1.2a Find the following limits using L’Hôpital’s rule:

\[
\lim_{n \to \infty} \frac{n^2 + n + 1}{2n + 3}, \quad \lim_{n \to \infty} \frac{ne^n}{2n + 3}, \quad \lim_{n \to \infty} \frac{\ln(n) + 4}{5n^4 + 7n^2 + 6}, \quad \lim_{n \to \infty} \frac{2^n}{\log_2(n)}, \quad \lim_{n \to \infty} \frac{n^{1.007}}{n \ln(n)}, \quad \lim_{n \to \infty} \frac{n^{1.99} + n + 1}{n^2 \ln(n) + n + 2}.
\]

1.2b Can you use L’Hôpital’s rule to determine the limit \( \lim_{n \to \infty} \frac{e^n}{2^n} \)?

1.2c Which of the following are equal?

i. \( 5^3 \) and \( 3^5 \)

ii. \( 8^2 \) and \( 4^3 \)

iii. \( 16^{\log(4)} \) and \( 4^{\log(16)} \)

1.2d Without doing any serious calculations:

i. approximately how many megabytes (MB) is \( 2^{24} \) bytes,

ii. how many gibibytes (GiB) is \( 2^{33} \) bytes, and

iii. approximately how many kibibytes (KiB) is \( 4000 \) bytes?

1.2e What is the sum of the first 50 integers? What is the sum of the integers from 51 to 100?

1.2f Quickly approximate the following:

i. \( \sum_{k=0}^{30} k^2 \), and

ii. \( \sum_{k=0}^{100} k^3 \).

1.2g Approximately, what is the sum \( \sum_{k=1}^{30} \frac{1}{2^k} \)?

1.2h Show, using a proof by induction, that \( \sum_{k=0}^{N} k2^k = 2 + (N-1)2^{N+1} \).
1.2f Consider the following weighted average of the values of a function on an interval:

\[ \frac{f(1) + 3f(1.5) + 3f(2) + f(2.5)}{8} \]

Why can we consider this a weighted average of the value of the function on the interval \([1, 2]\)? Should this multiplied by 1.5 approximate \( \int_{1}^{2.5} f(x) \, dx \)? Why? Check the validity of the approximation using the sine function.

1.2k How many pairs of entries are there in a list of 100 integers?