How to Make an Argument

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This is not about the other type of argument: those based on emotional and irrational arguments supported by the loudest voices or angriest expressions. However, once you understand how to make this type of argument, you may have to less frequently rely on the other type of argument.

Everyone has opinions: At one extreme, you have conspiracy theorists who believe things without evidence: global warming is not happening, the United States did not land astronauts on the Moon, and 9/11 was the result of a conspiracy by George W. Bush and the United States government. Then you have more serious questions: How should people behave and interact in society? How do we regulate society through appropriate choice of laws? Is incarceration better than other forms of criminal rehabilitation? Is the purpose of the prison system more to protect Canadians, to punish criminal behaviour, or to allow for rehabilitation? Should we prevent others from seeking pardons for previous crimes simply because one notorious criminal was eligible for one? To what extent should individuals be free to live their lives as they choose? Who should be allowed to come to Canada as immigrants? Should students in professional programs pay more tuition than students in liberal arts programs? How should Canada respond to global warming? Some may even think there is still a debate open as to what was the best English-language novel of 1937.

In all these cases, people have opinions, but if you ask them to support those opinions, it would be quite difficult to support them. To form an argument, it requires more than emphatic statements that your particular opinions are true: otherwise, if the loudest voice held sway (as so often it does under a mob mentality), we would all simply believe that global warming is a fiction, the United States did not land astronauts on the Moon, cryptozoological (*e.g.*, Loch Ness monster, big foot) creatures actually exist, Roswell was the site of a UFO landing, chemtrails are poisoning the North American continent, fluoridation is evil, pseudoscientific alternative medicines (homeopathy, acupuncture, subluxation-based chiropractic practice, *etc*.) have an effect beyond that of a placebo, all high ranking government officials are reptilian aliens, HIV does not cause AIDS, vaccines cause autism, polio vaccines are an attempt by the United States to sterilize Africans or Muslims, and the United States government had J.F. Kennedy assassinated. Why do rational people reject such ideas? The lack of anything but the vacuous arguments not supported by any evidence made by the peddlers of these claims is becomes clear when their arguments are properly examined.

More applicable to this course, however, what behaviour is ethical and what is unethical for a professional engineer? What actions constitute professional misconduct and why? What differentiates unethical behaviour from unprofessional behaviour? In a given circumstance, should engineers risk their careers, income and reputation when they see egregious perversions of professional behaviour? Should an engineer wear perfumes or colognes to work if there are fellow employees who suffer serious allergic reactions? What if those reactions are only minor?

This report is about how to build arguments to support a particular opinion you may hold or one which you must provide support for. Additionally, the information here will help you understand the weaknesses in your arguments while simultaneously assist you in finding the weaknesses in the arguments of others. The goal of such arguments is, of course, differentiating truth from falsehoods. In some cases, the implications will be more definitive (often with ethics and professional conduct, this will be the case) while in others it may be less clear (as it is with many questions of policy). To achieve this, we will begin with a review of logic. This will be followed by a section on tautologies. We will then see how to apply logic to arguments about truths in society and this will be followed by common fallacies.

1. Introduction to Logic

Logic is based on assuming that we know which statements are true and which are not: in logic, we simply represent any true statement by *T* and any false statement by *F*. A statement that is not known to be true or false is represented by a symbol such as *x*, *y* or *z*. We will start by looking at Boolean operators.

1.1 Logical "and" and Logical "or"

From your course in logic, you are already aware of the Boolean operations of "*and*" and "*or*". For the purpose of this essay, we will represent these by the operators ∧ and ∨, respectively.

Aide Memoire

You can remember that ∧ represents *and* by thinking of the first letter **A**nd.
Alternatively, you can remember ∧ and ∨ by recalling set theory:

Set intersection	$S \cap T$	all elements that are in *S* **and** also in *T*	$S \cap T = \{x : (x \in S) \wedge (x \in T)\}$
Set union	$S \cup T$	all elements that are in *S* **or** in *T*	$S \cup T = \{x : (x \in S) \vee (x \in T)\}$

The statement $x \wedge y$ is true if both *x* **and** *y* are true and false otherwise; whereas, the statement $x \vee y$ is true if either *x* **or** *y* are true and false only when both are false.

1.2 Logical Negation

The other common unary operation is negation, represented by ¬. For example $\neg T = F$ and $\neg F = T$. In English, we represent this by *not*: If the statement "It is not raining" is true, this is the same as saying the statement "It is raining" is false. Again, for an unknown statement, to negate it, we would simply write, for example, $\neg x$.

1.3 Logical Implication

Okay, so who cares? The most important operator for arguments, however, is *implication* represented by $\rightarrow$. For example, $x \rightarrow y$. In English, this can be said in many ways. For example, all of the following English sentences represent the statement "It is raining" $\rightarrow$ "There are clouds in the sky."

If it is raining, there are clouds in the sky.
It is either not raining, or there are clouds in the sky.
There are clouds in the sky if it is raining.

In this case, all of these statements of the form $x \rightarrow y$ are equally true. Here is one that is not true:

"If it is not raining, there are no clouds in the sky."

This represents the statement "It is not raining" $\rightarrow$ "There are no clouds in the sky." Such a statement is false—the author is looking outside right now and while there are clouds in the sky, it is not raining.

On the other hand, the reverse statement,

"If there are no clouds in the sky, it is not raining."

appears to be very related to the original statement, "If it is raining, there are clouds in the sky."

1.4 Logical Equivalence

The last logical operator we will look at is equivalence. The statement $x \leftrightarrow y$ is true only if the one statement always has the same truth value as the other and can be read as "if and only if". One common example is:

"I see lightning if and only if I hear thunder."

"I see lightning" is equivalent to "I hear thunder" because the two are different manifestations of the same phenomenon.

2. Implications, Deductions and Refuting Implications

The implication is the one most useful tool in logic for arguments. From them, we will be able to make deductions, but at the same time, it may be necessary to falsify implications that are incorrectly assumed to be true.

2.1 Use of Implications in Arguments

As an introductory example of implications, we should ask why would George W. Bush claim that we should "lower taxes on the rich"? It is because this is an action he claims that should be taken and he is making this claim because there is the implication that this will have beneficial results:

"If we lower taxes on the very rich, we will stimulate the economy and create jobs."

The claim is that performing the action of lowering the taxes on the very rich will **both** stimulate the economy **and** create jobs. We can rewrite this mantra of the political right using logical operators:

$$\text{"}\textit{Lower taxes on the rich}\text{"} \to (\text{"}\textit{stimulates the economy}\text{"} \wedge \text{"}\textit{creates jobs}\text{"}) \qquad (1)$$

In a similar case, our previous example is similar, though simpler:

"It is raining; therefore, there are clouds in the sky."

The claim that an implication is true says nothing about the current state of things. It simply says that if the left-hand side is true, then the right hand side must be true, too. For the author at this minute, it is not raining—consequently, this implication doesn't say anything about whether or not there are clouds in the sky. However, earlier this week, it was raining, and, indeed, there were clouds in the sky. All the implication says is that ***if*** it is raining, there are clouds in the sky.

You will notice that all mathematical theorems are essentially in the form of implications. For example, the mean-value theorem is one that you were probably presented with the following form:

Mean-value theorem

If a function f is continuous on a closed interval $[a, b]$ where $a < b$, and it is differentiable on the open interval (a, b), there exists a point $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Again, this is can always be rewritten as an implication:

$$[(a < b) \wedge (f \text{ is continuous on } [a, b]) \wedge (f \text{ is differentiable on } (a, b))] \to$$

$$\exists c : \left[\left(c \in (a,b) \right) \wedge \left(f'(c) = \frac{f(b) - f(a)}{b - a} \right) \right].$$

Quick Review

The statement $\exists x{:}f(x)$ is true if for at least one relevant value of x, the statement $f(x)$ is true.
For example, "There exists a Waterloo undergraduate who was born on the moon." is a false statement, but "There exists a Waterloo undergraduate who was born in Seoul, South Korea." is a currently a true statement. You can read "$\exists x{:}f(x)$" as "there exists an x such that $f(x)$ is true".

The statement $\forall x{:}f(x)$ is true if that for all relevant values of x, the statement $f(x)$ is also true
For example, "All Waterloo undergraduates are co-op students." is a false statement, but "All Waterloo engineering undergraduates are co-op students." is a true statement. You can read "$\forall x{:}f(x)$" as "there exists an x such that $f(x)$ is true".

A different case is a hypothesis: we may not know if there is a solution, but we may make a guess as to what is true:

The Goldbach Conjecture

Any integer greater than 1 is the average of two prime numbers.

If we represent the set of all integers as **Z** and the set of all prime numbers as **P**, we may then write this as

$$\forall z{:}(z \in \mathbf{Z} \wedge z > 1) \to \exists p_1, p_2 : \left[(p_1 \in \mathbf{P}) \wedge (p_2 \in \mathbf{P}) \wedge \left(z = \frac{p_1 + p_2}{2} \right) \right].$$

Thus, we have established that whether we are attempting to make a mathematical argument or a rhetorical argument, we require implications that indicate what must be done to achieve what end.

2.2 Deduction

Suppose, therefore, that we have an implication that we understand to be true. Now we use the next tool of logic: logical deduction. Suppose that we have shown (or assumed or, at least, argued) that the implication $x \to y$ is true, then if we show, assume, or argue that x is true, we may logically deduce that y is also true.

For example, given the implication:

"If is raining, there are clouds in the sky."

If I am standing outside and getting wet (presumably by rain), the above implication gives us that there must be clouds in the sky; that is,

"It is raining; therefore, by this implication, there are clouds in the sky."

In the mantra of some on the politically right,

"If we lower taxes on the very rich, we will stimulate the economy and create jobs."

In this case, lowering taxes on the rich must therefore have two consequences: not only will the economy be stimulated, but it will also create jobs, too. Here is a different example:

"Skipping all classes will either cause you to study on your own time or you will fail."

This may be written as:

$$\text{“Skipping all classes”} \rightarrow (\text{“You will study on your own time”} \vee \text{“You will fail”})$$

In this case, skipping class will result in either one or the other or both of the consequences of the right-hand side. One may argue, however, that this is not true: there may be another consequence:

$$\text{“Skipping all classes”} \rightarrow (\text{“You will study on your own time”} \vee \text{“You will fail”} \vee \text{“You will drop out”})$$

The terms on the right-hand side are not mutually exclusive: you could try to study on your own time, but still fail or still drop out. Notice also that it is not therefore a necessary consequence that students who skips all of their classes will fail: they may just end up studying at home on their time and still passing—indeed, one student who never showed up to ECE 250 once passed with 100 %. Thus, any student who satisfies the left-hand side will therefore have one of the three outcomes on the right-hand side.

2.3 Refuting an Implication

Suppose someone wishes to use an implication such as $x \rightarrow y$ in an argument. For example,

“Anything that has been used for over one hundred years is good.”

This is really the argument

“If x has been used for over one hundred years, x is good.”

Next, one may argue that “homeopathy has been used since 1796”, consequently, “homeopathy is good”. Aside from the absurdity of such a claim, how do we refute such an argument? In this case, all that need be shown is that in one case, you can find something that is old but claim that it is not good.

“Capital punishment has been used for over one hundred years but capital punishment is not good.”

By demonstrating this one point (although some may claim that this is an invalid statement), the implication is demonstrated to be false. Thus, if someone wants to claim that homeopathy is good, he or she will need to use a different line of reasoning.

Similarly, consider the statement similar to the one previously presented:

“If there are clouds in the sky, it is raining.”

It is easy to refute this, too. One need look up in the sky on almost any day it is not raining: almost certainly, “There are clouds in the sky yet it is not raining.” With this one observation, the implication is shown to be wrong. Similarly, one may claim:

“If it is raining, the sun is hidden behind clouds.”

During most showers, this statement is true, but it is possible to have a *sunshower* (and they are often very beautiful). Consequently, one observation of one event that contradicts the implication disproves the implication in general and therefore we may not use it. One may, however, now attempt to tighten the requirements:

“If it is raining at least 10 mm/hr, the sun is hidden behind clouds.”

This says that “If it is raining and the rate of rainfall is more than ten millimeters per hour, the sun is hidden behind clouds.” This is an incredibly heavy rate of rainfall (not likely but not impossible) equalling ten liters of water per square meter per hour. At this point, most individuals would concede that if the implication is not universally true, it may never-the-less be used in a valid argument.

Let us go back to the mantra of some on the political right:

$$\text{“}\textit{Lower taxes on the rich}\text{”} \rightarrow (\text{“}\textit{stimulates the economy}\text{”} \wedge \text{“}\textit{creates jobs}\text{”}) \qquad (1)$$

How can you prove (or disprove) this statement? The easiest way is to set up the left-hand side: lower taxes and see what happens. Despite having done this during the time of George W. Bush, one may try to argue that jobs were created but the jobs lost by far exceeded that, so consequently, as both the terms must be true, disproving either one is sufficient to refute the implication. What does do those on the political right now do seeing that the mantra has been disproven? They can modify the terms of the implication:

$$(\text{“}\textit{Lower taxes on the rich}\text{”} \wedge \text{“}\textit{right incentives}\text{”}) \rightarrow (\text{“}\textit{stimulates the economy}\text{”} \wedge \text{“}\textit{creates jobs}\text{”}) \qquad (2)$$

Now we have two conditions that must be satisfied before the right-hand side will be satisfied: not only must we lower taxes on the rich, but we must also create the right incentives. At this point, anyone can now say that George W. Bush was not able to stimulate the economy and did not create jobs because while he did lower taxes, he did not provide the right incentives. Of course, the definition of “right incentives” is much more difficult to define. Indeed, some on the political right may simply always assume that Statement 2 is true and any time taxes on the rich are lowered but the economy is not being stimulated and we are not creating jobs, the fault lies squarely on not being able to provide the correct incentives.

Now, why am I picking such a divisive topic? To hopefully keep you interested. Whether or not you are left, center or right politically speaking, you might like to find a different way of creating such arguments. The next thing we will look at are tautologies: logical statements that are always true which we can therefore use to refine our arguments.

3. Tautologies

At this point, you understand the

1. implication “If x, y.”, “y if x”, “Either not x or y.”, or $x \rightarrow y$.
2. deduction “If x is true and $x \rightarrow y$, it follows that y is true.”
3. refutation “If x is true but y is false, it follows that $x \nrightarrow y$.”

Now, the next step is to define tautologies: a statement that is always true no-matter what truth values the variables take on. For example, $x \wedge y$ is not a tautology, because if either x or y is false, then the entire statement is false. We will, however, look at statements that are always true:

3.1 Double Negation

Consider the statement, $\neg\neg x \leftrightarrow x$. Whether x is true or false, the statement is true. Because the operator in between is the equivalence operator, this means that whenever we see one, we can replace it with the other. For example, the statement

“It will not happen that it will never rain.”

contains a double negative, and therefore we may remove the double negative:

“It will rain.”

Because it is a tautology, you need not worry about the actual truth value, all we are saying is that you can always replace one by the other and if one is true, so must the other; for example

"It is impossible that the sky is not purple." is equivalent to "The sky is purple."

Either statement is equally false.

3.2 Implication Rewritten

We have already noticed that the two statements

"If it is raining, there are clouds in the sky."

and

"Either it is not raining, or there are clouds in the sky."

both appear to be saying the same thing. In fact, the equivalence

$$(x \rightarrow y) \leftrightarrow (\neg x \lor y)$$

is true regardless of the values of x or y. Consequently, we can, like before, always replace one with the other. In general, the right-hand side will be reinterpreted as the left-hand side for arguments; while for logic gates, it will likely be the other way around: there is no IMPLICATION gate but there are OR, NOT, NAND, NOR, and XOR gates (mostly because these are commutative and n-ary—they can take an arbitrary number of arguments; whereas $x \rightarrow y$ can only take two arguments (it is binary) and it is not even commutative: $x \rightarrow y$ is different from $y \rightarrow z$).

In English, this tautology says:

"x implies y" is identical to saying "Either x is false, or y is true."

Similarly, our mantra of some on the political right may be rewritten as

"Either we are not lowering taxes on the very rich, or we will stimulate the economy and create jobs."

3.3 Two Trivial Tautologies

Here are two simpler tautologies:

$$x \lor \neg x \quad \text{and} \quad \neg(x \land \neg x).$$

The first is the equivalent of a statement like "It is raining or it is not raining" while an example of the second is "It is not true that it is both raining and not raining". Logically, either of these are both true, so we may replace such statements with the general statement "It is always true".

3.4 Reductio Ad Absurdum

A more interesting tautology is

$$(x \rightarrow \neg x) \rightarrow \neg x$$

This has a name: *reduction ad absurdum* or, more commonly, proof by contradiction. If the assumption that the statement x is true implies that x is false, x is false. You have already used this in mathematics:

"Assuming there are a finite number of prime numbers implies that that there are not a finite number of prime numbers; consequently, there are not a finite number of prime numbers."

The proof is actually quite easy: p is prime if p divided by any number from 2 to $p - 1$ has a non-zero remainder. Thus, we may argue:

1. Any integer is either a prime number or it is divisible by a prime number less than it.
2. Any integer is the unique product of prime numbers.
3. If there are a finite number of objects, we can write them all down as $a_1, a_2, ..., a_n$.
4. Assume that the statement **"there are a finite number of prime numbers"** is true.
5. Therefore, we can write all prime numbers as $p_1, p_2, ..., p_n$.
6. Any two integers can be multiplied or added producing another integer.
7. Therefore, we can create a new integer $q = p_1 \times p_2 \times \cdots \times p_n + 1$.
8. At this point, $q > p_k$ for each $k = 1, 2, ..., n$.
9. If we divide q by any number from p_1 to p_n, the remainder will always be 1.
10. Therefore, q cannot be the product of numbers from p_1 to p_n.
11. Therefore, q is either a prime number, or it is divisible by another number that is also prime but not in the list $p_1, p_2, ..., p_n$.
12. Therefore, there are at least $n + 1$ prime numbers.
13. As n was chosen arbitrarily, there is no finite number n which gives us the number of prime numbers.
14. Therefore, **"there are a finite number of prime numbers"** is false.
15. Therefore, by *reduction ad absurdum*, **"there are not a finite number of prime numbers"** is true.

Therefore, because each step in this sequence is true, we may therefore conclude that there are infinitely many prime numbers. If you look closely at the arguments, however, we are making certain assumptions: Points 1, 2, and 6, are assumed, a rigorous mathematician may wish to go and prove these, too; however, most engineering students will happily assume that these are true statements.

Now, mathematicians do follow these rules, but after they have followed the rules and proven new statements, they may, for evermore, take those statements as true. Mathematicians call such statements that have been proven to be true *theorems*.

Theorem:

There are infinitely many prime numbers.

Unlike the mean-value theorem, this is simply a statement of fact.

3.5 De Morgan's Rules

Another two tautologies are:

$$\neg(x \wedge y) \leftrightarrow (\neg x \vee \neg y)$$

$$\neg(x \vee y) \leftrightarrow (\neg x \wedge \neg y)$$

You have likely seen these as *De Morgan's* rules. Examples of this in English include

"It is false that it is both raining and the sun is shining."

is equivalent to

"It is not raining or the sun is not shining."

3.6 De Morgan's Rules

You may have also noticed that the statement

"If it is raining, there are clouds in the sky."

seems to say the same thing as

"If there are no clouds in the sky, it is not raining."

Indeed, these are saying the exact same thing, for the following is another tautology:

$$(x \rightarrow y) \leftrightarrow (\neg y \rightarrow \neg x).$$

This says that the implication $x \rightarrow y$ is true if and only if the implication $\neg y \rightarrow \neg x$ is also true. Thus, any implication can always be reworded in such a way. This may actually be useful at times when refuting an implication: it may be easier to think of a counter-example. For example,

"If inflation is up, it will be a bear market."

and

"If it is a bull market, inflation will be down."

are similar (check out *bear* and *bull markets* in wikipedia). If you are trying to prove this implication wrong, it may be easier to find evidence of a bull market existing while inflation is still up.

3.7 The Syllogism

The most important tautology in arguments, however, is the *syllogism*:

$$((x \to y) \wedge (y \to z)) \to (x \to z).$$

If x can be shown to imply y, and y can be shown to imply z, we may simplify the expression so that we may conclude that x implies z. Note that this is not an equivalence: showing the second does not imply the first:

"If it is raining, there are clouds in the sky."

while being true, does not imply both of the statements:

"If it is raining, the sky is pink" and "If the sky is pink, there are clouds in the sky."

However, the syllogism can be used to prove that an implication is true: suppose you want to prove that

$$x \to z.$$

If you can find an intermediate y such that you can show that both $x \to y$ and that $y \to z$, you can therefore conclude that your original implication, $x \to z$, is true. Indeed, very often, if you are trying to prove that $a \to z$, you may have to find an entire sequence such as:

$$(a \to b) \wedge (b \to c) \wedge (c \to d) \wedge \cdots \wedge (x \to y) \wedge (y \to z)$$

and therefore you may conclude that $a \to z$. The catch, however, is that you must show that each of the intermediate steps is also true. On the other hand, if someone comes to you and claims that $a \to z$ and provides all the intermediate implications, you need only disprove one of them and the entire argument falls apart. In some cases, it may be more difficult, for example, after significant examination, one may find that $j \to k$ fails to hold true; however, in other cases, you may be able to catch the first implication $a \to b$ after which you can nip the entire argument in the bud.

3.7.1 Complex Implications

Now, very seldom will you find a simple causal chain of implications as $(v \to w) \wedge (w \to x) \wedge (x \to y) \wedge (y \to z)$ when trying to prove $w \to z$. Instead, you may have intermediate arguments such as:

1. $(v \vee a) \to w$ either v and a implies w,
2. $(w \wedge b) \to x$ both w and b together imply x,
3. $x \to (y \vee c)$ x implies either y or c or both, and
4. $y \to (z \wedge d)$ x implies both y and d.

In the first case, if you can successfully argue that $(v \vee a) \to w$ is a true implication, this isn't an issue for your argument: if either v or a implies w, then v by itself also implies w. That is, we have the simplification that is itself a tautology:

$$((v \vee a) \to w) \to (v \to w).$$

In the second, if you can successfully argue that $(w \wedge b) \rightarrow x$ is a true implication, not only must you argue that w is true, but you must simultaneously show that b is also true before you can conclude x is true.

In the third, if you can successfully argue that $x \rightarrow (y \vee c)$ is a true implication, having shown that x is true, it is not enough to assume that therefore y is also true, to use this in your argument, you must show that c is false.

In the fourth, if you successfully argue that $y \rightarrow (z \wedge d)$ is a true implication and you were able to show that y is true, then you may assume that both z and d are true and, as you are only interested in z, you need not worry. That is, we have the simplification that is itself a tautology:

$$(y \rightarrow (z \wedge d)) \rightarrow (y \rightarrow z).$$

3.7.2 Refuting Implications

Suppose that in building an argument, an individual claims that $x \rightarrow y$ or that $y \rightarrow z$ is true. What could you do to refute these?

You could argue that $x \rightarrow y$ isn't entirely true because there are more possible requirements; for example, there is another condition a that must also be satisfied so the correct implication is $(x \wedge a) \rightarrow y$; the onus is now on the other person to demonstrate that a is also true.

Alternatively, you could argue that $y \rightarrow z$ isn't entirely true because there are more possible outcomes; for example, there may be another consequence of y which may be different from z so the correct statement is $y \rightarrow (z \vee b)$; the onus is now on the other person to demonstrate that b is false.

3.8 Example, in Detail

Let's go back to our proof by contradiction that there are a finite number of prime numbers.

1. "n is a positive integer greater than 1"
   $\rightarrow$ either "n is a prime number" or "it is divisible by a prime number less than it" but not both.

2. "If, for a given n, for all integers m between 2 and n the ratio n/m has a remainder"
   $\rightarrow$ "n is a prime number."

3. "There are a finite number of objects in a collection"
   $\rightarrow$ "There is an n such that we can write those objects down in a list $a_1, a_2, ..., a_n$"

4. "Given any set of positive integers"
   $\rightarrow$ "We can perform an arbitrary number of addition or multiplication operations on them and the result will be a positive integer"

5. "Given a positive integer n"
   $\rightarrow$ "$n + 1 > n$ and given any positive integer m, $m \times n + 1 > n$"

6. "Given a positive integer n"
   $\rightarrow$ "Given any other positive integer m, $(nm + 1) \div n$ will have a remainder of 1"

7. "Given positive integers n, p, and q where $n \div p$ and $n \div q$ have different remainders"
   $\rightarrow$ "$p \neq q$"

8. "Given a set of objects"
   $\rightarrow$ "There are either a finite number of objects or there is an infinite number of objects"

Assume that the statement "There are a finite number of prime numbers" is true.

The integer 2 is a prime numbers. Therefore the set of prime numbers is not empty.
By 3, we can write down a list of all $N \geq 1$ prime numbers, $p_1, p_2, ..., p_N$.
By 4, we can now create the positive integer $q = p_1 \times p_2 \times \cdots \times p_N + 1$.
Because 2 and 3 are prime numbers and any prime number is greater than 1, $q \geq 3$.
By 5, $q > p_k$ for each $k = 1, 2, ..., N$.
By 6, $q \div p_k$ will have a remainder of 1.
By 1, either q is a prime number or there is a prime number r such that $q \div r$ has remainder 0.
In the first case, where q is a prime number, $q > p_k$ so q is not in our list of prime numbers $p_1, p_2, ..., p_N$.
In the second case where r is a prime number, $q \div r$ has remainder 0 and therefore $r \neq p_k$ for each value of $k = 1, 2, ..., N$ because $q \div p_k$ has a remainder of 1 (by 7).
In either case, we have a new positive integer which is not in our list of prime numbers.
Therefore, the list of prime numbers is $p_1, p_2, ..., p_N, q$ or $p_1, p_2, ..., p_N, r$.
In either case, the number of prime numbers is at least $N + 1$.
From our assumption, we concluded that there were N prime numbers.
Because 3 is a valid implication but the statement that there are N prime numbers is false, it follows that the left-hand side of 3 is false. Therefore "There is a finite number of prime numbers" is false.
Therefore, by *reduction ad absurdum* (assuming p implies $\neg p$),
the statement "There is a finite number of prime numbers is false" is true.
By 8, the other consequent must be true: "There must be an infinite number of prime numbers."

Incidentally, this method can be used to find prime numbers:

2,

2 + 1 = 3,

2 × 3 + 1 = 7,

2 × 3 × 7 + 1 = 43,

2 × 3 × 7 × 43 + 1 = 1807 = 13 × 139,

2 × 3 × 7 × 13 × 43 × 139 + 1 = 3263443,

2 × 3 × 7 × 13 × 43 × 139 + 1 = 10650056950807 = 547 × 607 × 1033 × 31051,

2 × 3 × 7 × 13 × 43 × 139 × 547 × 607 × 1033 × 31051 + 1 = 113423713055421844361000443
= 29881 × 67003 × 9119521 × 6212157481

next number = 181 × 1989 × 112374829138729 × 114152531605972711 ×
35874380272246624152764569191134894955972560447869169859142453622851

With the next step, we find that 5295435634831, 31401519357481261 and 77366930214021991992277 are prime numbers, and the step thereafter finds that 181, 1989, 112374829138729, 114152531605972711 and 35874380272246624152764569191134894955972560447869169859142453622851 are prime; however, the technique has not yet determined that 5 is prime.

3.9 "You're Joking, Right?", Informal Logic, and Fallacies

You are now thinking to yourself: how does anyone do this? Indeed, only two people tried to do this to the extent suggested. These two were Bertrand Russell and Alfred Whitehead. In the early 1900s, these two wrote three volumes entitled *Principia Mathematica*. In this, they attempted to demonstrate that all of mathematics could be deduced from simple logical axioms. It took them 379 pages in the first edition to prove that 1 + 1 = 2, or more precisely, on that page, they say

> "From this proposition it will follow, when arithmetical addition has been defined, that 1+1=2."

Most humans, however, make leaps of logic—often called *informal* logic—where the person making the argument makes many implicit assumptions and takes certain implications for granted. This is reasonable as long as these are understood by both parties but in many cases, the implicit assumptions and implications taken for granted turn out to be false. Such incorrect assumptions and implications are called fallacies and some occur so often in argumentative discourse that those that are given names.

The next two sections will look at proving implications about the physical world and named logical fallacies.

4. Proving Physical Implications

We will look at a number of examples where we will try to argue that some statement of the physical world is true. In mathematics, it is relatively easy: you make some base assumptions and you then draw the logical conclusions. The aforementioned work by Russell and Whitehead is one such example. As another example, all of Euclidean geometry can be deduced from twenty axioms, none of which mention either arithmetic or real numbers. Statements about the physical world are almost never entirely decidable. Never-the-less, let us consider an easy case.

4.1 Proving that Rain Requires Clouds

Someone challenges you on your claim "If it is raining, there are clouds in the sky." Now you must create a more detailed argument: you may then create the following sequence of implications:

1. If it is raining, there are raindrops.
2. A raindrop is formed by condensed atmospheric water vapour.
3. The size of condensation of atmospheric water vapour is exponentially distributed.
4. A raindrop is a droplet that has gained sufficient size to precipitate: that is, to be heavy enough to fall to Earth due to gravity.
5. If there is sufficient condensation to produce something as large as a raindrop, there must be significantly more condensation that has collected into droplets not sufficiently large to be attracted to Earth by gravity.
6. With such a quantity of droplets, these will form visible masses within the sky.
7. These visible masses are called clouds.
8. "Therefore, there will be clouds in the sky.

Even some of these steps may be challenged: What is the scientific evidence that the distribution of sizes of water droplets is exponentially distributed? What is visible? However, at some point within any argument, some facts must be taken for granted based on, for example, the mass of scientific evidence supporting it. As an example, there is so much scientific evidence that the Earth is approximately spherical in shape that it would be pointless to challenge such an assumption. Similarly, any other scientific theory has, by definition, such a mass of evidence that it would be either naive to attempt to challenge it. (Please note, *string theory* is incorrectly named: it should be more correctly called the *string hypothesis*—there has not been found sufficient evidence that string theory is actually true.) On the other hand, there are legitimate scientific hypotheses for which there is currently insufficient evidence to allow them to be called theories: for example, "hotspots" are known to exist within the Earth's mantle and are the source of significant volcanic activity. Two such examples include the Hawaiian Islands and Yellowstone National Park (the entire park is one massive caldera measuring 55 km by 72 km). One anomaly, however, is the significant appearance of hotspot pairs—two hotspots at opposite ends of the Earth. With over fifty confirmed or postulated locations of hotspots, the first observation must be that, by chance, some hotspots will be approximately opposite each other on the Earth. If, however, it is found that there are more opposing pairs of hotspots than would be statistically reasonable, we would need to attempt to find another explanation. One hypothesis is that such pairs are formed as a result of meteoric impacts; however, there is insufficient evidence to date to allow any such hypothesis to merit the name of a scientific theory.

4.2 Proving Professional Misconduct

Here is another claim:

> "Offering services within the practice of professional engineering without a Certificate of Authorization constitutes professional misconduct."

While this statement is true, how do we prove it is true?

1. Paragraph 12(2) of the Professional Engineers Act says that "No person shall offer to the public or engage in the business of providing to the public services that are within the practice of professional engineering except under and in accordance with a certificate of authorization."
2. This is in the form "If a person does not have a certificate of authorization, that person may not provide the public services within the practice of professional engineering."
3. Therefore, "If a person is providing the public services within the practice of professional engineering, that person must have a certificate of authorization."
4. As this is a statute, it is a statement that this implication must be true.
5. If there is a situation that causes an implication in a statute to be false, it is said to be a breach of that statute.
6. In this case, the person does not have a certificate of authorization, yet never-the-less, he or she is offering the public services within the practice of professional engineering.
7. Therefore, the individual has breached the Professional Engineers Act.
8. In Section 72(2)(g) of Ontario Regulation, "For the purposes of the Act and this Regulation, 'professional misconduct' means...breach of the Act or regulations, other than an action that is solely a breach of the code of ethics."
9. This says, "If an action is a breach of the Act, it constitutes professional misconduct."
10. Therefore, using the syllogism, "Offering services within the practice of professional engineering without a Certificate of Authorization constitutes professional misconduct."

Think about this: to definitively show that the statement is true required ten steps and even this is a significant jump in some cases: One may argue over what is the definition of "constitutes" or "breach". One may wish to understand why an action that causes a statement within of an act of a legislative body of a government is a breach. One may even want to know whether or not the Ontario Legislature has authority to enact statutes within Ontario with respect to the practice of professional engineering.

As you can see, at some point, we must take some points for granted. In this case, we always assume that the Professional Engineers Act is a valid statute. There may come a day when an individual challenges the constitutionality of a section or item within that statute; however, until that point, we may take it as law.

4.3 Economic Arguments

One much more difficult topic to argue on is on anything dealing with the economy. The issue here is that there are so many different factors that may be involved, or so many possible consequences. Therefore, both the left- and right-hand sides of any implications become more and more complex, and therefore, it is only possible to argue that certain consequences "may be likely" or "are almost certain" or "it is reasonably unlikely that".

5. Fallacies

We will now look at some fallacies that are so common that they are given names.

5.1 Argumentum ad antiquitatem, appeal to tradition

Any argument of the form "If *x* is old, *x* is good." is an argument from antiquity. It suggests that simply the age or longevity of *x* lends credence to its correctness. This can be refuted quite easily:

Cannibalism is an ancient practice, but cannibalism is wrong.

Even this is not correct: how can we say that cannibalism is wrong? One may argue that cannibalism need not include the murder of the individual being eaten, but rather the eating of those who have died of other causes. Thus, we could use a line of reasoning as follows:

> A parasite that has, as a host, a particular species may not adversely affect another species and the more distantly related two species are, the less likely that a parasite that affects one will also affect the other.

Now, I am asserting that this is true, but one could also look into the literature to find evidence for this statement. One could also observe that we normally do not get ill from eating fruits and vegetables, we are also unlikely to become ill from fish, we may get salmonella from birds, we may become ill from eating other mammals, and HIV is derived from the SIV in primates. While this is not a *proof*, it could be used in an informal argument and most people would accept it. Therefore, any parasite that infects one human is almost certainly capable of infecting another human. Therefore, consuming one human implies that almost all parasites in the consumed individual will be transferred and are viable in the one consuming the human.

1. Consuming humans will almost certainly transfer all parasites.
2. If the human being consumed is ill, the one consuming the human will become ill.
3. Any highly dangerous activity should be restricted.

But wait... Mountain climbing is dangerous—should it be restricted? Thus, creating a solid argument about the state of society is often very difficult, which is why politicians argue. In addition, individuals have different moral values: one might value liberty over authority, while others might see authority as a source of stability and thus value authority over liberty. It is, in fact, quite easy to make a shallow but convincing argument that strong central authority is essential to a stable society, as fascists do, but this requires the one making the argument to ignore significant reams of evidence that contradicts this.

Anyway, it is reasonably plausible that cannibalism is wrong. Therefore,

Despite cannibalism being an ancient practice, it is never-the-less wrong.

As the statement "Cannibalism is an ancient practice" is definitely true, the only other logical conclusion is that the implication "If *x* is old, *x* is good." is false. Hence, the *argumentum ad antiquitatem* is a fallacy. This is often used with many pseudoscientific, alternative and traditional medicines. There is a name for traditional medicine that has been demonstrated to be true: they are called *medicine*.

5.2 Argumentum ad populum, appeal to popularity

Another fallacy is *argumentum ad populum*:

If x is popular, x is good or right.

Once again, numerous things have been popular in the past and they have not always been right.

5.3 Summary

One reason many people prefer not to engage in debates is that they are not able to argue: they cannot correctly build up a successful argument, nor do they have the tools necessary to refute an argument of an opponent. Thus, they become frustrated in the entire process.

6. How Do We Prepare Arguments?

How do you prepare arguments to either support your case? A book of logic puzzles was written by Lewis Carroll in 1896 (yes, the same Lewis Carroll of “Alice in Wonderland”). In this, he provides a sequence of statements, each of which may be interpreted as an implication. One such example is given in Appendix B. It is your job to find the most general implication. For example, once you begin converting the statements into implications, you may find that

$$A \to \neg B,\ C \to D,\ E \to B,\ \neg C \to E$$

Therefore, using the contrapositive, where necessary, we may deduce that $A \to \neg B$, $\neg B \to \neg E$, $\neg E \to C$ and $C \to D$ and using the syllogism four times, we get that $A \to D$ or $\neg D \to \neg A$.

What is most interesting about the games by Carroll is that there appears to be a lot of information from which the implications are drawn. You should do the same: collect as much information on the subject matter as possible. Be very aware of *confirmation bias*: every human has a tendency to look for evidence that supports his or her point of view or even if that evidence is collected, it is more quickly to be dismissed by finding arguments to discount it.

Once you have evidence, you may see a number of implications:

$$A \to B,\ C \wedge D \to E,\ F \to G \vee H,\ etc.$$

7. Conclusions

We have summarized how to construct arguments. Mathematical arguments are, in a sense, easier to construct—all you must do is familiarize yourself with all of the assumptions. Arguments about the physical world are much more difficult to construct. Due to human nature, for any form of reasonable communication, it is absolutely necessary to take some assumptions as granted and implications as true; otherwise, our discourse would be never-ending. However, this leads individuals to make incorrect assumptions and use implications that are false. The skilled user of logic will be able to find flaws in the arguments of others but as humans we are biased: it is always easier to see the flaws in another individual's arguments than it is to find flaws in your own. Consequently, always keep an open mind: your goal should be to find truth, as opposed to just being right.

A. Logic Puzzles

How are logic puzzles related to creating arguments? In many cases, the logic puzzles are a sequence of consequences. Here is an example modified from Brain Bashers Logic Puzzles[1]:

> During a university reunion, four ECE graduates discussed their starting salaries. The starting salaries in question were 60, 70, 80, and 90 thousand dollars per year. The successful entrepreneur earned the most. Alexis earned more than Bailey and the consulting engineer earned more than Dylan, the design engineer. Casey could not remember what he started on. Bailey, the Bay Street analyst did not start with $70,000 nor did Dylan.

For such a puzzle, we can set up the following grid of possible associations. We can fix one of them: let us choose salary but any could work.

ABCD	ABCD	ABCD	ABCD
acde	acde	acde	acde
$60,000	$70,000	$80,000	$90,000

The successful entrepreneur earned the most.
Therefore, the entrepreneur is at $90,000 and not at any other salary.

ABCD	ABCD	ABCD	ABCD
acd~~e~~	acd~~e~~	acd~~e~~	~~acd~~**e**
$60,000	$70,000	$80,000	$90,000

Alexis earned more than Bailey.
Therefore, Alexis did not earn the least and Bailey did not earn the most.

~~A~~BCD	ABCD	ABCD	A~~B~~CD
acd	acd	acd	**e**
$60,000	$70,000	$80,000	$90,000

The consulting engineer earned more than Dylan.
Therefore, the consulting engineer did not earn the least and Dylan could not have earned $80,000 or $90,000. As Dylan was the design engineer, we can also remove the design engineer from $80,000.

BCD	ABCD	ABC~~D~~	AC~~D~~
a~~c~~d	acd	ac~~d~~	**e**
$60,000	$70,000	$80,000	$90,000

Bailey, the Bay Street analyst did not start with $70,000.
We can remove Bailey and the analyst from the $70,000 salary.

BCD	A~~B~~CD	ABC	AC
ad	~~a~~cd	ac	**e**
$60,000	$70,000	$80,000	$90,000

[1] http://www.brainbashers.com/logic.asp

Dylan did not start with $70,000.
We can remove Dylan from $70,000, as well. As Dylan is the design engineer, we can remove the design engineer from the $70,000 category. Therefore, the consulting engineer is making $70,000.

BCD	AC~~D~~	ABC	AC
ad	**c**~~d~~	ac	**e**
$60,000	$70,000	$80,000	$90,000

There is only one possibility now for Dylan the design engineer: $60,000.

~~BC~~**D**	AC	ABC	AC
~~a~~**d**	**c**	ac	**e**
$60,000	$70,000	$80,000	$90,000

There is only one possibility for Bailey now: $80,000. Therefore,

D	AC	~~A~~**B**~~C~~	AC
d	**c**	ac	**e**
$60,000	$70,000	$80,000	$90,000

As the consulting engineer is making $70,000, it follows the consultant is not making $80,000; therefore, Bailey is the analyst making $80,000.

D	AC	**B**	AC
d	**c**	**a**~~c~~	**e**
$60,000	$70,000	$80,000	$90,000

Finally, we return to the fact that Alexis made more than Bailey, and therefore Alexis is making $90,000 and Casey must be making $70,000.

D	~~A~~**C**	**B**	**A**~~C~~
d	**c**	**a**	**e**
$60,000	$70,000	$80,000	$90,000

Thus, we conclude that

Alexis	entrepreneur	$90,000
Bailey	analyst	$80,000
Casey	consulting engineer	$70,000
Dylan	design engineer	$60,000

Such logical games are processes of elimination and while they use implications, they do not necessarily aid in understanding how to construct logical arguments.

B. Symbolic Logic by Lewis Carroll

The author of Alice in Wonderland was also a mathematician and logician. He has an excellent book titled *Symbolic Logic* published in 1896. In this, he introduces a sequence of implications and the reader is asked to draw the most general consequence possible. For example,

1. I call no day “unlucky” when Robinson is civil to me.
2. Wednesdays are always cloudy.
3. When people take umbrellas, the day seldom turns out fine.
4. The only days when Robinson is uncivil to me are Wednesdays.
5. Nobody leaves his umbrella behind when it is raining.
6. My “lucky” days usually turn out fine.

Each of these can be interpreted as an implication, and we will represent each statement by a single word:

1.	If Robinson is civil to me, I do call that day “lucky”.	$\mathit{Civil} \rightarrow \mathit{Lucky}$
2.	If it is Wednesday, it is cloudy.	$\mathit{Wed} \rightarrow \mathit{Cloudy}$
3.	If people take umbrellas, the day seldom turns out fine.	$\mathit{Umbrellas} \rightarrow \neg \mathit{Fine}$
4.	If Robinson is uncivil to me, it is Wednesday.	$\neg \mathit{Civil} \rightarrow \mathit{Wed}$
5.	If it is raining, nobody leaves his umbrella behind.	$\mathit{Rain} \rightarrow \mathit{Umbrellas}$
6.	If it is a “lucky” day, it turns out fine.	$\mathit{Lucky} \rightarrow \mathit{Fine}$

We may also find the contrapositive of each of these:

1'.	If I call the day “unlucky”, Robinson is not civil to me.	$\neg \mathit{Lucky} \rightarrow \neg \mathit{Civil}$
2'.	If it is not cloudy, it is not Wednesday.	$\neg \mathit{Cloudy} \rightarrow \neg \mathit{Wed}$
3'.	If the day turns out fine, people do not take umbrellas.	$\mathit{Fine} \rightarrow \neg \mathit{Umbrellas}$
4'.	If it is not Wednesday, Robinson is civil to me.	$\neg \mathit{Wed} \rightarrow \mathit{Civil}$
5'.	If everyone leaves their umbrellas behind, it is not raining.	$\neg \mathit{Umbrellas} \rightarrow \neg \mathit{Rain}$
6'.	If a day does not turn out fine, it is an “unlucky” day.	$\neg \mathit{Fine} \rightarrow \neg \mathit{Lucky}$

Notice, that *Cloudy* and *Rain* appear only once. We may now pick a sequence of implications:

If it is not cloudy, it is not Wednesday.	5	$\mathit{Rain} \rightarrow \mathit{Umbrellas}$
If it is not Wednesday, Robinson is civil to me.	3	$\mathit{Umbrellas} \rightarrow \neg \mathit{Fine}$
If Robinson is civil to me, I do call that day “lucky”.	6'	$\neg \mathit{Fine} \rightarrow \neg \mathit{Lucky}$
If it is a “lucky” day, it turns out fine.	1'	$\neg \mathit{Lucky} \rightarrow \neg \mathit{Civil}$
If the day turns out fine, people do not take umbrellas.	4	$\neg \mathit{Civil} \rightarrow \mathit{Wed}$
If everyone leaves their umbrellas behind, it is not raining.	2	$\mathit{Wed} \rightarrow \mathit{Cloudy}$

Consequently, we have the general consequence $\mathit{Rain} \rightarrow \mathit{Cloudy}$:

If it is raining, it is cloudy.

or, its contrapositive, $\neg \mathit{Cloudy} \rightarrow \neg \mathit{Rain}$:

If it is not cloudy, it is not raining.

Notice a difficulty: this author initially interpreted “I call no day ‘unlucky’ when Robinson is civil to me.” as “If I do not call a day ‘unlucky’, Robinson is civil to me.” but this is not the case. It is the opposite: if Robinson is civil, it follows that I would not call that day “unlucky”. At this point, we must make the assumption that not calling a day “unlucky” suggests that one calls the day “lucky”. Incidentally, Carroll’s wording of the solution is “Rainy days are always cloudy.”

The page from the book containing this example is given here:

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52.

(1) Anything, that is not absolutely ugly, may be kept in a drawing-[room ;
(2) Nothing, that is encrusted with salt, is ever quite dry ;
(3) Nothing should be kept in a drawing-room, unless it is free from damp;
(4) Bathing-machines are always kept near the sea ;
(5) Nothing, that is made of mother-of-pearl, can be absolutely ugly ;
(6) Whatever is kept near the sea gets encrusted with salt.

Univ. "things" ; a = absolutely ugly ; b = bathing-machines ; c = encrusted with salt ; d = kept near the sea ; e = made of mother-of-pearl ; h = quite dry ; k = things that may be kept in a drawing-room.

53.

(1) I call no day "unlucky," when Robinson is civil to me ;
(2) Wednesdays are always cloudy ;
(3) When people take umbrellas, the day seldom turns out fine ;
(4) The only days when Robinson is uncivil to me are Wednesdays ;
(5) Nobody leaves his umbrella behind him when it is raining ;
(6) My "lucky" days usually turn out fine.

Univ. "days" ; a = called by me 'lucky' ; b = cloudy ; c = days when people take umbrellas ; d = days when Robinson is civil to me ; e = rainy ; h = usually turning out fine ; k = Wednesdays.

54.

(1) No shark ever doubts that it is well fitted out ;
(2) A fish, that cannot dance a minuet, is contemptible ;
(3) No fish is quite certain that it is well fitted out, unless it has three rows of teeth ;
(4) All fishes, except sharks, are kind to children.
(5) No heavy fish can dance a minuet ;
(6) A fish with three rows of teeth is not to be despised.

Univ. "fishes" ; a = able to dance a minuet ; b = certain that he is well fitted out ; c = contemptible ; d = having 3 rows of teeth ; e = heavy ; h = kind to children ; k = sharks.

55.

(1) All the human race, except my footmen, have a certain amount of common-sense ;
(2) No one, who lives on barley-sugar, can be anything but a mere baby ;
(3) None but a hop-scotch player knows what real happiness is ;
(4) No mere baby has a grain of common sense ;
(5) No engine-driver ever plays hop-scotch ;
(6) No footman of mine is ignorant of what true happiness is ;

Univ. "human beings" ; a = engine-drivers ; b = having common sense ; c = hop-scotch players ; d = knowing what real happiness is ; e = living on barley-sugar ; h = mere babies ; k = my footmen.

[Ans. 133 ; Sol. 162, 163.

CH. I. § 9.] *EXAMPLES.* 123

56.

(1) I can trust any animal that belongs to me ;
(2) Dogs gnaw bones ;
(3) I admit no animals into my study, unless they will beg when told to do so ;
(4) All the animals in the yard are mine ;
(5) I admit any animal, that I can trust, into my study :
(6) The only animals, that are really willing to beg when told to do so, are dogs.

Univ. "animals" ; a = admitted to my study ; b = animals that I can trust ; c = dogs ; d = gnawing bones ; e = in the yard ; h = my ; k = willing to beg when told.

57.

(1) Animals are always mortally offended if I fail to notice them ;
(2) The only animals that belong to *me* are in that field ;
(3) No animal can guess a conundrum, unless it has been properly trained in a Board-School ;
(4) None of the animals in that field are badgers ;
(5) When an animal is mortally offended, it always rushes about wildly and howls ;
(6) I never notice any animal, unless it belongs to me ;
(7) No animal, that has been properly trained in a Board-School, ever rushes about wildly and howls.

Univ. "animals" ; a = able to guess a conundrum ; b = badgers ; c = in that field ; d = mortally offended if I fail to notice them ; e = my ; h = noticed by me ; k = properly trained in a Board-School ; l = rushing about wildly and howling.

58.

(1) I never put a cheque on that file, unless I am anxious about it ;
(2) All the cheques received this year, that are not marked with a cross, are payable to bearer ;
(3) No cheques are ever brought back to me, unless they have been dishonoured at the Bank ;
(4) All the cheques received this year, that are marked with a cross, are for amounts of over £100 ;
(5) All the cheques received this year, that are not on that file, are marked "not negotiable" ;
(6) No cheque of yours has ever been dishonoured ;
(7) I am never anxious about a cheque, unless it should happen to be brought back to me ;
(8) None of the cheques received this year, that are marked "not negotiable," are for amounts of over £100.

Univ. "cheques received this year" ; a = brought back to me ; b = cheques that I am anxious about ; c = honoured ; d = marked with a cross ; e = marked 'not negotiable' ; h = on that file ; k = over £100 ; l = payable to bearer ; m = your.

Ans. 133 ; Sol. 163.]

This image is taken from http://archive.org/details/symboliclogic00carr where you can download a free pdf version of the full content. If you do choose to read this book, however, be aware that the book does use the occasional social prejudice that would be considered rude, absurd, offensive and wrong today. May I recommend number 58?

C. Sample Factsheet

This is a sample factsheet from the article *The "Factsheet" as a Tool for Teaching Logical Writing* by L.E. Modesitt, Jr. Please see http://wac.colostate.edu/journal/vol3/modesitt1.pdf .

Recreation in the United States

Thesis

People in the United States only think they are getting good exercise. In reality, they waste billions of dollars when walking, chopping wood, doing gardening and housework would provide better health at lower cost. The exercise game is nothing more than expensive escapism.

Facts

458 of the $35 billion spent annually on exercise and recreational goods by Americans goes for recreational vehicles, such as boats, snowmobiles, RVs, and bicycles. Sales of bicycles—the only human powered vehicle—comprise less than 8% of the $35 billion, although the average bicycle retails for around $150.

15% of recreational exercise expenditures go for sport clothing, and another 10% goes for athletic footwear, including golf shoes, jogging shoes, sneakers, and gym shoes. More than 35% of the $10 billion spent annually on athletic equipment goes toward firearms, golfing, and fishing equipment. More active sports, such as tennis, get less than 5% of equipment spending dollars.

More than 68million Americans bowl, but lessthan 20 million play tennis, and with 14 million pleasure boats owned in the U.S., boating captures more participants than tennis also.

The number of Americans playing active sports has declined.

Tennis players have dropped from 25 million to less than 20 million. Even bowling participants have dropped from 72 million to 68 million over the past 5 years.

Source: *Statistical Abstract of the U.S.*, 1989.