Legged Mechanism Design with Momentum Gains

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Abstract— **There are two main goals for any mobile, bipedal system: locomotion and balance. These behaviors both require the biped to effectively move its center of mass (COM). In this work, we define an optimization framework which can be used to design a biped that maximizes its ability to move its COM, without having to define an associated controller or trajectory. We use angular momentum gain in our objective function, a measure of how efficiently a system can move its COM based on its physical properties. As a comparison, we also optimize the model using a cost of transport-based objective function over a set of trajectories and show that it provides similar results. However, the cost of transport calculation requires slow hybrid dynamics equations and hand-designed trajectories, whereas the angular momentum gain calculation requires only the joint space inertia matrix at each configuration of interest.**

I. INTRODUCTION

The two critical performance objectives of most bipedal systems are its ability to locomote and to balance. Initially these appear to be fundamentally different behaviors:

- The goal of balancing is to keep the biped's center of mass (COM) in a desired upright pose or on a trajectory, without undesired changes in the contact surface(s).
- The goal of biped locomotion (gait) is to move the COM in space, via repeated changes in contact.

However, both of these behaviors can be more easily accomplished when a biped is able to efficiently move its COM relative to the contact(s). When balancing, the COM is moved towards a point above the contact(s), while for gait it is moved between a series of contact points.

This is true both in the static case, where the COM should be maintained above the contact surface(s), and the dynamic case, where the COM is moved between contact surfaces. In both of these cases, compensating for external disturbances also typically requires COM movement. Therefore, a biped which can efficiently move its COM relative to its contact(s) should be excellent at these behaviors.

The ability of a biped to balance and walk is impacted by both the physical properties of the biped and the controller used to achieve the desired behavior(s). In this paper, we develop an optimization framework which can be used to design a biped with excellent balance and gait capabilities within a specified motion space, regardless of the controller used to achieve those behaviors.

The objective function used in the optimization is based on the model's *angular momentum gain*, a measure of how effectively a mechanism can move its COM initially proposed in [1] for 2-link planar inverted pendulum models, and extended to general 2D and 3D models in [2].

Angular momentum gain is a measure of how efficiently an articulating system balancing on a passive (point or rolling) contact can move its COM via actuated joint motions. It is independent of the controller used, invariant to gravitational or velocity product dynamics, and (when balancing on a point or line contact) independent of the contact angle.

In this paper, we use angular momentum gain to quantify and optimize the efficiency of COM movement based purely on the physical properties of a mechanism. The proposed approach is demonstrated on a 5-link biped mechanism.

The rest of the paper is organized as follows: After summarizing the related work in Section II, we describe our optimization framework and our proposed objective function based on angular momentum gain in Section III. We demonstrate the capabilities of this framework using a 5-link planar biped in Section IV. We compare the results of our proposed objective function to a cost of transport-based objective in Section VI. Finally, conclusions and suggestions for future work can be found in Section VII.

II. RELATED WORK

Several research groups have used optimization to generate dynamic parameters for bipeds [3]–[9]. In these examples, the objective is to generate a gait for the biped in tandem with selecting its physical properties, using either the number of steps or the cost of transport as an optimization metric.

In [3], [4], evolutionary computing and genetic algorithms were used to generate dynamic properties and control parameters (or, somewhat equivalently, the trajectory) in tandem. These were the first examples of using optimization to generate the properties of biped mechanisms.

More recently, a general framework was developed to extract design principles from biology [5], [6]. After observation of a biological system, principles are transferred to a non-dimensionalized design space, which is then sampled and tested using an optimized controller to determine if the principle holds for the proposed design space.

An optimization has also been developed to simultaneously generate gait and design parameters in [7], although the only design parameter that is included in the optimization is a spring constant between the model's thighs. A hybrid zero dynamics approach [10] is used to reduce the biped to a 1 DOF system, controlled with a trajectory tracking controller.

In [8], a simulation framework was used to optimize the design and control of bipeds in parallel. The key differences between [8] and classic biped optimization methods such as

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[3], [4] are the inclusion of non-periodic gait and full 2D and 3D dynamics in the simulation and optimization.

This approach was then extended in [9] for the optimal design of compliant bipedal gait trajectories, where the robot's dynamic parameters were fixed and the spring constants and trajectories (state and input) were optimized. Although their goals are different, in our work we will also ignore the instantaneous dynamics of stepping and replace the stance ankle with a passive rotational joint as was done in [9].

These works incorporate an optimization of the dynamic properties of a biped either in parallel with optimizing a controller or using an existing controller. This leads to bipedal mechanisms which are optimized for that specific controller. They all also use the cost of transport as their objective function, with a few also using the squared norm of torque for comparison. Both of these cost functions are dependent on the controller and/or trajectory used for analysis.

An alternative metric for mechanism design was proposed by Featherstone in [1]: A set of dynamic ratios he defined as *velocity gains* for a given mechanism. These velocity gains are independent of the control scheme used, as they are functions only of the properties and configuratoin of the mechanism, and provide an upper bound on how well any controller could balance the given mechanism.

The gains are invariant to a scaling of the total mass of the system, and the angular velocity gain is also invariant to a scaling of length, allowing the balancing capabilities of an entire class of mechanisms to be quantified with a single metric [1]. He also defined a set of *momentum gains* for 2-link planar systems, which we have extended and expanded to general 2D and 3D systems in [2]. We propose to use these expanded momentum gains, specifically the generalized 2D angular momentum gain [2], as our objective metric.

III. Optimization Framework

In this paper, we develop a general optimization-based framework for designing parameterized mechanisms without the need for a controller. To achieve this goal, the objective function is formulated solely using the physical properties of the system within a desired motion space. This enables the framework to find a mechanism's fundamental limits on the desired behavior, independent of the controller used to achieve the behavior (typically gait or balance[1]).

Specifically, we find the parameters of the model which maximize the *potential* of the given mechanism to balance and locomote, independent of the controller used to achieve those goals. This gives an upper limit on how well the mechanism can balance and walk using any controller in a range of configurations near the desired motion subspace, avoiding overfitting to a specific controller and/or trajectory.

The framework requires five main elements to be defined:

- A *model*, defining the parameterized mechanism (including the number of links, joint details, etc.);
- The modifiable parameters $\boldsymbol{x}$ of the model (e.g., link mass, length, COM), with their allowable upper and lower bounds ($\boldsymbol{x}_{min}$ and $\boldsymbol{x}_{max}$, can be $\pm\infty$ if desired);
- A parameter map $\boldsymbol{X}$, which dictates how to assign a given set of parameters $\boldsymbol{x}$ to the model;
- A set of key poses $\boldsymbol{Q} = \{\boldsymbol{q}_1, \boldsymbol{q}_2, \dots\}$ in the model's configuration space (including the passive joint); and
- An objective function $J(\boldsymbol{x})$ which quantifies a model's ability to achieve a desired behavior for a given $\boldsymbol{x}$.

Once these five elements have been selected, a global optimization is used to find the optimal parameterization:

$$\max_{\boldsymbol{x}} J(\boldsymbol{x}), \quad \text{s.t.} \quad \boldsymbol{x}_{min} \le \boldsymbol{x} \le \boldsymbol{x}_{max} \tag{1}$$

The model must define the overall morphology of the desired mechanism. This will typically include the relative positions and orientations of the joints, motion freedoms of the joints, and the links' inertial properties. Any aspect of the mechanism which cannot be changed or modified as part of the optimization is included in the model definition.

Aspects of the mechanism to be optimized are then defined as the modifiable parameters $\boldsymbol{x}$ of the system. This will typically consist of link properties (e.g., mass, length, inertia), but can also include joint directions, relative orientations, or any other desired model property.

A set of upper and lower bounds on the parameters, labeled $\boldsymbol{x}_{min}$ and $\boldsymbol{x}_{max}$ must also be defined. For ease of specification and exploration, these limits can be set equal to each other to fix a parameter at a particular value or can be set to $\pm\infty$, as appropriate, to allow unbounded exploration.

Once the model and parameters are defined, a mapping $\boldsymbol{X}$ is required which assigns a given set of parameters onto the model. This enables the use of an existing modeling platform to be used in the objective function calculations.

As an example, in 2D the scalar inertia of each link about its COM can be parameterized using a ratio between the link's radius of gyration and its length. As part of the mapping, these ratios would be converted to an equivalent inertia, using the (possibly also parameterized) length and mass of the link, and applied to the dynamic model.

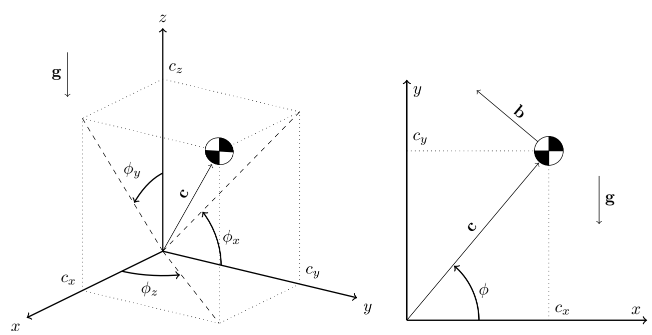
The key poses $\boldsymbol{q}_i$ in $\boldsymbol{Q}$ can be chosen to cover the entire configuration space of a given model, or can be used more selectively to focus the optimization on a particular subset of the space. Whether some or all of the joints are passive or active, the full configuration of the model must be specified for each key pose in the set $\boldsymbol{Q}$.

Finally, the objective function $J(\boldsymbol{x})$ generates a scalar value which quantifies the ability of a particular parameterization of the model to achieve a desired behavior. In general, this means that $J(\boldsymbol{x})$ uses the parameter map $\boldsymbol{X}$ to assign the parameters $\boldsymbol{x}$ to the given model, and then computes an objective value based on the model's kinematic or dynamic properties for the set of key poses $\boldsymbol{Q}$.

A. Objective Function

Motivated by our goal to optimize a biped without the need for a controller, we use the angular momentum gains for general 2D and 3D mechanisms defined in [2] as objective

[1]Here we assume that the angular momentum about the COM is negligible or regulated, so balance is only concerned with the COM motion.



(a) 3D Coordinates (b) 2D Coordinates

Fig. 1. In 3D, $\boldsymbol{c} = [c_x \; c_y \; c_z]^T$ is a vector from the contact point to the COM and the angles $\boldsymbol{\phi} = [\phi_x \; \phi_y \; \phi_z]^T$ are measured from the reference frame to $\boldsymbol{c}$. In 2D, the COM vector simplifies to $\boldsymbol{c} = [c_x \; c_y]^T$, the vector $\boldsymbol{b} = [-c_y \; c_x]^T/c$ is perpendicular to $\boldsymbol{c}$, and the angle ϕ is a scalar.

function candidates. Angular momentum gain $\boldsymbol{G}_o$ is defined as the change in angular momentum about the contact point due to COM motion caused by an instantaneous change in joint torque $\Delta\tau_a$, called an impulse and labeled $\boldsymbol{\iota}_a$ [2]:

$$\boldsymbol{G}_o(\boldsymbol{\iota}_a) = mc^2\Delta\dot{\boldsymbol{\phi}} = m\boldsymbol{c} \times \Delta\dot{\boldsymbol{c}} \tag{2}$$

where $\boldsymbol{c} = [c_x \; c_y \; c_z]^T$ is a vector from the contact point to the COM with length $c = ||\boldsymbol{c}||_2$, the change in COM velocity is $\Delta\dot{\boldsymbol{c}} = [\Delta\dot{c}_x \; \Delta\dot{c}_y \; \Delta\dot{c}_z]^T$, the change in angular COM velocity about the contact point is $\Delta\dot{\boldsymbol{\phi}} = [\Delta\dot{\phi}_x \; \Delta\dot{\phi}_y \; \Delta\dot{\phi}_z]^T = (\boldsymbol{c} \times \Delta\dot{\boldsymbol{c}})/c^2$, and the total mass is m (see Figure 1a).

Note that in this equation, it is assumed that $\boldsymbol{\iota}_a$ is a unit step impulse (i.e., $||\boldsymbol{\iota}_a|| = 1$) [2]. The gain is divided by the magnitude of the step impulse, making $\boldsymbol{G}_o$ dimensionless.

In the planar case, the gain (G_o) and change in angular COM velocity ($\Delta\dot{\phi}$) reduce to scalars, and the change in linear COM velocity ($\Delta\dot{\boldsymbol{c}}$) and COM vector ($\boldsymbol{c}$) reduce to 2D vectors. Defining $\boldsymbol{b} = [-c_y \; c_x]^T/c$, a 2D unit vector perpendicular to $\boldsymbol{c}$ (see Figure 1b), the gain for general planar models is (with $||\boldsymbol{\iota}_a|| = 1$ again, making G_o dimensionless):

$$G_o(\boldsymbol{\iota}_a) = mc^2\Delta\dot{\phi} = c\,(\boldsymbol{b} \cdot m\Delta\dot{\boldsymbol{c}}) \tag{3}$$

where $\boldsymbol{c} = [c_x \; c_y]^T$ is the vector from the contact point to the COM (with length $c = ||\boldsymbol{c}||_2$), the change in COM velocity is $\Delta\dot{\boldsymbol{c}} = [\Delta\dot{c}_x \; \Delta\dot{c}_y]^T$, the change in angular COM velocity about the contact point is $\Delta\dot{\phi} = (\boldsymbol{b} \cdot \Delta\dot{\boldsymbol{c}})/c$, and the impulse at the joints is $\boldsymbol{\iota}_a$ (which must satisfy $||\boldsymbol{\iota}_a|| = 1$).

Note that G_o is defined using the change in the angular momentum about the contact due only to COM motion. This is not the change in total angular momentum about the contact, which is always 0 for a passive rotary joint [1]: the total angular momentum about a passive rotary joint cannot be changed by any impulsive change in a robot's joint angles.

Based on the comparison in [2], we use angular momentum gain to define the objective function for four reasons:

- Momentum gains incorporate inertial information, quantifying the effort needed at the actuated joints to generate COM motion (either for balancing or gait).
- Angular gains are dimensionless, removing any dependence on the total length of the system and allowing link lengths to be parameterized as ratios.
- Angular gains for mechanisms with a point contact (line contact in 3D) are invariant to the passive contact angle.
- Angular momentum gain is defined everywhere, whereas the angular velocity gain approaches infinity as the COM nears the contact point and becomes undefined when the COM is at the contact.

The angular momentum gain is therefore an ideal candidate for use in our objective function: It is a dimensionless measure of how efficiently a system's actuated joints can move the COM around a passive contact, independent of the total mass, total length, gravity, and the passive contact angle between the stance leg and the ground.

Since the angular momentum gain is linear with respect to $\boldsymbol{\iota}_a$, we can define a gain vector $\boldsymbol{G}_{oa} = [G_{o2} \; G_{o3} \; \ldots]$ such that $G_o(\boldsymbol{\iota}_a) = \sum_{i \in a} G_{oi}\iota_i = \boldsymbol{G}_{oa}\boldsymbol{\iota}_a$. If we use the 2-norm to define the step impulse as $||\boldsymbol{\iota}_a||_2 = 1$, then the maximum angular momentum gain for any configuration and parameter pair $(\boldsymbol{q}, \boldsymbol{x})$ is given by $||\boldsymbol{G}_{oa}(\boldsymbol{q}, \boldsymbol{x})||_2$ [2].

The momentum gain based objective function J_G is then defined as the mean over $\boldsymbol{Q}$ of the maximum angular momentum gains (assuming $||\boldsymbol{\iota}_a||_2 = 1$):

$$J_G(\boldsymbol{x}) = \frac{1}{n_q}\sum_{\boldsymbol{q}} ||\boldsymbol{G}_{oa}(\boldsymbol{q}, \boldsymbol{x})||_2 \qquad \forall \boldsymbol{q} \in \boldsymbol{Q} \tag{4}$$

where n_q is the number of configurations in the set $\boldsymbol{Q}$.

IV. Example

In this section, the framework is illustrated on a 5-link planar biped (see Figure 2). This mechanism can be used as a simplified representation of a broad range of natural and artificial bipeds, including humans, ostriches, and others [6].

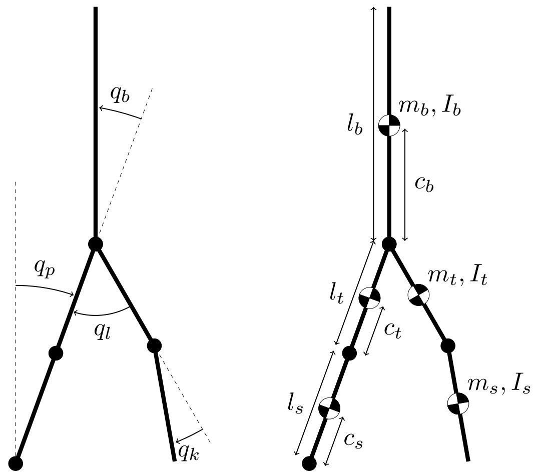
The parameters, parameter mapping and key configuration poses for this mechanism are outlined in the following subsections, along with an alternative objective function based on the Cost of Transport for comparison to our proposed objective function from Section III-A.

A. Parameters

We will use a modified version of the 5-link biped parameters defined by Haberland and Kim [6]. Since the model has a symmetric form, the legs are assumed to be identical so only one set of leg links are independently parameterized.

In [6], the mass and length of the body link are defined in units of kg and m, respectively, and all other lengths and masses are defined relative to these values. However, since our objective functions will be using measures which are invariant to a scaling of the total mass m or length l of the system, all parameters will be defined as ratios.

Four parameters are used for each independent link i: mass m_i, inertia $I_i = m_i r_i^2$, COM c_i, and length l_i. For each of the links, the COM and inertia (via the radius of gyration r_i) are defined relative to that link's length, while the link length and mass are defined relative to other links.



(a) Joint Labels (b) Parameters

Fig. 2. Diagram of the 5-link planar biped model used in the example. In (a), passive rotation about the contact point q_p is measured from a vertical axis, knee rotation q_k is measured relative to a straight leg, body link rotation q_b is measured relative to the stance thigh, and leg rotation q_l is measured from the swing thigh to the stance thigh. In (b), the link parameters are the mass m_i, length l_i, COM c_i, and inertia $I_i = m_i r_i^2$, where the index i is replaced with b, t, or s for the body, thigh, and shank links, respectively.

A minimal representation of these ratios is shown in Table I. We use the indexes b, s and t to indicate the body, shank and thigh links, respectively. The mass and length of the leg are then defined as $m_l = m_s + m_t$ and $l_l = l_s + l_t$, respectively, and the total mass and length of the biped are $m = m_b + 2m_l$ and $l = l_b + l_l$, respectively.

Note that in Table I, the body link's mass and length are scaled by the total mass and total length, respectively, while the thigh links' mass and length are scaled by the leg mass and length, respectively. This means that the shank mass and length do not need to be separately parameterized.

TABLE I
PARAMETER DEFINITIONS FOR 5-LINK BIPED

Parameter	**Equation**	**Min**	**Max**
Body Link Mass	m_b/m	1/2	3/4
Body Link Length	l_b/l	1/4	1/2
Thigh Link Mass	m_t/m_l	1/4	3/4
Thigh Link Length	l_t/l_l	1/3	2/3
All Link COMs	c_i/l_i	1/4	3/4
All Link Inertias	r_i/l_i	0	2/3

This defines a parameterization which can be satisfied strictly using lower and upper bounds, with 10 independent parameters: two mass ratios, two length ratios, and the COM and inertia for each of the three independent links.

The mapping $\boldsymbol{X}$ which converts these parameters to model quantities assumes $m = 50$ kg and $l = 2$ m for the purposes of calculating kinematic and dynamic properties. This allows the masses, lengths, COMs, and inertias to be defined in real quantities and applied to the model representation used in the objective function. However, the measures used in the objective function are invariant to m and l, so these values could be chosen arbitrarily with the same results.

B. Key Poses

For this example, the key poses are used to outline a set of typical motion paths within the configuration space of the biped model. This set of poses is chosen to be representative of the swing phase of a standard walking gait, where the swing leg starts on the ground behind the stance leg and finishes in the same pose but with the leg positions switched.

The leg poses are assumed to be symmetric in the starting/ending configuration. In this type of gait, the stance knee is fully extended for the duration of the stance phase following the results of [11], who showed that optimal periodic gaits for simple bipeds always involve pendular motion in the stance phase, due to the elimination of work when the system is acting as an inverted pendulum.

The considered motion subspace in the biped's configuration space is defined by the following joint angle ranges:

$$\begin{aligned} -\pi/6 \leq \quad & q_p \leq \pi/6 \\ 0 \leq \quad & q_k \leq \pi/3 \\ -\pi/3 \leq \quad & q_l \leq \pi/3 \\ -\pi/4 \leq \quad & q_b \leq \pi/4 \end{aligned} \tag{5}$$

To ensure a fair comparison to the cost of transport based objective function defined below, only configurations which are in the CoT trajectories will be sampled to generate key poses. Compared to the larger motion subspace above, this primarily limits the motion of the body link to within $\pi/12$ of vertical and limits the swing leg joint angles to remain within typical walking ranges.

C. Comparison Objective Function

In addition to the momentum gain-based objective function defined in Section III-A, we also evaluate an alternative objective function based on the Cost of Transport (CoT) (the most commonly used cost function in the literature, e.g., [7]–[9]). The CoT for a system is defined as a ratio between the energy consumed (W) and the product of the system's weight (mg) and the distance (d) it travels while consuming said energy: W/mgd. For this objective function, we evaluate the energy required to move along a given trajectory parameterized as shown in Equation (6) below.

Around a nominal swing phase pattern (with a fully extended stance knee and symmetric start/end configurations), the initial/final and midstance poses are varied to increase the effective motion subspace being optimized over.

These variations are parameterized using:

- The initial angle between the legs, $\pi/9 \leq 2\theta_p \leq \pi/3$;
- The swing knee angle at midstance, $0 \leq \theta_k \leq \pi/3$; and
- The body link angle at midstance, $-\pi/12 \leq \theta_b \leq \pi/12$.

If we define the joint positions as $\boldsymbol{q} = [q_p \; 0 \; q_b \; q_l \; q_k]$, using the labels shown in Figure 2a, the configurations at the start, middle, and end of the step (with smooth transitions between these poses based on quintic splines) are:

$$\begin{aligned} \boldsymbol{q}_0 &= [-\theta_p \quad 0 \quad -\theta_p \quad -2\theta_p \quad 0\ ]^T \\ \boldsymbol{q}_m &= [\ \ 0 \quad 0 \quad \theta_b \quad \theta_k \quad \theta_k]^T \\ \boldsymbol{q}_f &= [\ \ \theta_p \quad 0 \quad \theta_p \quad 2\theta_p \quad 0\ ]^T \end{aligned} \tag{6}$$

These trajectories are then used to generate the required joint torques to achieve required motions via hybrid dynamics, to ensure that the trajectories are dynamically feasible and satisfy the dynamic constraints of the model. These joint torques are not necessary for the momentum gain based optimization, but are used in the comparison objective function based on the cost of transport below.

For our purposes, since the ground is flat and impact is ignored, we assume the energy consumed is $W = \int \boldsymbol{\tau}_a \cdot \dot{\boldsymbol{q}}_a dt$, where $\boldsymbol{\tau}_a$ is a vector of actuated joint torques. This is similar to the CoT from [9] without the absolute power assumption, which itself is a modified version of the CoT from [8] when dealing with flat ground and no impact.

The objective function then minimizes the average of this CoT over the set of n_ψ trajectories $\boldsymbol{q}(t) \in \boldsymbol{Q}_\psi$:

$$J_{CoT}(\boldsymbol{x}) = -\frac{1}{n_\psi} \int \frac{\boldsymbol{\tau}_a^T \dot{\boldsymbol{q}}_a(t)}{mgd} dt \qquad \forall \boldsymbol{q}(t) \in \boldsymbol{Q}_\psi \tag{7}$$

where the step length d is defined as the distance between the feet at both the start and end of the step.

V. Results

The results of optimizing the 5-link biped defined in the previous section are shown in Table II and Figure 3. These results show that optimizing for the cost of transport over a set of trajectories provides almost identical results to optimizing for the average angular momentum gain over a comparable configuration space. The only difference between the two results is the mass of the body link: for the CoT results $m_b = m/2$, while for the G_o results $m_b = 3m/4$.

The resulting mechanism obtained by optimizing the angular momentum gain has several characteristic properties: First, the body link has the maximum possible mass and length, and its COM is as far from the hips as possible. This confirms the observations in [2], where the large mass near the top of the body link enables small changes in stance hip angles to produce large COM angular displacements.

Much like the body link, the swing leg has also been optimized to place most of the leg mass near the foot and the remaining mass very close to the hip, allowing both swing leg joints to move the system's COM around with minimal effort. The long shank length, relative to the thigh, ensures that the swing knee can produce maximal COM motion even if the swing hip is held fixed.

It is interesting to note that, although it was an available parameter, the inertia of all of the links has been eliminated. Although this is not realistic in a real world robot, it is feasible in a model like this as the addition and subtraction of virtual masses at the joints can add inertia back to the links (as described in [1]).

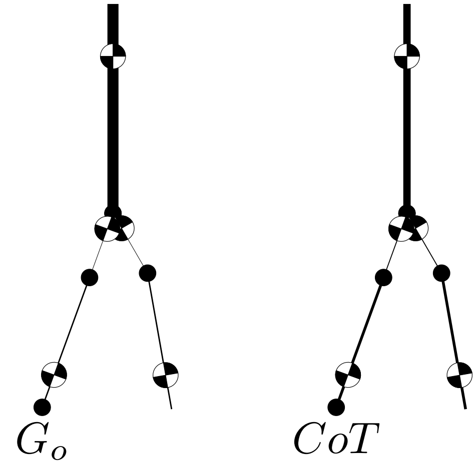


Fig. 3. Diagrams of the 5-link biped models which correspond to the optimization results given in Table II. In this diagram, the relative thicknesses of each link denote their relative masses, quartered circles show the locations of each link's COM, and solid circles are the joints.

TABLE II
RESULTS OF OPTIMIZATION

	Objective Function Basis	
Parameters	Angular Momentum	Cost of Transport
m_b/m	3/4	1/2
l_b/l	1/2	1/2
m_t/m_l	1/4	1/4
l_t/l_l	1/3	1/3
c_b/l_b	3/4	3/4
c_t/l_t	3/4	3/4
c_s/l_s	1/4	1/4
r_b/l_b	0	0
r_t/l_t	0	0
r_s/l_s	0	0

The only difference between the two optimizations is the relative mass of the body link to the total mass. The mass of the body link has been reduced to its minimum possible value in the model optimized for the cost of transport. This is likely due to the cost of transport having no concept of balancing outside of the given trajectories, as well as incorporating gravitational effects which make heavier feet slightly cheaper due to pendular swing leg motion.

VI. Discussion

The similarity in the results is expected, as both the CoT and the G_o optimizations should produce a mechanism which can efficiently move its COM around in the plane. Note that the G_o based optimization was able to achieve similar results to the CoT optimization, without the need for trajectories, torques, an integration over time, or a controller. Despite this similarity in results, there are four main differences between the angular momentum gain and CoT approaches:

First, the cost of transport approach requires a trajectory and/or a controller to be defined. These elements could be either specified [12], or co-optimized in parallel with the physical optimization [8], [9]. The angular momentum gain approach requires only a desired configuration space, defined using a set of key poses which span the space.

Second, the cost of transport approach must include some form of hybrid or inverse dynamics calculation over time to determine the work required to take a step. By comparison, the angular momentum gain approach requires only the calculation of the joint space inertia matrix (or generalized inertia matrix, for systems with kinematic loops) for each configuration of interest, and the inverse of a positive definite symmetric submatrix.

Third, the cost of transport depends directly on the scale of the mechanism (i.e., its total mass and length), which means that it is only effective for a specific design. The angular momentum gain, however, is invariant to scaling of the total mass and/or total length, as well as to various modifications of the inertial properties as discussed in [1], which enables it to optimize an entire family of mechanisms at once. This is primarily due to defining the gain using impulsive dynamics, which do not include gravitational terms or any velocity-product terms (e.g., Coriolis terms).

Finally, the cost of transport is concerned only with how much effort it takes the mechanism to follow the prescribed trajectory and/or use the prescribed controller. This may result in a system that has excellent performance near the nominal trajectory, but suffers from poor performance if disturbed away from the nominal trajectory. With angular momentum gain, the inherent physical ability of the mechanism to move the COM with minimal effort is maximized.

To improve the biped's balance not only along the specified trajectories generated for the CoT optimization but also throughout the configuration space near them, the motion subspace defined in Equation (5) should be uniformly sampled to generate a set of key poses for a subsequent application of the angular momentum gain optimization.

VII. Conclusion

In this paper, we have introduced a general optimization framework for the design of parameterized mechanisms using angular momentum gain. Since the gain is invariant to scaling of total mass and total length, entire families of mechanisms can be optimized in one application of the framework.

We demonstrated the usefulness of this framework and objective function using a 5-link biped mechanism, and showed that the results were very similar to those found using an objective function based on the cost of transport (the typical objective for mechanism optimization), while requiring less computational effort and not limited to a specific controller or trajectory.

One direction for future work would be the use of these gains, in tandem with other objective metrics, for the design of complex and 3D systems. An important step in this direction would be an examination of how to more effectively compare velocity and momentum gains in a mechanism's configuration space. Since the configuration space of typical complex and 3D systems are not human-readable in most cases, this work could augment the existing tools available for designing these types of systems with an understanding of how effectively the mechanism could move its COM.

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