5.3*b* When h is 0, there is only a single node, and $2^0 = 2^1 - 1 = 1$.

Assume that in general, a complete binary tree of height h has between 2^h and $2^{h+1} - 1$ nodes.

There are two cases for complete binary trees of height $h + 1$:

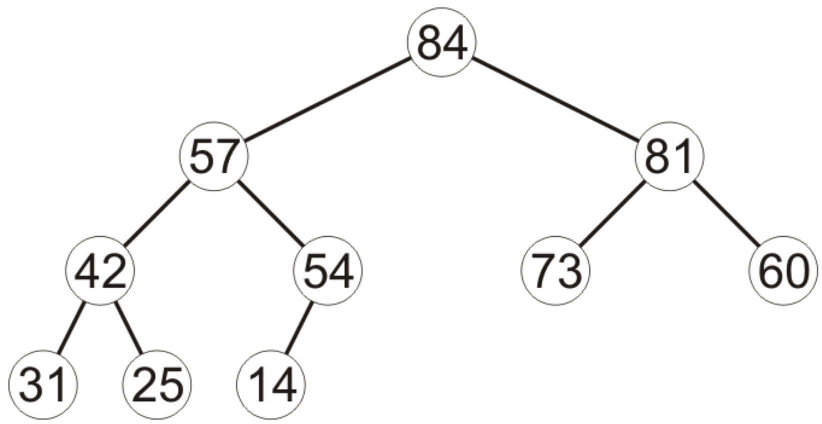
1. The left sub-tree has between 2^h and $2^{h+1} - 1$ nodes and the right sub-tree has $2^h - 1$ nodes, or
2. The left sub-tree has $2^{h+1} - 1$ nodes and the right sub-tree has between 2^h and $2^{h+1} - 1$ nodes.

Taking into account the root node,
the first case has between $1 + 2^h + 2^h - 1 = 2^{h+1}$ and $1 + 2^{h+1} - 1 + 2^h - 1 = 3\cdot 2^h - 1$ nodes, and
the second case has between $1 + 2^{h+1} - 1 + 2^h = 3\cdot 2^h$ nodes and $1 + 2^{h+1} - 1 + 2^{h+1} - 1 = 2^{h+2} - 1$ nodes.

Thus, the number of nodes runs between 2^{h+1} and $2^{h+2} - 1$, which is the expected result.

5.3*d* $\left\lceil \frac{n}{2} \right\rceil$

5.3*f* The actual tree is



42 is at index 4, so its parent is at index 4/2 = 2 and its children are at $2\cdot 4 = 8$ and $2\cdot 4 + 1 = 9$

54 is at index 5, so its parent is at index 5/2 = 2 and its children are at indices $2\cdot 5 = 10$ and $2\cdot 5 + 1 = 11$, but the size of the tree is 10, so it has only one child.

5.3*g* Some implementations are:

```
template <typename Type, int N>
Type Complete_binary_tree::parent( Type const &obj ) {
    int n = find( obj );

    if ( n == 0 ) {
        throw illegal_argument();
    }

    if ( n == 1 ) {
         throw underflow();
    }

    return array[n/2];
}

template <typename Type, int N>
Type Complete_binary_tree::parent( Type const &obj ) {
    int n = find( obj );

    if ( n == 0 ) {
        throw illegal_argument();
    }

    if ( 2*n + 1 > complete_size ) {
         throw underflow();
    }

    return array[2*n + 1];
}
```