ECE 250 *Algorithms and Data Structure*
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with the subject ECE 250 Questions 2.4.
Assistances and comments will be acknowledged.

5.2*a* Prove that a perfect binary tree of height h has $2^{h+1} - 1$ nodes by using induction together with the recursive definition that a perfect tree of height h has two sub-trees that are themselves perfect binary trees of height $h - 1$.

5.2*b* Prove that a perfect binary tree of height h has 2^h leaf nodes by using induction together with the recursive definition that a perfect tree of height h has two sub-trees that are themselves perfect binary trees of height $h - 1$.

5.2*c* Prove that a perfect binary tree of height h has $2^h - 1$ internal nodes by using induction together with the recursive definition that a perfect tree of height h has two sub-trees that are themselves perfect binary trees of height $h - 1$.

5.2*d* A perfect binary tree has 2^k nodes at depth k for $k = 0, \ldots, h$. Use this to prove that a perfect binary tree of height h has $2^{h+1} - 1$ nodes.

5.2*e* Explain why we use the formula

$$\sum_{k=0}^{h} k2^k = 2 + (h-1)2^{h+1}$$

when finding the average depth of a node in a perfect binary tree?

5.2*f* With a perfect binary tree of height h, if you randomly select a node within the tree, what is the average length of the path from the root node to that node?

5.2*g* The height of a perfect binary tree is $\lg(n + 1) - 1$. Show that this is $\Theta(\ln(n))$ by using l'Hôpital's rule.

5.2*h* The height of a perfect binary tree is $\lg(n + 1) - 1$. Given binary trees with $n = 1000$, $n = 6\,000\,000$, and $n = 20\,000\,000\,000$ nodes, what is the minimum possible height of binary trees that stores these number of nodes? Do not use your calculator.