ECE 250 *Algorithms and Data Structure*
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with the subject ECE 250 Questions 2.3.
Assistances and comments will be acknowledged.

2.3*a* What is the relative error between $n^2 + 2n + 5$ and an approximation n^2 when $n = 1000$ and when $n = 1000000$? The relative error is the difference between the actual value and the approximation over the actual value.

2.3*b* By finding the limit (using l'Hôpital's rule where necessary), determine the most appropriate Landau symbol to describe the relationship between the following pairs of functions:

1. $n + \ln(n)$ and $n + 5$
2. $n^2 + 2$ and e^n
3. $2n^2 + 3n + 1$ and $3n^{\lg(3)} + n$
4. $2n + 4$ and $5n \ln(n) + 3n + 2$
5. $n \ln(n)$ and $n \ln(n^5)$
6. $\ln(n)$ and $\lg(n)$
7. $n^2 \lg(n)$ and $n^2 \log_{10}(n)$

2.3*c* Sort the following lists of numbers those that are in the same equivalence class and then order those equivalence classes.

1. $n^5 \ln(n)$, $n^{0.5}$, $n \log_{10}(n)$, $n \ln(n^5)$, n, $n^{\ln(4)}$¸ 1¸ $n \ln(n)$, $n^2 \ln(n)$, n^3, n^2, $n^{\lg(4)}$

2.3*d* In each case, determine the appropriate relationship between $f(n) + g(n)$ and $h(n)$.

1. $f(n) = \Theta(h(n))$ and $g(n) = \Theta(h(n))$
2. $f(n) = \Theta(h(n))$ and $g(n) = \Omega(h(n))$
3. $f(n) = \Theta(h(n))$ and $g(n) = \mathbf{o}(h(n))$
4. $f(n) = \mathbf{O}(h(n))$ and $g(n) = \mathbf{o}(h(n))$
5. $f(n) = \omega(h(n))$ and $g(n) = \mathbf{O}(h(n))$

In some cases, you may be able to say nothing.

2.3*e* In each case, determine the appropriate relationship between $f_1(n) + f_2(n)$ and $g_1(n) + g_2(n)$ if it is known that $g_1(n) = \omega(g_2(n))$.

1. $f_1(n) = \Theta(g_1(n))$ and $f_2(n) = \Theta(g_2(n))$
2. $f_1(n) = \Theta(g_1(n))$ and $f_2(n) = \Omega(g_2(n))$
3. $f_1(n) = \mathbf{o}(g_1(n))$ and $f_2(n) = \Theta(g_2(n))$
4. $f_1(n) = \mathbf{O}(g_1(n))$ and $f_2(n) = \mathbf{o}(g_2(n))$
5. $f_1(n) = \omega(g_1(n))$ and $f_2(n) = \mathbf{O}(g_2(n))$

In some cases you may not be able to say anything.

2.3*f* Find the most appropriate representative element that describes each of the following rates of growth. For example, the most appropriate representative of $3n^2 + 4n \ln(n) + 5n + 2$ is n^2.

1. $5n^2 + 4n + 3n^{\lg(6)} + 4 + \ln(n)$
2. $6n + 7 \ln(n) + 8n \ln(n) + 9$
3. $4n^3 + 7n + 514n^4 + 35n^2 + 2n^6 + 5624$

2.3*g* Given the relationships between $f(n)$ and $g(n)$, and between $g(n)$ and $h(n)$, attempt to deduce a relationship between $f(n)$ and $h(n)$ if possible. Write the relationship as $f(n) = \mathbf{?}(h(n))$.

1. $f(n) = \mathbf{o}(g(n))$ and $h(n) = \boldsymbol{\omega}(g(n))$
2. $g(n) = \boldsymbol{\Theta}(f(n))$ and $g(n) = \mathbf{o}(h(n))$
3. $f(n) = \mathbf{o}(g(n))$ and $h(n) = \mathbf{O}(g(n))$
4. $g(n) = \boldsymbol{\Omega}(f(n))$ and $h(n) = \boldsymbol{\Theta}(g(n))$
5. $f(n) = \boldsymbol{\Omega}(g(n))$ and $g(n) = \boldsymbol{\omega}(h(n))$
6. $g(n) = \boldsymbol{\Omega}(f(n))$ and $g(n) = \mathbf{O}(h(n))$

Hint, if you're having difficulty, translate the question into, for example, $f < g$ and $h > g$ and ask the same question if f, g, and h were real numbers. In some cases, you may find no reasonable deduction, for example, if $f < g$ and $h < g$, no information is given about the relationship between f and h.