In a nutshell: The Hooke-Jeeves method

Given a continuous real-valued function $f$ of a vector variable with one initial approximation of a minimum $u_0$, the Hooke-Jeeves method steps towards a minimum by using the canonical unit vectors without relying on the ability to differentiate the function.

We will assume the dimension of the vector variable is $n$ and the canonical vectors are $e_1, \ldots, e_n$.

Parameters:

- $\varepsilon_{\text{step}}$ The maximum error in the value of the minimum cannot exceed this value.
- $\varepsilon_{\text{abs}}$ The difference in the value of the function after successive steps cannot exceed this value.
- $h$ An initial step size.
- $N$ The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. If $k > N$, we have iterated $N$ times, so stop and return signalling a failure to converge.
3. Let $\Delta u_k \leftarrow 0$ and letting $j$ take the values from 1 to $n$ do the following:
   a. If $f(u_k + \Delta u_k + he_j) < f(u_k + \Delta u_k)$, $f(u_k + \Delta u_k - he_j)$, set $\Delta u_k \leftarrow \Delta u_k + he_j$.
   b. otherwise, if $f(u_k + \Delta u_k - he_j) < f(u_k + \Delta u_k)$, set $\Delta u_k \leftarrow \Delta u_k - he_j$.
4. If $\Delta u_k = 0$, we are done for this step, increment $k$ and divide $h$ by 2: $h \leftarrow h/2$, and return to Step 2.
5. Let $u_{k+1} \leftarrow u_k + \Delta u_k$.
   a. If $f(u_{k+1} + \Delta u_k) < f(u_{k+1})$, set $u_{k+1} \leftarrow u_{k+1} + \Delta u_k$ and return to this Step 5a.
6. If $||u_{k+1} - u_k||^2 < \varepsilon_{\text{step}}$ and $|f(u_{k+1}) - f(u_k)| < \varepsilon_{\text{abs}}$, return $u_{k+1}$.
7. Return to Step 2.

Acknowledgement: Jakob Koblinsky noted I was referring to $\Delta x_k$ and not $\Delta u_k$ in Step 5. This has been corrected.