Outline

- In this lesson, we will:
  - Introduce the concept of repetition and the for loop
  - Look examples using the loop variable
  - Author a program to determine if an integer is prime
  - Consider the different variations of a for loop
  - Look at three examples of a loop within a loop

Repetition statements

- Suppose we wanted to repeat an action a fixed number of times

```cpp
#include <iostream>

// Function declarations
int main();

// Function definitions
int main() {
  for (int k{1}; k <= 10; ++k) {
    std::cout << "Hello!" << std::endl;
  }

  return 0;
}
```

Output:
Hello!
Hello!
Hello!
Hello!
Hello!
Hello!
Hello!
Hello!
Hello!
Hello!

The components of a for loop

- Looking at the for loop:

```cpp
for (int k{1}; k <= 10; ++k) {
  std::cout << "Hello!" << std::endl;
}
```
Performing a loop

- Working through this example:
  ```
  for ( int k=1; k <= 5; ++k ) {
    // The loop body
    std::cout << "Hello!" << std::endl;
  }
  ```
  Output:
  ```
  Hello!
  Hello!
  Hello!
  Hello!
  Hello!
  ```
  - A loop variable k is initialized with the value

Using the loop variable

- You can also use this loop variable:
  ```
  for ( int k=1; k <= 5; ++k ) {
    // The loop body
    std::cout << k << std::endl;
  }
  ```
  Output:
  ```
  1
  2
  3
  4
  5
  ```
  - A loop variable k is initialized with the value

Calculating the sum of the first n integers

- Here we use this loop variable in a calculation:
  ```
  int sum=0;
  for ( int k=1; k <= 5; ++k ) {
    // The loop body
    sum += k;
  }
  ```
  Output:
  ```
  15
  ```
  std::cout << sum << std::endl;

Calculating n!

- Here we calculate the value of 5!:
  ```
  int factorial=1;
  for ( int k=1; k <= 5; ++k ) {
    // The loop body
    factorial *= k;
  }
  ```
  Output:
  ```
  120
  ```
  std::cout << factorial << std::endl;
While loops

Is $n$ prime?

- Let us determine if an integer $n$ is prime
  - By definition, $n$ is prime if it is divisible only by 1 and $n$
  - In other words, $n$ is prime if it is not divisible by 2, 3, ..., $n-1$
  - If $n$ is divisible by $k$,
    - The remainder of $n \div k$ of zero
  - In C++, we find the remainder of $n \div k$ by calculating $n \% k$
  - Therefore, test if $n \% k == 0$ for $k$ going from 2 to $n - 1$

Is $n$ prime?

- Do we have to test all integers?
  - If $n$ is divisible by 14,
    - then $n$ must be divisible by at least one of 2 or 7
  - Therefore, we only have to test if $n$ is divisible
    - by all prime numbers $k$ between 2 and $n - 1$
  - Problem: we don’t have a list of all prime numbers...
  - We do know, however, that all even numbers after 2 are not prime
    - Can we avoid calculating $n \% k$ for even values of $k$?

- Strategy: test if $n \% 2 == 0$,
  - if not, test $n \% k == 0$ for $k$ from 3, 5, 7, ..., up to $n - 1$

Is $n$ prime?

- Implementing this in a program:

```cpp
int main() {
    int n{};
    std::cout << "Enter an integer: ";
    std::cin >> n;
    bool is_prime(true);
    for (int k{2}; k < n - 1; k++) {
        if (n % k == 0) {  // is_prime = false;
        }
    }
    if (is_prime) {  
        std::cout << "The integer " << n << " is prime" << std::endl;
    } else {
        std::cout << "The integer " << n << " is not prime" << std::endl;
    }
    return 0;
}
```

- We could use the following condition statement and for loop:

```cpp
if (n % 2 == 0) {
    is_prime = false;
} else {
    for (int k{3}; k < n; k++) {
        // Only test if n is divisible by k for odd k
        if (n % k == 0) {  
            // k must be odd
            if (n % k == 0) {  // is_prime = false;
                }
            }
    }
}
```
While loops

Is \( n \) prime?

• Recall that \( ++k \) is the same as \( k += 1 \), so this is also valid:
  ```c++
  if ( n%2 == 0 ) {
    is_prime = false;
  } else {
    for ( int k=3; k < n; k += 1 ) {
      // Only test if \( n \) is divisible by \( k \) for odd \( k \)
      if ( k%2 != 0 ) {
        // \( k \) must be odd
        if ( n%k == 0 ) {
          is_prime = false;
        }
      }
    }
  }
  
  return is_prime;
  ```

Is \( n \) prime?

• Therefore, we could just use the following:
  ```c++
  if ( n%2 == 0 ) {
    is_prime = false;
  } else {
    for ( int k=3; k < n; k += 2 ) {
      if ( n%k == 0 ) {
        is_prime = false;
      }
    }
  }
  ```

Different update statements

• Let us determine if an integer \( n \) is prime
  ```c++
  bool is_prime=true;
  if ( n%2 == 0 ) {
    is_prime = false;
  } else {
    for ( int k=3; k < n; k += 2 ) {
      if ( n%k == 0 ) {
        is_prime = false;
      }
    }
  }
  
  if ( is_prime ) {
    std::cout << "The integer " << n << " is prime" << std::endl;
  } else {
    std::cout << "The integer " << n << " is not prime" << std::endl;
  }
  ```

• Here we use this loop variable in a calculation:
  ```c++
  int sum=0;
  for ( int k=1; k <= 20; k += 2 ) {
    // The loop body
    sum += k;
  }
  
  std::cout << sum << std::endl;
  ```

Output: 31
While loops

• You can also use this loop variable:
  ```cpp
  for ( int k(4); k >= 0; --k ) {
    // The loop body
    std::cout << k << std::endl;
  }
  ```
  Output:
  ```
  4
  3
  2
  1
  0
  ```
  – A loop variable k is initialized with the value

The most important loop

• The most important loop you will see in this course:
  ```cpp
  for ( int k(0); k < N; ++k ) {
    // The loop body
    std::cout << k << std::endl;
  }
  ```
  loops with k taking values from 0 up to N – 1

  – This is equivalent to:
  ```cpp
  for ( int k(0); k != N; ++k ) {
    // The loop body
    std::cout << k << std::endl;
  }
  ```

Arbitrary starting and ending points

• Of course, your end-points need not be 0 or 1:
  ```cpp
  for ( int k(256); k < 1024; ++k ) {
    // The loop body
    std::cout << k << std::endl;
  }
  ```
  – Loops with k taking values from 256 up to 1023

  ```cpp
  for ( int k(256); k > 128; --k ) {
    // The loop body
    std::cout << k << std::endl;
  }
  ```
  – Loops with k taking values from 256 down to 129

Loops within loops

• Loops within loops:
  ```cpp
  for ( int k(0); k < N; ++k ) {
    // Inner loop body
    for ( int k(0); k < M; ++k ) {
      // Outer loop body
    }
  }
  ```
  – Given two integers, m and n, create the following ASCII art:

```cpp
even
```
Loops within loops

- We will require a loop that prints each of the m rows
  - This outer loop must run from 1 to m
- For each row, we must print n asterisks
  - This requires an inner loop from 1 to n
  - At the end of each execution of the inner loop, we must print an end-of-line

n rows

n columns

Loops within loops

- Loops within loops:
  - Given one integer, n, create the following ASCII art:
    
    n columns
    
    n rows

Loops within loops

int main()
{
    int m;
    int n;
    std::cout << "Enter the number of rows: ";
    std::cin >> m;
    std::cout << "Enter the number of columns: ";
    std::cin >> n;
    
    for (int rows = 1; rows <= m; ++rows)
    {
        for (int columns = 1; columns <= n; ++columns)
        {
            std::cout << "*
        }
        std::cout << std::endl;
    }
    return 0;
}
### Loops within loops

- Note, however,
  - In row 1, we print 1 asterisk
  - In row 2, we print 2 asterisks
  - In row 3, we print 3 asterisks

```
for (int rows = 1; rows <= n; ++rows) {
    for (int columns = 1; columns <= rows; ++columns) {
        std::cout << "*
    }
    std::cout << std::endl;
}
```

### Conditional statements within loops within loops

- Loops within loops:
  - Given one integer, n, create the following ASCII art:

```
for (int rows = 1; rows <= n; ++rows) {
    for (int columns = 1; columns <= n; ++columns) {
        if (columns < rows) {
            std::cout << " ";
        } else if (columns == rows) {
            std::cout << "o ";
        } else {
            std::cout << "* ";
        }
    }
    std::cout << std::endl;
}
```
Applications of loops within loops

- These sound like silly games, but these algorithms are all essential for implementations of linear algebra algorithms
  - Initializing the entries of an $m \times n$ matrix
  - Multiplying an $n$-dimensional vector by a $m \times n$ matrix
  - Performing Gaussian elimination on a system of $n$ linear equations in $n$ unknowns
  - Using backward substitution to find a solution to such a system in row-echelon form
  - Multiplying an $\ell \times m$ matrix and a $m \times n$ matrix

Summary

- Following this lesson, you now
  - Understand how to construct and run a for loop
  - Know how to use the loop variable within the loop body
  - Understand how we can determine if an integer is prime in C++
    - We will see more efficient algorithms later
  - Know that the initial value, the conditional statement, and the update statement can all be modified as necessary
  - Understand why a loop may be used inside another loop
    - Especially with applications in linear algebra
    - This includes some that require loops within loops within loops
  - Know that the inner loop can also depend on the loop variable of an outer loop

References

   https://en.wikipedia.org/wiki/For_loop
[2] cplusplus.com
   http://www.cplusplus.com/doc/tutorial/control/

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