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Constrained Dynamic Parameter Estimation using the Extended Kalman Filter

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Abstract— **In this paper we present a real-time method for identification of the dynamic parameters of a manipulator and its load using kinematic measurements and either joint torques or force and moment at the base. The parameters are estimated using the Extended Kalman Filter and constraints are imposed using Sigmoid functions to ensure the parameters remain within their physically feasible ranges, such as links having positive masses and moments of inertia. Identified parameters can be used in model based controllers. The presented approach is validated through simulation and on data collected with the Barret WAM manipulator. Using the estimated parameters instead of ones provided by the manufacturer greatly improves joint torque prediction.**

I. INTRODUCTION

Dynamic parameters (masses, centers of masses, inertia tensors, and motor friction) describing a manipulator are imperative for accurate model based control. Unfortunately manufacturers often do not provide parameters for each robot. One approach would be to measure the parameters of each link separately, but this requires disassembly of the robot and is time consuming. Another approach is the use a CAD model but the variability in materials and manufacturing differences make this method inaccurate, Rackl showed that using weighted least-squares optimization for identification outperforms using the CAD model for torque prediction [1]. Furthermore neither disassembly nor CAD modeling approaches provide accurate actuator friction coefficients. A single manipulator may be required to perform a variety of tasks and thus handle changing external loads. This leads to a constantly changing set of parameters at the end effector. An online parameter estimation method is desired so the model based controller can be adapted as the manipulator handles different objects.

Equations of motion for a manipulator depend on the kinematic variables (position, velocity and acceleration) as well as the parameters to be identified. Therefore any identification method relies on joint trajectories such that the parameters have an effect on the joint torques, these are called *exciting* trajectories [2]. Rackl proposed using static poses to identify static parameters (mass and center of mass) and generating exciting trajectories based on B-Splines to estimate the inertia tensor [1]. Swevers used finite Fourier series as joint trajectories to allow for analytical differentiation of the trajectory, periodic excitation, and calculation of noise characteristics [3]. Depending on the kinematic structure of the robot some parameters may always appear in linear combinations and will not be identifiable uniquely [4]. In that case either the equations of motion in terms of the linear combination must be derived and the linear combination identified or a non unique solution to the parameters in the linear combination must be found to best predict the joint torques.

A number of previous offline and online identification methods using equations of motion have been proposed. Atkeson *et al.* used least squares to estimate the inertial parameters of the load and the links and verified the approach on the PUMA robot [5]. Gautier and Poignet compared weighted least squares and Extended Kalman Filter approaches for parameter estimation of a 2 degrees of freedom robot [6] however they did not consider constraining the parameters. Both of the methods rely on inverting potentially singular matrices or deriving the reduced set of identifiable parameters, also they do not guarantee that the estimates will be physically feasible. Mata proposed using the Gibbs-Appell dynamic equations and optimization to ensure physical feasibility of the parameters [7], unfortunately optimization techniques are computationally expensive and are difficult to run online.

Other approaches to dynamic model identification consider the robot as a non linear system and find the mapping between input joint positions, velocities, and accelerations, and the output torques. Jiang *et al.* used a neural network to aid the dynamic model after an initial set of parameters has been identified [8]. Duy Nguyen-Tuong *et al.* used Local Gaussian Process Regression to learn the mapping online [9]. While the non linear approaches can accurately predict the torques given kinematic measurements they do not compute the parameter values explicitly and thus may not generalize to all regions of the manipulator state space.

In this paper we present a real time method to estimate the dynamic parameters of a manipulator and its load using kinematics measurements (joint positions, velocities, and accelerations) as well as joint torques or force and moment at the base of the robot. The parameters to be estimated are set as the state vector of the Extended Kalman Filter (EKF) and we propose the use of Sigmoid functions to satisfy constraints. EKF ensures that segment inertial parameters will converge to a steady value and only parameters which are excited by the motion will update. Derivation of the complex linear inverse dynamic model is avoided by relying on the Recursive Newton Euler method. The algorithm is validated in simulation to show convergence to the correct

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parameters of links and load as well as on real data from the Barret WAM robot showing that the estimated parameters significantly improve torque prediction.

The paper is organized as follows: Section II introduces the equations of motion for manipulators showing why parameter estimation is necessary for model based control. Section III describes the formulation of the Extended Kalman Filter with constraints to estimate the dynamic parameters. Section IV describes the results using both simulated and actual robot data. Finally Section V presents the conclusion and discussion.

II. BACKGROUND

A common approach to model manipulators is to use the Euler Lagrange equations of motion to express joint torques in terms of the difference between kinetic and potential energy. The energies can be computed from the joint positions, velocities, accelerations and the dynamic parameters, mass m_i, first-order moment $m_i r_i$ where r_i are the coordinates of the center of mass, and inertia tensor I_i, of each link. In matrix form the equations are written as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

Where $q, \dot{q}, \ddot{q}, \tau$ are the joint positions, velocities, accelerations, and torque, $D(q)$ is configuration dependent inertia matrix, $C(q,\dot{q})$ contains Coriolis and centrifugal effect terms, and $G(q)$ is the gravitational force [10]. Many previous methods rely on reformulating (1) as a linear equation of the dynamic parameters

$$y(\tau, \dot{q}) = \phi(q, \dot{q}, \ddot{q})p \tag{2}$$

where p is the vector of parameters [5].

To control a manipulator equation (1) is re-written in terms of control input u_k at each joint

$$(D(q) + J)\ddot{q} + C(q,\dot{q})\dot{q} + B\dot{q} + G(q) = u \tag{3}$$

where J and B describe the motor inertia and damping friction coefficients respectively and u is the control input. Thus for model based control it is important to accurately estimate the dynamic parameters of each link and friction coefficients of each joint.

III. PROPOSED APPROACH

Extended Kalman Filter (EKF) is used to adapt the dynamic parameters of the manipulator model based on noisy joint position, velocity, acceleration, and torque measurements. In order to constrain the parameters we propose to use a Sigmoid function to map the unconstrained EKF state vector into constrained parameter space. The measurement Jacobian of the Kalman filter ensures that only the parameters which are excited by the trajectory of the manipulator are estimated.

A. *Extended Kalman Filter formulation*

The Extended Kalman Filter is a common sensor fusion technique used to estimate the state of a system given noisy measurements by minimizing the trace of the error covariance matrix P. The next state estimate $\mathbf{s_t}$ and measurement update $\mathbf{z_t}$ are defined by

$$s_t = f(s_{t-1}) + w_{t-1} \tag{4}$$

$$z_t = h(s_t) + v_t. \tag{5}$$

where $\mathbf{f}$ is the process relating the previous state to the next and $\mathbf{h}$ relates the measurement to the state. $\mathbf{w}$ and $\mathbf{v}$ are zero mean Gaussian process and measurement noise with covariances $\mathbf{Q}$ and $\mathbf{R}$ respectively. Linearizing the state update and measurement prediction equations about the operating point EKF approximates the system as

$$\mathbf{z_t} \approx \tilde{\mathbf{z}}_\mathbf{t} + \mathbf{M}(\mathbf{s_t} - \tilde{\mathbf{s}}_\mathbf{t}) + \mathbf{v_t} \tag{6}$$

$$\mathbf{s_t} \approx \tilde{\mathbf{s}}_\mathbf{t} + \mathbf{A}(\mathbf{s_t} - \tilde{\mathbf{s}}_\mathbf{t}) + \mathbf{w_{t-1}}, \tag{7}$$

where $\mathbf{A}$ and $\mathbf{M}$ are the Jacobians of the state update and measurement equations with respect to the state $\mathbf{s}$, $\tilde{\mathbf{s}}$ is the noiseless state estimate, $\tilde{\mathbf{z}}$ is the noiseless measurement estimate [11].

B. *EKF State Vector*

The subset of the inertial parameters we want to estimate is the mass m_i, first-order mass moment $m_i r_i$, friction coefficients B, and the moments of inertia $[I_{i,xx} I_{i,yy} I_{i,zz}]$ at the center of mass for each link i of the manipulator. Writing the inertia tensor in the frame of the principal axes of inertia, we can omit estimating the off diagonal elements. It is important to note that the masses and moments of inertia must be positive to be physically consistent and guarantee asymptotic tracking using PD control [12]. For this reason we propose to use a Sigmoid function to map from the EKF state to the inertial parameters.

Consider the mapping from EKF state variable x_k to the inertial parameter p_k through the Sigmoid function

$$p_k = Sig(x_k) = \frac{a_k - b_k}{1 + e^{-cx_k}} + b_k \tag{8}$$

where a_k is the upper bound, b_k is the lower bound and c controls the slope. This constrains p_k within the (b_k, a_k) region without solving the optimization problem. To initialize EKF state using initial guess of the inertial parameters for each link the inverse of the Sigmoid functions is used

$$x_k = Sig_k^{-1}(p_k) = \frac{ln(a_k - p_k) - ln(p_k - b_k)}{-c}. \tag{9}$$

Thus state vector s_t is defined as

$$s_t = \left[Sig^{-1}(m_{1:n}\ I_{1:n,xx}\ I_{1:n,yy}\ I_{1:n,zz}\ m_{1:n}r_{1:n}\ B_{1:n})\right] \tag{10}$$

where separate Sigmoid functions are chosen to place bounds on each dynamic parameter and friction coefficient. The effect of the Sigmoid bounds on EKF tracking is shown in figure 1.

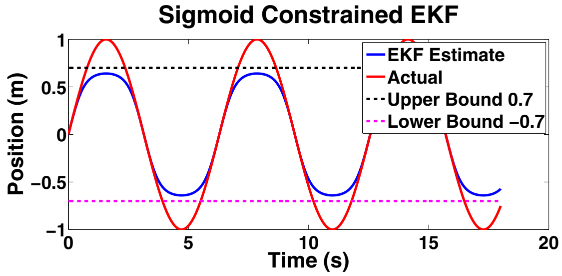

Fig. 1. Demonstrates constraining EKF using the Sigmoid function. In this example EKF is set up to track the position and velocity of one dimensional particle experiencing sinusoidal motion. The state vector s maps through Sigmoids with bounds (-0.7, 0.7) and (10,-10) to the position x and velocity $\dot{x}$ respectively. It is assumed that actual position and velocity can be directly measured. The EKF tracked position does not leave the imposed bounds and provides accurate tracking within.

Since the estimated parameters are constant the state update equation is

$$s_t = s_{t-1} \tag{11}$$

and it's Jacobian A is the identity matrix.

C. EKF Measurement Vector

We first consider manipulators that have encoders and torque sensors providing a measurement of the position q_t, velocity $\dot{q}_t$, acceleration $\ddot{q}_t$, and torque τ_t of each joint at each time step. While encoders are relatively accurate, especially for direct-drive manipulators, torque measurements often contain a lot of noise. Thus we define the noisy measurement vector of EKF as the torque at each joint and assume position, velocity, and acceleration are known. Given q_t, $\dot{q}_t$, $\ddot{q}_t$, the Recursive Neuton-Euler (RNE) algorithm works as follows to predict the force f_t and torque $\tilde{z}_i$ experienced by each joint:

First, the forward kinematics algorithm is used with q, $\dot{q}$, and $\ddot{q}$ from the encoders to compute the linear acceleration a_i, angular velocity w_i, and angular acceleration $\dot{w}_i$ of each link. Then starting at the end effector and working backwards to the base, the torque τ_i and force f_i experienced by each link are computed as

$$f_i = R^i_{i+1} f_{i+1} + m_i a_i - m_i g_i \tag{12}$$

$$\tau_i = R^i_{i+1}\tilde{z}_{i+1} - f_i \times r_i + (R^i_{i+1} f_{i+1}) \times r_{i+1} + \dot{w}_i + w_i \times (\hat{I}_i w_i) + B_i \dot{q}_i \tag{13}$$

where R^i_{i+1} is the rotation matrix from frame $i+1$ to frame i, g_i is gravity expressed in frame i and $\hat{I}_i$ is the moment of inertia tensor about the joint computed using parallel axis theorem

$$\hat{I}_i = I_i + m_i(r_i^T r_i \mathbf{1_{3x3}} - r_i r_i^T) \tag{14}$$

$\mathbf{1}$ is the identity matrix [13].

The parameters m_i, r_i, I_i, and B_i are computed as $Sig(s_t)$ using appropriate Sigmoid functions. The RNE algorithm has the benefit over the full equations of motion shown in (1) since only the structure of the robot must be known and the derivation of the inertia and Coriolis matrices is not necessary making it easy to use for multiple different manipulators.

Unfortunately many manipulators do not have torque sensors built into the joints but only encoders. However as long as measurement prediction and measurement Jacobian equations are available EKF can update the state. For example it is also possible to rewrite equation (1) in terms of the force experienced by the end effector:

$$\hat{J}^T(q)[D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)] = F \tag{15}$$

where $\hat{J}$ is the generalized Jacobian inverse [14]. Therefore while the manipulator is not in a singular configuration the force and moment at the end effector can also be used as the measurement vector for the EKF. Furthermore Ayusawa and Venture [15] showed that the set of inertial parameters appearing in the under-actuated base equations of motions is equivalent to whole body equations of motions. Thus it is also possible to estimate the inertial parameters for manipulators using only the force and moment at the base as the measurement vector for EKF.

D. Measurement Jacobian

The measurement Jacobian M is defined as the derivative of the measurement z_t with respect to the state vector s_t. In terms of estimated parameters the derivative can be written as

$$\frac{\partial z_t}{\partial s_t} = \frac{\partial z_t}{\partial p}\frac{Sig(s_t)}{\partial s_t} = \frac{\partial z_t}{\partial p}\frac{c(a-b)e^{cs_t}}{(e^{cs_t}+1)^2} \tag{16}$$

If a parameter is not excited by the trajectory of the manipulator, M will have a zero column and thus that parameter will not be updated using the measurement. This is beneficial as it is difficult to generate trajectories which excite all of the parameters simultaneously. Similarly as a parameter p_k approaches it's bounds (b_k, a_k) the derivative of the Sigmoid approaches zero leading to a zero column in M preventing the state from diverging further.

E. EKF Parameter Settings for Steady State

The EKF algorithm requires setting the initial error covariance matrix P_0 as well as the measurement and process noise parameters R and Q. By using finite Fourier series for identification trajectories the measurement noise variance can be computed as described in [3]. Initial error covariance is set to a diagonal matrix $P_0 = \mathbf{1}$ to represent some certainty in the initial parameters. Scaling P, Q, and R equally does not change EKF's results thus we only consider Q and R as tuning parameters and always set P_0 to identity [16].

It can be shown that EKF will drive the diagonal entries of P which correspond to states without any noise to zero and stop updating those states [17]. However initially the state vector should quickly adapt to the correct value. We set Q to $k\mathbf{1}$ where k starts as a large value and is annealed to zero to allow for quick parameter adaptation at the beginning of identification and ensuring steady state convergence [16].

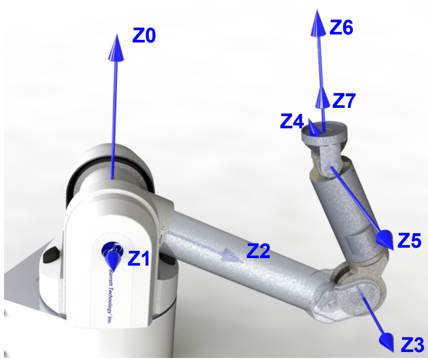

Fig. 2. Model of the 7 degrees of freedom Barret WAM [19].

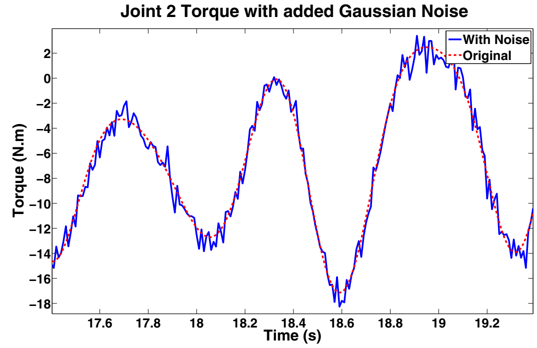

Fig. 3. Describes the amount of noise added for simulation experiments.

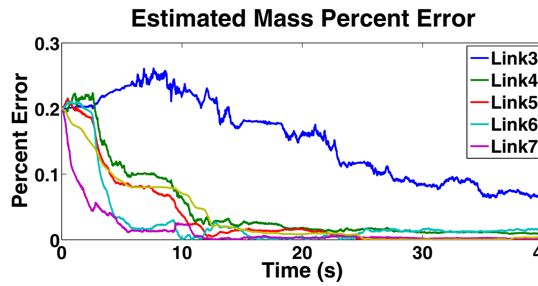

Fig. 4. EKF successfully estimates masses of each link for the Barret WAM using joint torques as measurement vector. Due to the structure of the robot masses of the first two links have no effect on joint torques thus EKF does not modify those parameters and the error stays at 20 percent, they are omitted from the figure. Since centers of mass and masses appear in linear combinations in the equations of motion for this test the centers of mass were excluded from the EKF state and were assumed known. A link's parameters are only excited by the torques applied to the joints before the link thus Kalman gain is zero for any torque in the joint after the link. This directly effects the rate of convergence and parameters of links appearing later in the chain converge faster.

IV. Experimental Results

The algorithm is evaluated in simulation and on real robot data using the 7 degrees of freedom, antrhopomirphic, direct drive Barret WAM manipulator [18] shown in figure 2. First we show that the algorithm converges to accurate inertial parameters given noisy joint torques as the EKF measurement vector and that it converges given noisy force and moment measurement at the base as the EKF measurement vector. Then we demonstrate that the algorithm can adapt to estimate a load with changing inertial parameters. Finally real robot data is used to show convergence of both dynamic parameters and friction coefficients to values that significantly improve prediction of torques at the joints.

A. Convergence

A simulated Barrett WAM was used to verify that the algorithm can estimate the inertial parameters in order to correctly predict torques at the joints. To make sure all of the inertial parameters are excited a finite Fourier series as described in [3] was used to generate the joint trajectories. Using these joint trajectories and the analytical Barrett WAM model the resulting torques at each joint are computed.

The EKF is initialized to have an error of 20 percent in the mass and inertia estimates. The Sigmoid functions are chosen such that mass and inertia parameters are constrained between zero and twice the initial value, first-order moments are constrained by the link dimensions multiplied by initial mass. First the algorithm is tested using the generated joint torques as the measurement vector for the EKF. Random Gaussian noise of 1 percent of maximum torque was added to the torques to simulate sensor noise, due to high peak torques, 1 percent results in significant measurement noise as shown in figure 4. Figure 3 shows the error in mass estimates after running the algorithm for 40 seconds with a sampling rate of 100Hz. Table I shows the root mean squared error (RMSE) of joint torque estimates using the initial parameters with 20 percent error and those estimated by EKF. Next the algorithm was tested using generated force and moment at the base as the measurement vector, similarly random Gaussian noise of 1 percent of maximum was added. Using force and moment at the base allows to identify mass of the base and the first link but light links have very little effect on the force and moment thus EKF does not update their mass and error stays close to the initial 20 percent, figure 5 shows the results.

B. Adapting to Changing Dynamics

Manipulators are designed to interact with the environment thus the dynamics of the end effector are expected to change during the performance of a task. To demonstrate the adaptability of the algorithm we consider the Barrett WAM picking up an object of unknown inertial parameters. For this scenario it is assumed that the parameters of the other links were estimated beforehand. Accordingly the EKF state is modified to only represent the end effector's dynamic parameters $[m^{ee}\ I^{ee}_{xx}\ I^{ee}_{yy}\ I^{ee}_{zz}]$. The upper and lower bounds of the Sigmoid functions for mass and inertia are set to $(0, 4)$ and $(0, 0.2)$ respectively to allow for the maximum payload rating for the manipulator of 4kg. The measurement vector is kept as the joint torques. The process noise Q is not annealed over time as it is undesirable for the state to converge to a steady value since the mass at the end effector may change as the robot picks up and releases the object.

In the experiment at first the robot is not moving and is in a configuration such that the mass of the end effector is observable. Thus even without any trajectory the mass

TABLE I

COMPARISON BETWEEN PREDICTED TORQUES AT EACH JOINT USING INITIAL GUESS OF INERTIAL PARAMETERS AND THE PARAMETERS ESTIMATED BY EKF.

Joint	Initial RMSE (Nm)	EKF RMSE (Nm)
1	2.9287	0.3124
2	6.3368	0.5034
3	1.2659	0.1167
4	1.5763	0.1831
5	0.1781	0.0425
6	0.3962	0.0471
7	0.003	0.0013

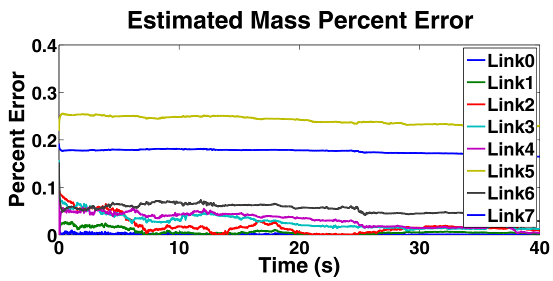

Fig. 5. EKF quickly adapts the mass parameters to reduce the error from 20 percent using only force and moment on the base as measurement. Note that in this scenario parameters of the base itself are observable unlike when torque is used as the measurement vector. However EKF does not successfully identify the masses of the two lightest links 5 and 7, 0.1238 and 0.0686 kg respectively as they have negligible effect on the moment and force at the base.

is tracked while the unobservable inertia parameters remain constant. Once the manipulator lifts the object it begins to perform an exiting trajectory. At this time the inertia parameters are excited by the motion and become observable. Figure 6 shows the ability of EKF to track load parameters. For the 7 degrees of freedom Barret WAM the EKF can update end effector parameters at 128Hz.

C. Real Data

To evaluate the accuracy of the algorithm on a real robot we used the Barret WAM data-set described in [9]. This data-set contains torques and kinematic measurements designed for learning the dynamic model using non-linear methods. Excitation of each parameter was verified by looking for non-zero entries in the measurement Jacobians throughout the trajectory. Due to the kinematic structure of the robot, the parameters of the base as well as the center of mass along the y axis, and moments of inertia in the x and z axes of the first link were not identifiable. The EKF ran using 9000 training samples. Next, the steady state parameter values were evaluated on 3000 previously unseen points of the same motion as well as 3000 points of the motion preformed 4 times faster. EKF successfully converges to a set of parameters that greatly improve torque prediction at each joint and performance remains consistent between different motions showing generalization across the manipulator state space. The estimated parameters are successfully kept in the bounds imposed by the Sigmoid function. Figure 7 compares

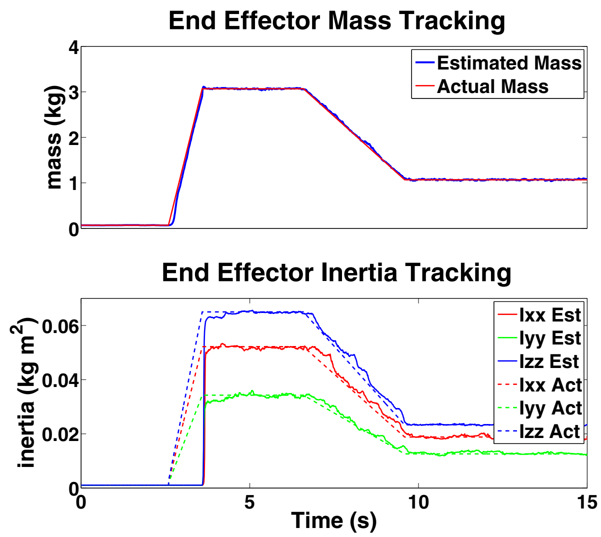

Fig. 6. In this simulation at first the robot is set in a stretched out arm configuration and is not moving, at 2.6 seconds the end effector parameters are changed to simulate lifting a box of dimensions 0.4m by 0.3m by 0.2m and weighing 3kg, at this point mass is observable but inertia parameters are not. This results in the mass being estimated correctly even while the robot is not moving, while the inertia parameters cannot be estimated in the segment from 2.6 to 3.6s, while the robot is at rest. At 3.6 seconds the box is lifted and the robot begins moving by performing an exciting trajectory. As soon as this occurs inertia parameters become observable and quickly converge to the correct value. At 6.6 seconds the box's mass begins to decrease until it reaches 1kg at 9.6 seconds.

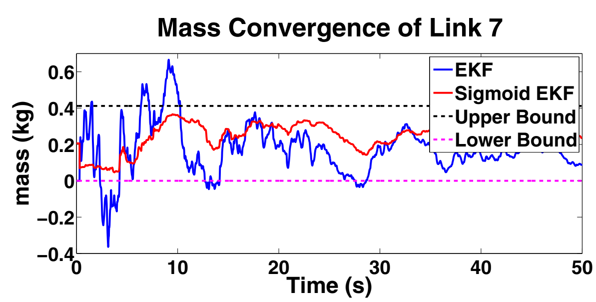

Fig. 7. Without the Sigmoid function mapping, applying EKF to real robot data leads to negative mass and inertia estimates or estimates outside of acceptable bounds. The Sigmoid function mapping ensures the mass and inertia stay positive and the center of mass is within the link's bounds.

mass estimation with the proposed approach to regular EKF, figure 8 shows the parameters converging to a steady state value, and figure 9 shows the resulting improvement in torque prediction for two of the joints. Table II contains the numerical results. We attribute the remaining error to parameters that are not estimated by EKF such as link lengths, stiction, motor backlash, and possibly non zero mean sensor noise.

V. DISCUSSION AND CONCLUSION

In this paper we presented a new approach to estimate the dynamic parameters of a manipulator and its load using the Extended Kalman Filter. To impose constraints a mapping between the filter's state and parameters through a Sigmoid function was introduced. The Kalman filter ensures convergence to steady state and that only parameters activated by

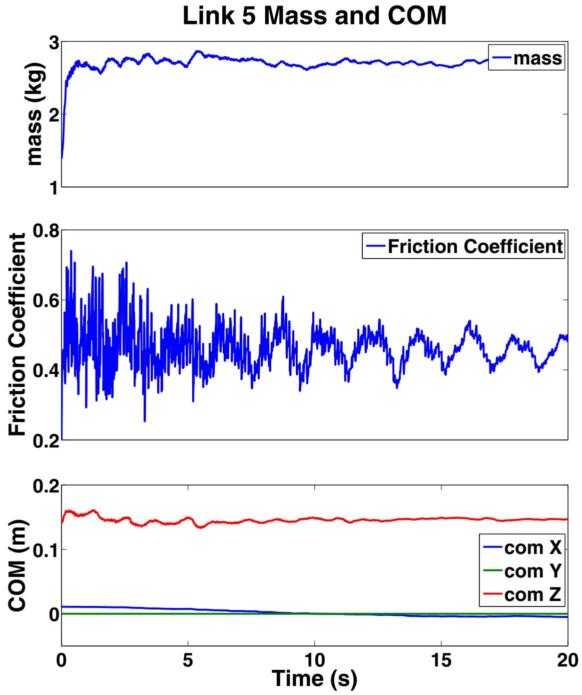

Fig. 8. This figure describes the typical performance on real robot data. Each of the parameters quickly adapts to a value that significantly improves torque prediction as seen in table II.

TABLE II

ROOT MEAN SQUARED ERROR BETWEEN PREDICTED TORQUES AT EACH JOINT USING INITIAL PARAMETERS AND PARAMETERS ESTIMATED BY EKF USING REAL ROBOT DATA. IN THIS DATASET ONLY JOINTS 1, 2, 3, AND 4 EXHIBIT TORQUES GREATER THAN 1 NM.

	Regular Motion	
Joint	**Initial RMSE Nm**	**EKF RMSE Nm**
1	2.88	1.74
2	7.48	1.04
3	0.95	0.51
4	2.85	0.41
5	0.24	0.13
6	0.19	0.13
7	0.09	0.06
	4x Fast Motion	
	Initial RMSE Nm	**EKF RMSE Nm**
1	2.88	1.76
2	7.60	1.93
3	0.95	0.51
4	2.87	0.64
5	0.25	0.13
6	0.19	0.13
7	0.09	0.06

the trajectory will be updated. The approach was validated both in simulation and real robot data. Future work includes using Dual EKF to deal with noisy kinematic measurements and testing the approach on real robot with a force and moment sensor on the end effector as well as combining joint torques with forces on the base as measurement to improve parameter estimation of the base links.

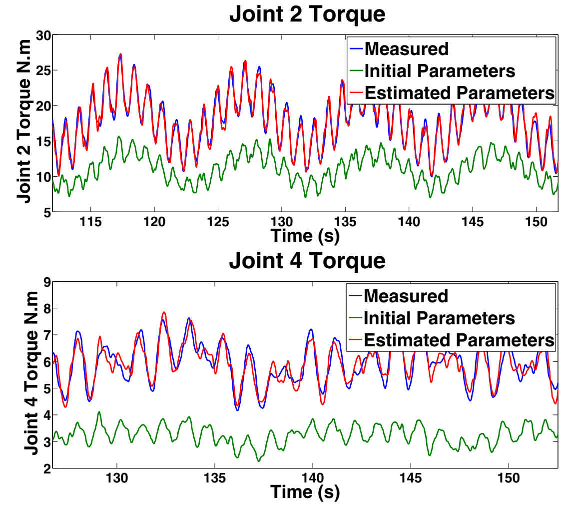

Fig. 9. This figure compares torque prediction using parameters obtained from Barret WAM documentation (Initial Parameters) and EKF estimated parameters for real robot data. See table II for numerical results of all the joints.

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