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1.2*a* Find the following limits using L'Hôpital's rule:

$$\lim_{n\to\infty}\frac{n^2+n+1}{2n+3},\ \lim_{n\to\infty}\frac{ne^n}{2n+3},\ \lim_{n\to\infty}\frac{\ln(n)+4}{5n^4+7n^2+6},\ \lim_{n\to\infty}\frac{2^n}{\log_2(n)},\ \lim_{n\to\infty}\frac{n^{1.007}}{n\ln(n)},\ \lim_{n\to\infty}\frac{n^{1.99}+n+1}{n^2\ln(n)+n+2}.$$

1.2*b* Can you use L'Hôpital's rule to determine the limit $\lim_{n\to\infty}\frac{e^n}{2^n}$?

1.2*c* Which of the following are equal?

i. 5^3 and 3^5
ii. 8^2 and 4^3
iii. $16^{\lg(4)}$ and $4^{\lg(16)}$

1.2*d* Without doing any serious calculations:

i. approximately how many megabytes (MB) is 2^{24} bytes,
ii. how many gibibytes (GiB) is 2^{33} bytes, and
iii. approximately how many kibibytes (KiB) is 4000 bytes?

1.2*e* What is the sum of the first 50 integers? What is the sum of the integers from 51 to 100?

1.2*f* Quickly approximate the following:

i. $\sum_{k=0}^{30} k^2$, and
ii. $\sum_{k=0}^{100} k^3$.

1.2*g* Approximately, what is the sum $\sum_{k=1}^{30}\frac{1}{2^k}$?

1.2*h* Show, using a proof by induction, that $\sum_{k=0}^{N} k2^k = 2+(N-1)2^{N+1}$.

1.2*j* Consider the following weighted average of the values of a function on an interval:

$$\frac{f(1)+3f(1.5)+3f(2)+f(2.5)}{8}.$$

Why can we consider this a weighted average of the value of the function on the interval [1, 2]? Should this multiplied by 1.5 approximate $\int_1^{2.5} f(x)\,dx$? Why? Check the validity of the approximation using the sine function.

1.2*k* How many pairs of entries are there in a list of 100 integers?