU's reward for contract n is determined by one of the following cases:

- If $Z_d \leq U_n \leq B$, then $P(U_n) \geq \Delta_d$, $Q(U_n) \leq \Delta_c$, and $f_n(R_n, r_n) = (a_n\Delta_d + b_nQ(U_n))\, D$.
- If $Z_c \leq U_n \leq Z_d$, then $P(U_n) \leq \Delta_d$, $Q(U_n) \leq \Delta_c$, and $f_n(R_n, r_n) = (a_nP(U_n) + b_nQ(U_n))\, D$.
- If $0 \leq U_n \leq Z_c$, then $P(U_n) \leq \Delta_c$, $Q(U_n) \geq \Delta_d$, and $f_n(R_n, r_n) = (a_nP(U_n) + b_n\Delta_c)\, D$.

Let $P_n = \left(a_n\beta + \frac{b_n\rho\alpha}{\eta_c\overline{D}}\left(1-\Gamma^K\right)\right)$ and $Z_n = \left(a_n\beta + \frac{b_n}{\eta_c}\left(\frac{\rho\alpha}{\overline{D}} - \eta_d\beta\right)\right)$. The reward in contract n is bounded as follows:

- If $Z_d \leq U_n \leq B$, then $\Delta_cD\min\{Z_n, P_n\} \leq f_n(R_n, r_n) \leq \Delta_cD\max\{Z_n, P_n\}$.
- If $Z_c \leq U_n \leq Z_d$, then $\Delta_cD\min\{Z_n, W_n\} \leq f_n(R_n, r_n) \leq \Delta_cD\max\{Z_n, W_n\}$.
- If $0 \leq U_n \leq Z_c$, then $\Delta_cD\min\{b_n, W_n\} \leq f_n(R_n, r_n) \leq \Delta_cD\max\{b_n, W_n\}$.

Case 5: Let us assume that $\eta_d\Delta_d < \frac{B}{D}$, $\eta_c\Delta_c < \frac{\epsilon B}{D}$, and $Z_c > Z_d$ (i.e., $\overline{D} < \rho\frac{\alpha}{(\eta_c+\eta_d\beta)}$). Given Δ_d, Δ_c, K, δ, η_c, η_d, and B, depending on the value of U_n in contract n, the SRU's reward for contract n is determined by one of the following cases:

- If $Z_c \leq U_n \leq B$, then $P(U_n) \geq \Delta_d$, $Q(U_n) \leq \Delta_c$, and $f_n(R_n, r_n) = (a_n\Delta_d + b_nQ(U_n))\, D$.

- If $Z_d \leq U_n \leq Z_c$, then $P(U_n) \geq \Delta_d$, $Q(U_n) \geq \Delta_c$, and $f_n(R_n, r_n) = (a_n\Delta_d + b_n\Delta_c)\, D$.
- If $0 \leq U_n \leq Z_d$, then $P(U_n) \leq \Delta_d$, $Q(U_n) \geq \Delta_c$, and $f_n(R_n, r_n) = (a_n P(U_n) + b_n\Delta_c)\, D$.

Therefore, the reward in contract n is bounded as follows:

- If $Z_c \leq U_n \leq B$, then $\Delta_c D \min\{(a_n\beta + b_n), P_n\} \leq f_n(R_n, r_n) \leq \Delta_c D \max\{(a_n\beta + b_n), P_n\}$.
- If $0 \leq U_n \leq Z_d$, then $\Delta_c D \min\{(a_n\beta + b_n), b_n\} \leq f_n(R_n, r_n) \leq \Delta_c D \max\{(a_n\beta + b_n), b_n\}$.

The results can be presented in terms of the storage parameters (i.e., α, β, γ, η_c, η_d, and ρ). This completes the proof. □

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