ECE 250 *Algorithms and Data Structure*
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with the subject ECE 250 Questions 2.4.
Assistances and comments will be acknowledged.

**5.2*b*** The base case: a perfect binary tree with height $h = 0$ has one leaf node, and $2^0 = 1$.

Assume that a perfect binary tree of height $h$ has $2^h$ leaf nodes.

For $h > 0$, a perfect binary tree of height $h + 1$ has two sub-trees of height $h$. As the root is not a leaf node, the total number of leaf nodes is the sum of the number of leaf nodes in each of the children:

$$2^h + 2^h = 2 \cdot 2^h = 2^{h+1}$$

which is the formula we expect.

**5.2*d*** Just sum the nodes at each of the depths and see that this is a geometric sum:

$$\sum_{k=0}^{h} 2^k = \frac{2^{h+1} - 1}{2 - 1} = 2^{h+1} - 1 \ .$$

**5.2*f*** As $h$ becomes large, the average path length is $h - 1$ based the definition of depth and on the calculations given in class.

**5.2*h***
$\lceil \lg(1001) \rceil - 1 = 10 - 1 = 9$
$\lceil \lg(6000001) \rceil - 1 = 23 - 1 = 22$ because $4 < 6 < 8$.
$\lceil \lg(20000000001) \rceil - 1 = 35 - 1 = 34$ because $16 < 20 < 32$.