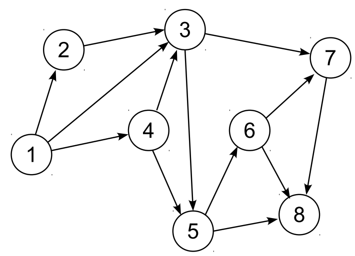
ECE-250 – Algorithms and Data Structures (Winter 2012)

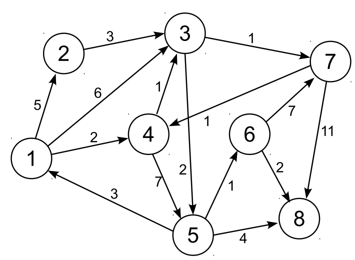
Assignment 5

Due on Friday, March 30, in class.

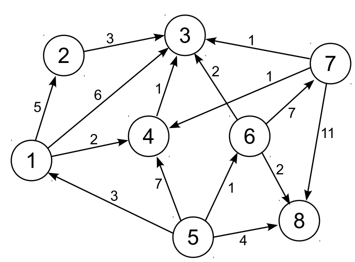
1 – (30 pts) Given the following Directed Acyclic Graph (DAG), run a topological sort, showing the steps and the contents at each step of any arrays used.



2 – (30 pts) Run Dijkstra's shortest path algorithm to find the shortest path from vertex 1 to vertex 5 in the following graph. You must show the steps, and the contents at each step of any arrays used (including the pointers to "previous" node).



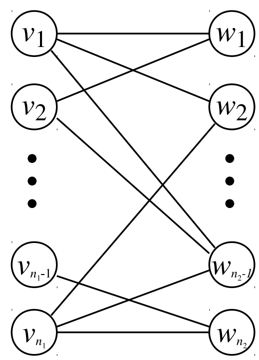
3 – (30 pts) Run Dijkstra's shortest path algorithm to find the shortest path from vertex 1 to vertex 5 in the following graph, to show that there is no such path. Explain the stop condition for the algorithm and how it reveals that no such path exists.



4 – (10 pts) We recall that a Hamiltonian cycle in a graph is a simple path that is a cycle (that is, a cycle that visits each vertex exactly once, except for the first and last vertex, which are the same vertex since it is a cycle).

We have an undirected graph where the vertices can be grouped into two sets $\mathcal{V} = \{v_1, v_2, \cdots, v_{n_1}\}$ and $\mathcal{W} = \{w_1, w_2, \cdots, w_{n_2}\}$ and every edge has the form (v_i, w_j) or (w_i, v_j); that is, every edge in the graph can only associate a vertex from $\mathcal{V}$ with a vertex from $\mathcal{W}$.

The figure below shows an example of such a graph (you can think of a graph where we have vertices at the left and vertices at the right, and edges always go between a vertex in the left group and a vertex in the right group).



Prove that a graph with the above characteristic and an odd number of vertices can not have a Hamiltonian cycle.

Hint/Suggestion: Use mathematical induction on the number of vertices; with the somewhat tricky aspect that in the induction step you only need to show that the statement being true for n vertices implies that it must be true for $n+2$ (since the statement only holds for n odd).

To prove the induction step, you could try proving the contrapositive — that is, show that if a graph of $n+2$ vertices (n odd) with the above characteristic has a Hamiltonian cycle, then a graph with n vertices also has a Hamiltonian cycle.

10% Bonus Marks – The subset sum problem is known to be NP-complete. However, in this problem we work with an array of arbitrary numbers (in particular, unsorted values). Suppose that someone attempted to solve the problem efficiently if the input values are in order.

Prove that this problem (subset sum with the input values in ascending order) is also NP-complete.