ECE 203
Probability Theory and Statistics I
Tutorial 1

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UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

Problem 1

- Describe the sample spaces of the following random experiments:

a) Draw a ball from an urn containing balls numbered 1 to 10.

b) Draw a card from a deck that only contains the following four cards: i) 2 of hearts, ii) the three of spades, iii) the four of diamonds, and iv) the three of clubs.

c) Measure the lifetime of a memory chip.

d) Measure the value of two $1k\Omega$ resistors with tolerances of 10%.

Problem 1 - Solution

- Describe the sample spaces of the following random experiments:

a) Draw a ball from an urn containing balls numbered 1 to 10.

b) Draw a card from a deck that only contains the following four cards: i) 2 of hearts, ii) the three of spades, iii) the four of diamonds, and iv) the three of clubs.

c) Measure the lifetime of a memory chip.

d) Measure the value of two 1kΩ resistors with tolerances of 10%.

a) $S = \{1, 2, \ldots, 10\}$.

b) $S = \{(2, \heartsuit), (3, \spadesuit), (4, \diamondsuit), (3, \clubsuit)\}$

c) $S = \{x \in \mathbb{R} \mid x \geq 0\}$

d) $S = \{(x, y) \in \mathbb{R}^2 \mid 900 \leq x \leq 1100, 900 \leq y \leq 1100\}$

Problem 2

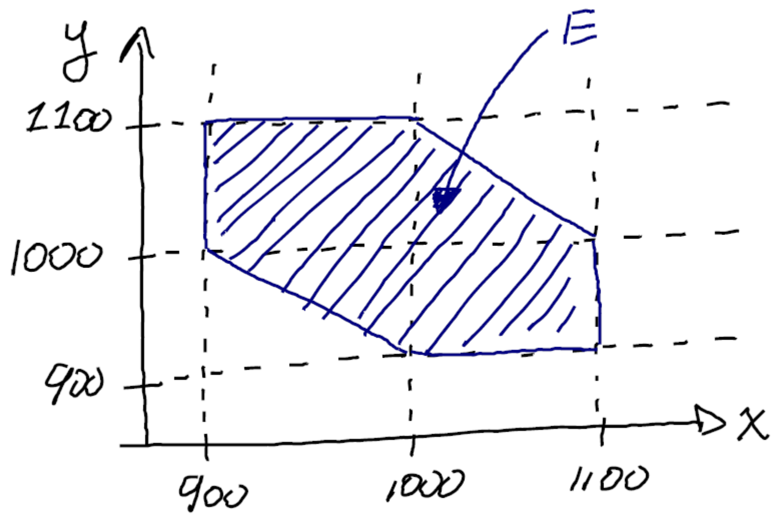
- In problem 1, sketch the event that the sum of the two resistors is within 5% of 2kΩ.

Problem 2 - Solution

- In problem 1, sketch the event that the sum of the two resistors is within 5% of 2kΩ.

$$S = \{(x,y) \in \mathbb{R}^2 \mid 900 \leq x \leq 1100, 900 \leq y \leq 1100\}$$

$$E = \{(x,y) \in S \mid 1900 \leq x + y \leq 2100\}$$



Problem 3

- Let A, B, and C be three events. Show that $P[AB^cC^c] \geq P[A] - P[AB] - P[AC]$.

Problem 3 - Solution

- Let A, B, and C be three events. Show that $P[AB^cC^c] \geq P[A] - P[AB] - P[AC]$.

$$
\begin{aligned}
A &= A(B \cup B^c)(C \cup C^c) \\
&= ABC \cup AB^cC \cup ABC^c \cup AB^cC^c
\end{aligned}
$$

and each of these four sets are disjoint. So

$$P[A] = P[ABC] + P[AB^cC] + P[ABC^c] + P[AB^cC^c]$$

and hence

$$
\begin{aligned}
P[AB^cC^c] &= P[A] - P[ABC] - P[AB^cC] - P[ABC^c] \\
&= P[A] - (P[ABC] + P[AB^cC]) - P[ABC^c] \\
&= P[A] - P[ABC \cup AB^cC] - P[ABC^c] \\
&= P[A] - P[AC] - P[ABC^c] \\
&\geq P[A] - P[AC] - P[AB]
\end{aligned}
$$

where we used the fact that $P[AB] \geq P[ABC^c]$.

Problem 4

- An urn has 10 balls numbered from 1 to 10. If we draw 3 balls at random. what is the probability that one of them is even and the other two are odd?

Problem 4 - Solution

- An urn has 10 balls numbered from 1 to 10. If we draw 3 balls at random. what is the probability that one of them is even and the other two are odd?

$$|S| = \binom{10}{3} = 120$$

$$|E| = \binom{5}{2}\binom{5}{1} = 50$$

and therefore $P[E] = 5/12$

- Question: Does order matter or not?

Problem 5

- Show that $P[A \cup B \cup C] = P[A] + P[A^c B] + P[A^c B^c C]$

Problem 5 - Solution

- Show that $P[A \cup B \cup C] = P[A] + P[A^c B] + P[A^c B^c C]$

We first show that $P[A \cup B] = P[A] + P[A^c B]$. Now

$$
\begin{aligned}
A \cup B &= A \cup (\ SB\ ) \\
&= A \cup (\ (A \cup A^c)B\ ) \\
&= A \cup (\ AB \cup A^c B\ ) \\
&= (\ A \cup AB\ ) \cup A^c B \\
&= A \cup A^c B
\end{aligned}
$$

Since A and $A^c B$ are disjoint, then

$$P[A \cup B] = P[A \cup A^c B] = P[A] + P[A^c B] \qquad (*)$$

Problem 5 - Solution

- Show that $P[A \cup B \cup C] = P[A] + P[A^c B] + P[A^c B^c C]$

To get the desired result, we apply $(*)$ twice:

$$
\begin{aligned}
P[A \cup B \cup C] &= P[A \cup (B \cup C)] \\
&= P[A] + P[A^c(B \cup C)] \\
&= P[A] + P[A^c B \cup A^c C] \\
&= P[A] + P[A^c B] + P[(A^c B)^c A^c C] \\
&= P[A] + P[A^c B] + P[(A \cup B^c) A^c C] \\
&= P[A] + P[A^c B] + P[A A^c C \cup A^c B^c C] \\
&= P[A] + P[A^c B] + P[\emptyset \cup A^c B^c C] \\
&= P[A] + P[A^c B] + P[A^c B^c C]
\end{aligned}
$$