ECE 250 *Algorithms and Data Structure*
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Please send any comments or criticisms to dwharder@alumni.uwaterloo.ca
with the subject ECE 250 Questions 1.4.
Assistances and comments will be acknowledged.

1.4*a* Prove by induction that the sum of the first *n* even numbers is $n^2 + n$.

1.4*b* Prove by induction that the sum of the first *n* numbers of the form 1, 4, 7, 10, … is $\frac{n}{2}(3n-1)$.

1.4*c* Prove by induction that $2\sum_{k=0}^{n} 3^k = 3^{n+1} - 1$.

1.4*d* Prove by induction that $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$.

1.4*e* Prove by induction that $\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$.

1.4*f* Prove by induction that $\sum_{k=0}^{n} (-1)^k r^k = \frac{1-(-r)^{n+1}}{1+r}$.

1.4*g* Prove that $n^2 - n$ is even for all integers *n*.

1.4*h* Show that $\sum_{k=0}^{n} k^2 \geq \frac{n^3}{3}$ for all $n \geq 0$.

1.4*i* Show that $\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \geq 0$.

1.4*j* Show that $\sum_{k=1}^{n} 3k^2 - 3k + 1 = n^3$ for all $n \geq 1$.

1.4*k* Show that $\sum_{k=1}^{n} \frac{1}{\sqrt{k}} \geq \sqrt{n}$ for all $n \geq 1$.

1.4*l* Show that 4 divides $3^{2n-1} + 1$ for all $n \geq 1$.

1.4*m* Show that $\left|\sum_{k=1}^{n} x_k\right| \leq \sum_{k=1}^{n} |x_k|$ for all $n \geq 1$.

1.4*n* Show that $2^{n-1} \leq n!$ for all $n \geq 1$.

1.4*o* Show that 133 divides $11^{n+1} + 12^{2n+1}$ for all $n \geq 0$.

1.4*p* If $F(n)$ is the n^{th} Fibonacci number where $F(0) = F(1) = 1$, show that $F(n)$ and $F(n + 1)$ are relatively prime (that is, they share no common factors) for all $n \geq 0$.

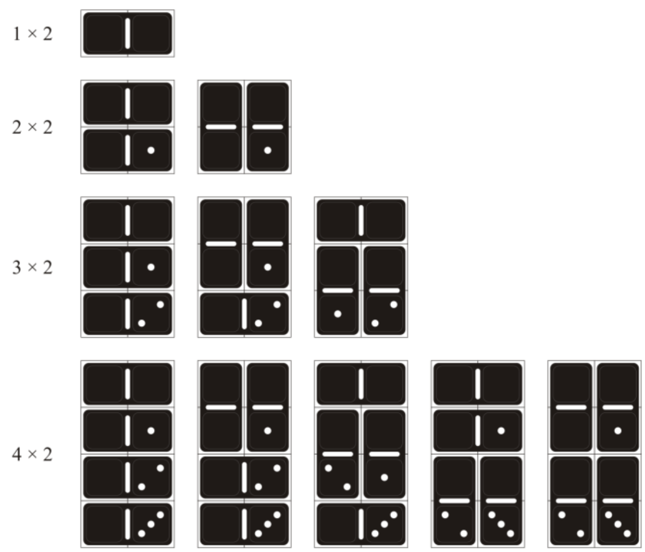
1.4*q* Show that every third Fibonacci number is even.

1.4*r* Show that $x^n - y^n$ is divisible by $x - y$ for all $n \geq 0$.

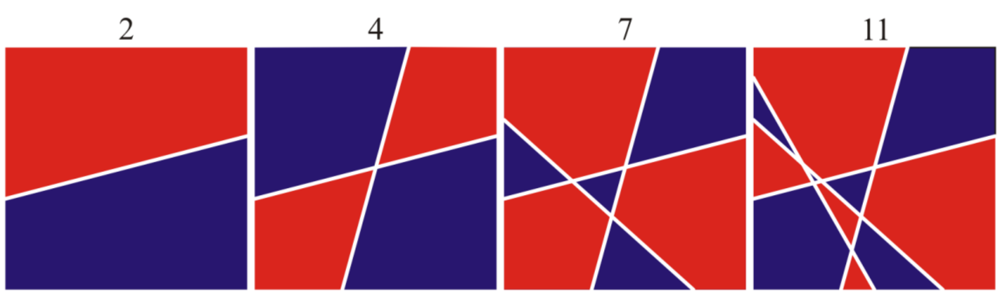
1.4*s* Can you use a proof by induction to prove that $n^2 \geq 3n - 2$ for all n?

1.4*t* Can you use a proof by induction to prove that $n^2 \geq 7n - 11$ for all n?

1.4*u* Come up with a formula that gives d_n, the number of unique ways in which dominos can be used to tile an $n \times 2$ grid and then demonstrate that your formula is correct using induction. The image shows that $d_1 = 1$, $d_2 = 2$, $d_3 = 3$ and $d_4 = 5$.



1.4*v* A set of n lines can be used to divide a plane into a maximum of $\frac{n^2 + n + 2}{2}$ regions, as is shown in the image. Come up with a recursive formula and show your formula is correct using induction.



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1.4*w* Using induction, demonstrate that any $2^n \times 2^n$ grid with one square deleted can be tiled with triominos, as is shown for $n = 0$, 1 and 2 and suggested for $n = 3$ in the following image.