7.2a Insert the following $n$ objects, in the order given, into a binary min-heap and place your answer into the following table

$$
5,3,9,7,2,4,6,1,8
$$


7.2b Is the following array-representation of a heap a valid min-heap-as-complete-tree? Why or why not?

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 | 6 | 9 | 15 | 7 | 35 | 12 | 19 | 31 | 10 | 17 | 42 | 39 | 54 | 13 | 20 | 40 | 30 | 32 |

7.2c Update the following min-heap-as-complete-tree data structure to demonstrate the state following a pop operation and place your answer in the following table.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 7 | 4 | 12 | 10 | 5 | 9 | 13 | 17 | 19 | 11 | 6 | 21 | 11 | 15 | 14 | 19 | 18 | 20 |


7.2d The formulas for the parent and two children of a node at index $k$ are $k / 2,2 k$ and $2 k+1$, respectively. This requires that the root of the tree be stored at index $k=1$. How do these formulas differ if we store the root of the tree at index $k=0$ ?
7.2e Just like we can represent a complete binary tree using an array, we could similarly represent a complete quaternary tree as an array. For example, we could store


| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 14 | 4 | 5 | 12 | 17 | 29 | 15 | 18 | 7 | 23 | 21 | 11 | 15 |  |  |  |  |  |  |

Why, in this representation, do we start the index at $k=0$ ? Hint: consider the formula used to find both the parent and children of the node at index $k$.
7.2 $f$ The quaternary tree in Question $7.2 e$ is also a min-heap: each node is less than or equal to all four of its children. Draw the intermediate states of the tree if

1. the value 8 is pushed onto this min-heap,
2. followed by four pops.
