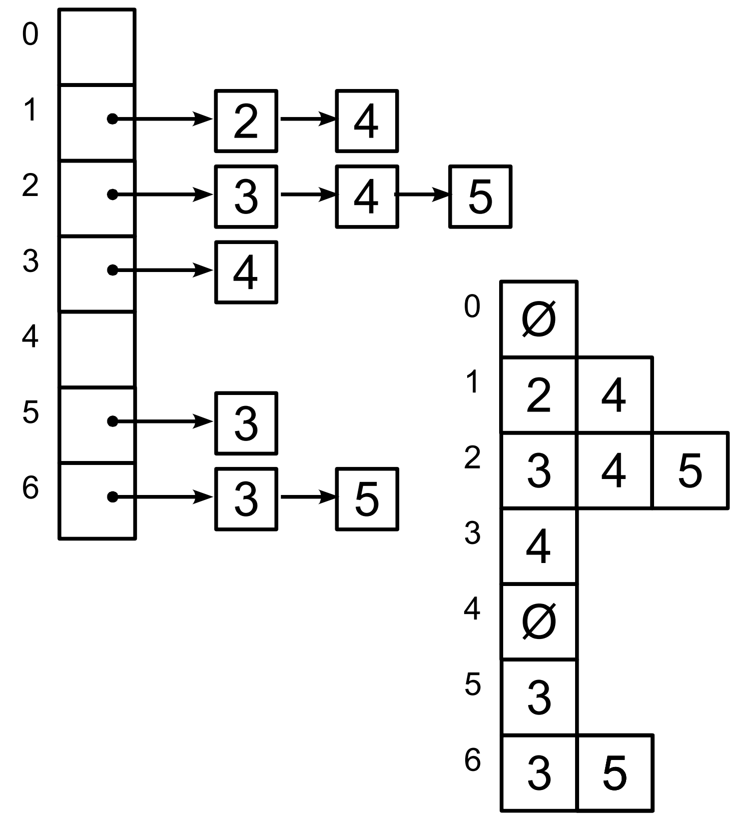
Graphs – Implementation Tips



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Graphs – Implementation Tips

Standard reminder to set phones to silent/vibrate mode, please!

Graphs – Implementation Tips

- Today's lesson:
  - Finish off the topic of Graphs with some tips on the various techniques to implement them
    - Should be helpful for your lab work this week!
  - Look into the C++ Standard Library **`vector`** class
    - Generic array class that handles memory management "behind the scenes"
  - Take a quick look a **`list`** class, also from the Standard Library.

Graphs – Implementation Tips

- We'll be using my own introductory tutorial on C++ vectors: http://www.mochima.com/tutorials/vectors.html

Graphs – Implementation Tips

- Copyright / academic integrity statement:
  - The code samples are only for illustration purposes.
  - You are NOT ALLOWED to directly copy fragments of code into your lab project.
  - You are of course allowed to use the ideas; but directly copying from these slides to your project would constitute an academic offence.

Graphs – Implementation Tips

- We recall our two typical implementation strategies — adjacency lists and adjacency matrix
- We briefly discussed this when talking about topological sort.

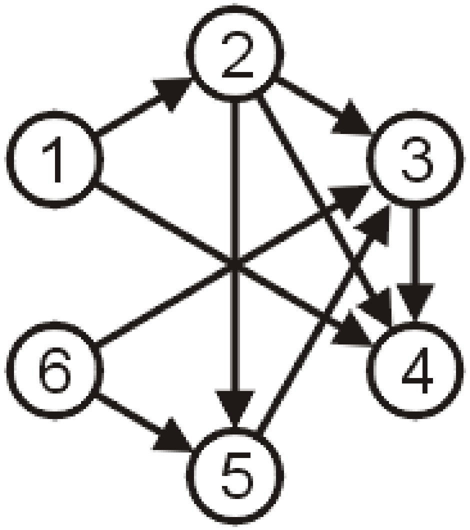
Graphs – Implementation Tips

- With *adjacency lists*, vertices are associated with a number between 0 and $|V|-1$, or between 1 and $|V|$, disregarding element 0.
- An array of adjacencies is defined — each element of the array is a list (either a dynamic array or a linked list) of the vertices adjacent to the vertex corresponding to that subscript.

Graphs – Implementation Tips

- Ajacency list — example:

$$V = \{1, 2, 3, 4, 5, 6\}$$
$$E = \{(1,2), (1,4), (2,3), (2,4), (2,5), (3,4), (5,3), (6,3), (6,5)\}$$

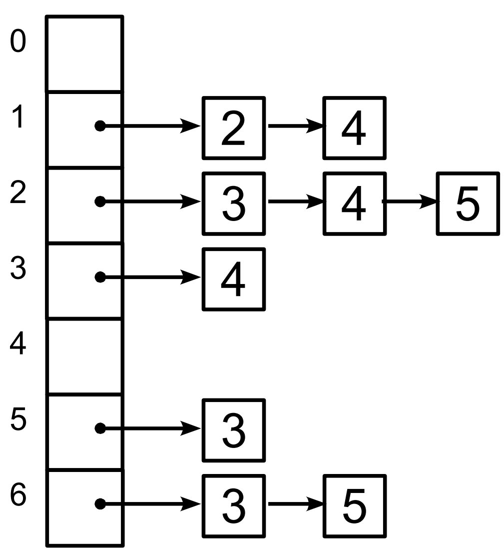


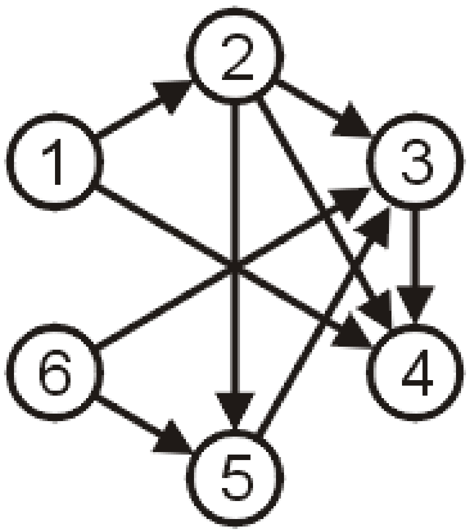
Graphs – Implementation Tips

- Ajacency list — example:

$$V = \{1,2,3,4,5,6\}$$
$$E = \{(1,2),(1,4),(2,3),(2,4),(2,5),(3,4),(5,3),(6,3),(6,5)\}$$

Representation:





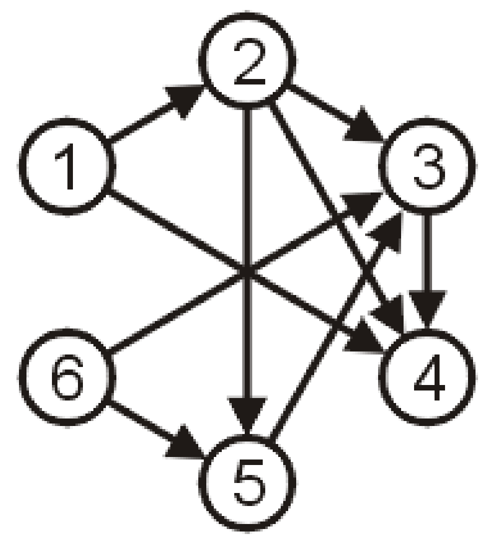
Graphs – Implementation Tips

- Ajacency list — example:

$$V = \{1, 2, 3, 4, 5, 6\}$$
$$E = \{(1,2), (1,4), (2,3), (2,4), (2,5), (3,4), (5,3), (6,3), (6,5)\}$$

Could also be a "jagged array" (an array of arrays):

0	∅		
1	2	4	
2	3	4	5
3	4		
4	∅		
5	3		
6	3	5	



Graphs – Implementation Tips

- The former could be implemented as a vector of lists:

```
std::vector< std::list<int> >
```

Graphs – Implementation Tips

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std::vector< std::list<int> >
```

- Notice a little “paper cut” feature in C++ (fixed in the new C++11 standard), the space between the two > > at the end is not optional!! (if you omit it, the compiler would parse the >> as the bitshift operator, and would result in a syntax error!)

Graphs – Implementation Tips

- The latter is similar — a vector of vectors:

```
std::vector< std::vector<int> >
```

Graphs – Implementation Tips

- Adding edges works similarly with both vector of vectors and vector of lists — the following example inserts edge (u,v):

```
vector< vector<int> > adjacencies(n+1);
  // ...

adjacencies[u].push_back (v);
```

(the sample assumes a `using std::vector;` directive, or `using namespace std;` — the latter generally not recommended on a header file)

Graphs – Implementation Tips

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  - The fact that the value is in the “row” array means that that vertex is adjacent to the vertex corresponding to the subscript of the given row.

Graphs – Implementation Tips

- This trick works for unweighted graphs, since we only need to denote the presence of an edge.
  - The fact that the value is in the "row" array means that that vertex is adjacent to the vertex corresponding to the subscript of the given row.
- If we need to indicate a weight (e.g., a `double` value), we could use an additional trick.

Graphs – Implementation Tips

- Create a simple class Edge that has two data members — target vertex, and weight:

```
class Edge
{
    int d_vertex;
    double d_weight;
public:
    Edge (int vertex, double weight)
      : d_vertex(vertex), d_weight(weight)
    {}

    int vertex() const { return d_vertex; }
    double weight() const { return d_weight };
};
```

Graphs – Implementation Tips

- With this, you would declare a vector of vectors of Edges:

```
vector< vector<Edge> > adjacencies;
```

- Adding edges is similar — the following example inserts edge (u,v) with weight w:

```
vector< vector<int> > adjacencies(n+1);
  // ...

adjacencies[u].push_back (Edge(v,w));
```

Graphs – Implementation Tips

- As an example, if we wanted to compute the weight of the graph (the sum of the weights of all edges):

```
for (vector< vector<Edge> >::size_type v = 1;
     v < adjacencies.size();
     ++v)
{
    for (vector<Edge>::size_type e = 0;
         e < adjacencies[v].size();
         ++e)
    {
        sum += adjacencies[v][e].weight();
    }
}
```

Graphs – Implementation Tips

- If we're representing a directed graph, we notice that this representation provides a very efficient (constant time) way to determine the out-degree of a vertex — for example, the out-degree of vertex v is given by:

```
adjacencies[v].size()
```

Graphs – Implementation Tips

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adjacencies[v].size()
```

- (BTW... Why only for directed graphs?)

Graphs – Implementation Tips

- Removing an edge is also simple (plus/minus complications with first locating the edge). To remove elements from a vector (inefficient, but simple in terms of syntax):

  http://www.mochima.com/tutorials/vectors.html#insert_remove

Graphs – Implementation Tips

- If you want to use a linked list (std::list) for the list of edges, removing is more efficient, but the code as a whole gets slightly more complicated — see my STL tutorial for details: http://www.mochima.com/tutorials/STL.html
- The code to remove edge (u,v) goes more or less like:

Graphs – Implementation Tips

```
for (list<Edge>::iterator e = adjacencies[u].begin();
                          e != adjacencies[u].end();
                          ++e)
{
    if (e->vertex() == v)
    {
        adjacencies.erase (e);
        break;
    }
}
```

Graphs – Implementation Tips

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- Additionally, they have the advantage of more efficient access for things like operations on each of the adjacent vertices to a given vertices (such as Prim's and Dijkstra's algorithms)
  - We just need to iterate over the elements of the linked list or array of edges (the “row”)

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- Additionally, they have the advantage of more efficient access for things like operations on each of the adjacent vertices to a given vertices (such as Prim's and Dijkstra's algorithms)
  - We just need to iterate over the elements of the linked list or array of edges (the “row”)
  - So this is also an advantage only when we have few edges.

Graphs – Implementation Tips

- Adjacency lists are not particularly efficient, for example, to test whether a vertex is adjacent to another vertex — the sample below checks whether vertex v is adjacent to vertex u:

```
for (vector<Edge>::size_type e = 0;
     e < adjacencies[u].size();
     ++e)
{
    if (adjacencies[u][e].vertex() == v)
    {
        return true;
            // assuming a function/method
    }
}
return false;
```

Graphs – Implementation Tips

- With a linked list, the loop would go more or less like:

```
for (list<Edge>::iterator e = adjacencies[u].begin();
                          e != adjacencies[u].end();
                          ++e)
{
    if (e->vertex() == v)
    {
        return true;
    }
}
return false;
```

Graphs – Implementation Tips

- Let's take a look at the implementation using the Adjacency matrix approach...

Graphs – Implementation Tips

- Two-dimensional dynamic arrays in C++ can be conveniently implemented as an array of arrays (that is, a vector of vectors).

Graphs – Implementation Tips

- Two-dimensional dynamic arrays in C++ can be conveniently implemented as an array of arrays (that is, a vector of vectors).
- The main difference is that we want to have the allocated full-size for all rows right from the start.
  - So, in the constructor we would do something like:

Graphs – Implementation Tips

```
Graph::Graph (int n)
    : adjacencies(n+1)
{
    for (vector<···>::size_type i = 1; i <= n; ++i)
    {
        adjacencies[i].resize(n+1);
    }
}
```

See http://www.mochima.com/tutorials/vectors.html#resize for more details.

Graphs – Implementation Tips

- The data type of `adjacencies` in this case would be `vector< vector<bool> >` if an unweighted graph (we just store `true` in `adjacencies[u][v]` to indicate that there is an edge from u to v), or `vector< vector<double> >` if weighted.

Graphs – Implementation Tips

- The data type of `adjacencies` in this case would be `vector< vector<bool> >` if an unweighted graph (we just store `true` in `adjacencies[u][v]` to indicate that there is an edge from u to v), or `vector< vector<double> >` if weighted.
  - In this sense, the implementation for a weighted graph is a little bit simpler (really, just a liiitle bit)

Graphs – Implementation Tips

- So, what types of operations are efficient with an adjacency matrix?

Graphs – Implementation Tips

- So, what types of operations are efficient with an adjacency matrix?
  - Checking if two given vertices are adjacent is quite trivial — example to check if vertex v is adjacent to vertex u:

```
bool Graph::adjacent (int u, int v) const
{
    return adjacencies[u][v];
}
```

Graphs – Implementation Tips

- That was for an unweighted graph (adjacencies stores bool values). For a weighted graph, assuming non-negative weights:

```
bool Graph::adjacent (int u, int v) const
{
    return adjacencies[u][v] >= 0;
}
```

Graphs – Implementation Tips

- Things that require going over each adjacent vertex tend to be less efficient:

```
for (vector<double>::size_type i = 1;
     i < adjacencies[u].size();
     ++i)
{
    if (adjacencies[u][v] >= 0)
    {
        // do whatever is required
    }
}
```

Graphs – Implementation Tips

- With either approach (adjacency list or adjacency matrix), one important advantage is that a big portion of the memory management is taken care of for you — classes vector and list encapsulate all the memory management aspects.
  - Their constructor, destructor, copy-constructor, and assignment operators handle all the details.

Graphs – Implementation Tips

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  - If you create your linked list class, you'd provide a constructor, destructor, copy-constructor, etc.

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  - If you create your linked list class, you'd provide a constructor, destructor, copy-constructor, etc.
  - So, if you use that linked list as a data member in your class Graph, you wouldn't need to provide a destructor for Graph (since the data member encapsulates all the functionality required).

Summary

- During today's class, we:
  - Finished off the topic of graphs.
  - Discussed some implementation details and tips.
  - Looked into standard library facilities vector and list
    - Vector useful for both adjacency lists and adjacency matrix
    - List requires iterators for accessing elements.