University of Waterloo
Department of Electrical and Computer Engineering
Spring, 2025

ECE 203: Probability and Statistics 1

MIDTERM EXAMINATION

Surname								
Legal Given Name(s)								
UW Student ID Number								

Instruction:

1. There are 100 points in total (plus 10 bonus points).

2. This is a written exam. Please turn off all electronic media and store them under your desk.

3. Be neat. Poor presentation will be penalized.

4. **No questions will be answered during the exam**. If in doubt, state your assumption(s) and proceed.

5. Do not leave during the examination period without permission.

6. Do not stand up until all the exams have been picked up.

Do well!

Question 1: 30%

A manufacturing plant has three machines: Machine A, Machine B, and Machine C. Machine A produces 30% of the total output, Machine B produces 50%, and Machine C produces 20%. The probability that a randomly selected item produced by each machine is defective is 5% (Machine A), 2% (Machine B) and 1% (Machine C).

Let D be the event that a randomly selected item is defective.

(a) Compute the probability that a randomly selected item from the overall production is defective.

(b) If an item is found to be defective, what is the probability that it was produced by Machine A?

Question 2: 35%

A company is hiring new software engineers through a three-stage process:

- Stage 1: Application screening. 60% of applicants pass this stage.

- Stage 2: Technical interview. 70% of those who pass the screening pass the interview.

- Stage 3: Final HR round. 80% of those who pass the interview succeed in the HR round.

(a) What is the probability that a randomly selected applicant passes all three stages?

(b) Given that an applicant passed both the screening and the technical interview, what is the probability that they also pass the HR round?

Question 3: 35%

A binary communication system transmits either a 0 or a 1 with equal probability. However, due to noise in the channel, the received bit may be corrupted. If a 0 is sent, it is correctly received as 0 with probability 0.9, and flipped to 1 with probability 0.1. If a 1 is sent, it is correctly received as 1 with probability 0.8, and flipped to 0 with probability 0.2.

(a) A bit is received as 0. What is the probability that the bit that was actually sent was a 0?

(b) Based on your result, would you trust that a received 0 was really a transmitted 0? Explain briefly.

Bonus Question: 10%

A digital sensor records the number of signal pulses it detects in a fixed time interval. The number of pulses, represented by the random variable X, can take the values 0, 1, 2, or 3, with the following probability mass function: $P(X = 0) = 0.1$, $P(X = 1) = 0.3$, $P(X = 2) = 0.4$ and $P(X = 3) = 0.2$.

(a) Verify that this is a valid PMF.

(b) Compute the mean (expected value) of X, i.e., $\mathbb{E}[X]$.

(c) Briefly interpret what this mean value represents in the context of the sensor.