ECE 250 *Algorithms and Data Structure*
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with the subject ECE 250 Questions 5.5.
Assistances and comments will be acknowledged.

5.5*a* Because the worst case height of a tree where the tree is essentially a linked list (each node having at most one child), the height cannot be greater than n, but the height can be less than n, so $O(n)$ properly describes an upper bound on the height of a tree.

Similarly, the best we can do is store the nodes in a complete tree, so the height cannot be any less than a scalar multiple of $\ln(n)$. However, the height can be greater than a scalar multiple of n, so $\Omega(\ln(n))$ properly describes a lower bound on the height of a tree.

5.5*b* Any operation that begins at the root of the tree must traverse the tree to a leaf node. If the height is $\omega(\ln(n))$, this means that the run time would grow asymptotically faster than if the nodes were stored in a tree that saw its height grow $\Theta(\ln(n))$.

5.5*c* If you were to look only at one node, what prevents both children from being essentially linked lists?

5.5*d* For nodes A, B, C, and E, the answers are:

A. 2, 3 and 3, 8 and 4, respectively.
B. 1, 3 and 2, 4 and 4, respectively.
C. 0, 2 and 2, 2 and 2, respectively.
E. 2, 2 and 1, 3 and 1, respectively.

5.5*e* The height of a null sub-tree is -1. The height of any other node is one plus the maximum of the heights of the two sub-trees.

The null-path length of a null sub-tree is 0. The null-path length of any other node is one plus the minimum of the null-path lengths of the two sub-tree.

The null sub-tree count of a null sub-tree is 1. The null sub-tree count of any other node is the sum of the null-path tree counts of the two sub-trees.

5.5*f* In general, a general tree is used to store a hierarchy. In an organizational tree, one child of a parent might represent a large manufacturing organization while the other may be an accounting department. In the military, a company may have three platoons (each comprised of three sections, a machine gun squad, a mortar squad, a platoon headquarters section, etc.) of around 36 soldiers each together with a sniper detachment of only two. In the definition of classes within Java, given that one sub-class of Object may have significant sub-classes each possibly having additional sub-classes does not affect that there may be a complex number class `Complex` which has no sub-classes.