ECE 203
Probability Theory and Statistics I
Tutorial 2

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UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

Problem 1

- Two fair four-sided dice are rolled. What is the probability that both dice roll the same number, given that the sum is 3 or less?

Problem 1 - Solution

- Two fair four-sided dice are rolled. What is the probability that both dice roll the same number, given that the sum is 3 or less?

$$S = \{(a, b) \in \mathbb{N}^2 \mid a \in \{1, \ldots, 4\}, b \in \{1, \ldots, 4\}\}$$
$$A = \{(1,1), (2,2), (3,3), (4,4)\}$$
$$B = \{(1,1), (1,2), (2,1)\}$$

So

$$P[A|B] = P[AB]/P[B] = P[(1,1)]/P[B] = \frac{\frac{1}{n}}{\frac{3}{n}} = \frac{1}{3}$$

Problem 2

- A pouch contains a four-sided die and a six-sided die. You pick one of the die at random and roll it.

a) What is the probability of rolling i, for i = 1,2, …,6?

b) If you roll a 3, what is the probability that you picked the 6-sided die?

Problem 2 - Solution

- A pouch contains a four-sided die and a six-sided die. You pick one of the die at random and roll it.

a) What is the probability of rolling i, for i = 1,2, …,6?

$A = \{\text{4-sided die picked}\}$

$B = \{\text{6-sided die picked}\} = A^c$

$E_i = \{\text{roll is } i\}.$

By law of total probability,

$$P[E_i] = P[E_i|A]P[A] + P[E_i|A^c]P[A^c]$$
$$= P[E_i|A]P[A] + P[E_i|B]P[B]$$

$$P[E_1] = 1/4 \times 1/2 + 1/6 \times 1/2 = 5/24$$
$$P[E_2] = 5/24$$
$$P[E_3] = 5/24$$
$$P[E_4] = 5/24$$
$$P[E_5] = 0 \times 1/2 + 1/6 \times 1/2 = 2/24$$
$$P[E_6] = 2/24$$

Problem 2 - Solution

- A pouch contains a four-sided die and a six-sided die. You pick one of the die at random and roll it.

b) If you roll a 3, what is the probability that you picked the 6-sided die?

$$A = \{\text{4-sided die picked}\}$$

$$B = \{\text{6-sided die picked}\} = A^c$$

$$E_i = \{\text{roll is } i\}.$$

$$\begin{aligned} P[B|E_3] &= \frac{P[E_3 B]}{P[E_3]} \\ &= \frac{P[E_3|B]P[B]}{P[E_3|B]P[B] + P[E_3|A]P[A]} \\ &= \frac{1/6 \times 1/2}{1/6 \times 1/2 + 1/4 \times 1/2} \\ &= \frac{2}{5} \end{aligned}$$

Problem 3

- Let V be the event that a random person is vaccinated against a disease, and D be the event that the random person develops the disease. The effectiveness, e, of a vaccine is defined to be

$$e = \frac{P[D|V^c] - P[D|V]}{P[D|V^c]} = 1 - \frac{P[D|V]}{P[D|V^c]}$$

Say 80% of the population is vaccinated, and 40% of all cases of the disease occur in people who are vaccinated. What is e?

Problem 3 - Solution

$$\mathrm{e} = 1 - \frac{P(D|V)}{P(D|V^C)} = 1 - \frac{\frac{P(DV)}{P(V)}}{\frac{P(DV^C)}{\mathrm{P}(V^C)}} = 1 - \frac{\mathrm{P(DV)}}{\mathrm{P(DV^C)}} \frac{P(V^C)}{P(V)}$$

$$= 1 - \frac{P(D)P(V|D)}{P(D)P(VC|D)} \frac{\mathrm{P(VC)}}{P(V)} = 1 - \frac{\mathrm{P(V|D)}}{\mathrm{P(V^C|D)}} \frac{\mathrm{P(V^C)}}{\mathrm{P(V)}}$$

$$= 1 - \frac{0.4}{0.6} \frac{0.2}{0.8} = 1 - \frac{8}{48} = \frac{5}{6}$$

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