Chapter 5

Fundamentals of Cellular Communications

And Multiple Access Techniques
Outline

- Cellular concept
- Frequency reuse factor
- Co-channel and adjacent channel interference
- Trunking and Grade of Service (GoS)
- Multiple Access Techniques for the Uplink:
  - FDMA
  - TDMA
  - CDMA
Cellular Concept

- In cellular systems, the total service area is divided into a number of smaller areas, each of which is defined as “cell”.

- Regular polygons may be used to represent the cell coverage. Hexagonal cells are popular because
  - their shapes are closest to circle
  - allows tight cellular packing and perfect partitioning of the service area

- In cellular systems, “frequency re-use” allows efficient use of scarce frequency spectrum.

- Cells which use the same frequency are called “co-channel cells” (e.g., the cells labeled as A). Frequency re-use is limited by co-channel interference.
Cellular Concept (cont’d)

$N$: Number of cells in each “cluster”  
(also known as “cluster size” or  
“frequency re-use factor”)

$J$: Number of channels in each cell

$K$: Number of channels within one cluster  
(=Total number of frequency carriers  
assigned to the system)

$\rightarrow K=J\times N$

$M$: Number of cell clusters in the system

$C$: Total number of channels in the system (also known as “system capacity”)

$\rightarrow C=M\times K$

➤ When the frequency reuse factor decreases (i.e., repeating the same  
frequencies more often), the system capacity increases. However, there is a  
minimum value of $N$ that depends on the co-channel interference level.
Frequency Reuse Factor

Consider hexagonal cells

- A hexagonal cell has exactly 6 equidistant “nearest” co-channel neighbours
- The lines joining the centers of any cell and each of its neighbours are separated by multiples of 60 degrees
- To find the nearest co-channel neighbours, follow two steps:
  - Move over $i$ cells along any chain of hexagons
  - Turn 60 degrees counter-clockwise and move over $j$ cells

where $i$ and $j$ are system parameters for determining cluster size

$$N = i^2 + ij + j^2$$
Frequency Reuse Factor (cont’d)

(a) $i = 2$ and $j = 0$

(b) $i = 1$ and $j = 2$

(c) $i = 2$ and $j = 2$

$N=4$

$N=7$

$N=12$
Frequency Reuse Factor (cont’d)

<table>
<thead>
<tr>
<th>$i$</th>
<th>$j$</th>
<th>$N = i^2 + ij + j^2$</th>
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Frequency Reuse Factor (cont’d)

$R$: Radius of the cell (from center to vertex)

$D_1$: Distance between the centers of two “adjacent” neighbour cells

$$D_1 = 2R \cos 30 = 2R \sqrt{3}/2 = \sqrt{3}R$$

$D$: Distance between the centers of two “co-channel” neighbour cells

$$D^2 = (jD_1 \cos \theta)^2 + (iD_1 + jD_1 \sin \theta)^2$$

$$= (i^2 + ij + j^2)D_1^2$$

$$= 3NR^2$$

$$\Rightarrow D = \sqrt{3NR}$$

$q$: Frequency reuse ratio

$$q \overset{\text{def}}{=} \frac{D}{R} = \sqrt{3}N$$
Frequency Reuse Factor (cont’d)

$q \downarrow \Rightarrow$ Same channels are used more frequently. System capacity increases.

- Minimum value of $q$ depends on the co-channel interference level. Defining “signal-to-co-channel interference ratio” as

\[
S/I = S/\sum_{k=1}^{N} I_k
\]

- We need to determine the power for each interference term. Consider only large scale path-loss effect (See Chapter 2)

\[
L(d) = L(d_0) \left( \frac{d}{d_0} \right)^{-n} \quad d_0 : \text{Reference distance}
\]

\[
n : \text{Path loss exponent}
\]

- Consider the downlink (i.e. forward link) and assume that the transmitted power from all the BSs are the same, then

\[
I_k \propto D_k^{-n} \quad D_k : \text{Distance from the } k^{th} \text{ co-channel cell BS to the MS.}
\]
Frequency Reuse Factor (cont’d)

Assume the MS is located at the cell boundary (i.e. the worst case scenario) and use the approximation, $D_k \approx D$ i.e. $D_k \gg R$

$$\frac{S}{I} = \frac{S}{N_I \sum_{k=1}^{N_I} I_k} = \frac{R^{-n}}{N_I} = \frac{(D/R)^n}{N_I} = \frac{q^n}{N_I} = \frac{(\sqrt{3N})^n}{N_I} \quad \Rightarrow \quad q = \left( N_I \cdot \frac{S}{I} \right)^{1/n}$$

Example: Assume that the interference comes only from the first-tier co-channel cells. Consider a path-loss model with path loss exponent $n=4$ and calculate the frequency reuse factor to achieve a minimum acceptable level of $S/I=20\text{dB}$.

In this problem, we neglect interference terms from second and higher order tiers, therefore $N_I=6$.

$$q = (6 \cdot 100)^{1/4} = 4.9492 \quad N = q^2 / 3 = 8.165 \quad \rightarrow \quad 9$$

Verify your choice

$$\frac{S}{I} = \frac{\left( \sqrt{3N} \right)^n}{N_I} \bigg|_{N=9} = 20.84\text{dB}$$
Consider a better approximation rather than the crude assumption of $D_k \approx D$

$$\frac{S}{I} \approx \frac{R^{-n}}{2(D - R)^{-n} + 2D^{-n} + 2(D + R)^{-n}}$$

$$= \frac{1}{2(q - 1)^{-n} + 2q^{-n} + 2(q + 1)^{-n}}$$

- $n=4, N=7$ \hspace{0.5cm} $q = \sqrt{3N} = 4.6 \Rightarrow S/I = 17.3\text{dB}$
- $n=4, N=9$ \hspace{0.5cm} $q = \sqrt{3N} = 5.2 \Rightarrow S/I = 19.8\text{dB}$
Adjacent Channel Interference

- Adjacent channel interference results from signals which are adjacent in frequency to the desired signal. To reduce this effect,

  - Use proper channel interleaving by assigning adjacent channels to different cells. For example if you assign the frequency carrier $f_1$ to cell A, the next available carrier frequency (which is adjacent to $f_1$ in frequency spectrum) should be assigned to another cell instead of A.

  - Use modulation schemes which have small out-of-band radiation (i.e. sidelobes are smaller). For example, MSK is better than QPSK, see Chapter 3

  - Carefully design the receiver front end filter
Trunking

- In cellular systems, a relatively small number of radio channels are used to serve a large population of mobile users, which is made possible by cellular design (i.e., frequency reuse) and by trunking. We have already studied frequency reuse concept. Now, we will briefly summarize some basic ideas behind trunking:

- Trunking allows the mobile users share the radio channels in each cell on a demand basis.

- Based on a traffic load, the number of radio channels in each cell should be determined in such a way that
  - All the channels are utilized efficiently,
  - Call blocking rate is below a pre-determined threshold.

- The measure of traffic efficiency: 1 Erlang is defined as the amount of traffic intensity carried by a channel that is completely occupied, e.g., a radio channel that is occupied for 30 minutes during an hour carries 0.5 Erlangs of traffic per hour.
Grade of Service (GoS)

- **Grade of service (GoS)** is a measure of the ability of a user to access a trunked system during the busiest hour. GoS is typically given as
  - the likelihood that a call is blocked (for *Erlang B* systems) or
  - the likelihood that a call experiences a delay larger than a certain pre-determined system queueing delay (for *Erlang C* systems)

**Basic definitions**

- **Blocked call (Lost call):** Call which can not be completed at the time of request, due to congestion
- **Average holding time** ($H$): Average duration of a typical call
- **Traffic intensity** ($A$): Measure of channel time utilization, which is the average channel occupancy measured in Erlangs
- **Load:** Traffic intensity across the entire trunked radio system, measured in Erlangs
- **Request rate** ($\lambda$): The average number of call requests per unit time per user
The traffic intensity offered by each user is (in Erlangs)

\[ A_u = \lambda \cdot H \]

For a system with \( u \) users, the total offered traffic intensity (traffic load) is (in Erlangs)

\[ A = u \cdot A_u = u \cdot \lambda \cdot H \]

In a \( C \)-channel trunked system, assuming the traffic is equally distributed among the channels, then the traffic intensity per channel is

\[ A_C = u \cdot A_u / C = u \cdot \lambda \cdot H / C \]

Difference between “offered” traffic and “carried” traffic
Types of Trunked Systems

- If no channels are available,
  - The requesting user is blocked without access, the call request is cleared and the user is free to try again later → **Blocked Calls Cleared**
  - The call request is delayed until a channel becomes available → **Blocked Calls Delayed**

Assume that call arrivals $X$ follow a *Poisson distribution*.

$$P(X = i) = e^{-\lambda} \frac{\lambda^i}{i!}, i = 0, 1, 2...$$  \hspace{1cm} \lambda : \text{Request rate}$$

$$E[X] = \lambda \quad \text{Var}[X] = \lambda$$

Assume that holding time $Y$ follows an exponential distribution.

$$f_Y(y) = \begin{cases} 
\alpha e^{-\alpha y}, & y \geq 0 \\
0, & y < 0 
\end{cases}$$  \hspace{1cm} \alpha : \text{exponent parameter}$$

$$E[Y] = H = 1/\alpha \quad \text{Var}[Y] = 1/\alpha^2$$
Blocked Calls Cleared (Erlang-B Formula)

- Under the following assumptions:
  - There are an infinite number of users
  - There are “memoryless” arrivals of calls, implying that all users, including blocked users, may request a channel at any time
  - There are a finite number of channels available in the trunked channels’ pool

- The call blocking probability is given by the so-called Erlangs-B formula

\[ P_B = \frac{A^C / C!}{\sum_{k=0}^{C} A^k / k!} \]

\( A \): Total offered traffic intensity
\( C \): Total number of channels in the system

- The assumption that there are an infinite number of users in the system results in a pessimistic estimate of the GoS. The blocking probability with a finite number of users is smaller than that obtained by the Erlang-B formula.
Blocked Calls Delayed (Erlang-C Formula)

- Under the same assumptions in our previous model, we use the *Erlang-C* formula to find the probability of a call not having immediate access to a channel

\[ P(\text{Delay} > 0) = \frac{A^C}{A^C + C!(1 - \frac{A}{C})^{C-1} \sum_{k=0}^{C-1} \left( \frac{A^k}{k!} \right)} \]

- If no channels are immediately available the call is delayed and the probability that the delayed call is forced to wait for more than \( t \) seconds

\[ P(\text{Delay} > t) = P(\text{Delay} > 0)P(\text{Delay} > t|\text{Delay} > 0) = P(\text{Delay} > 0)\exp[-(C - A)t/H] \]
**Example:** Consider a cellular system with 416 radio channels. Suppose 21 of these channels are designated as control channels. Let the average channel holding time of a call be 3 minutes, the call blocking probability be 2% and the frequency reuse factor be 9. Determine the number of calls per cell per hour.

The number of voice channels is \(416 - 21 = 395\).

The number of voice channels per cell is

\[
\frac{395}{N} \bigg|_{N=9} \approx 44
\]

With 44 available channels and \(P_B = 0.02\), the traffic intensity is 34.683 Erlangs. See Erlang-B table in Appendix F.

\[
\frac{34.683}{3} \approx 12 \text{ calls per cell per minute}
\]

\[
\frac{34.683}{3} \times 60 \approx 693 \text{ calls per cell per hour}
\]
Multiple Access in a Radio Cell

- A radio cell is the geographical coverage area in which the services of mobile stations (MSs) are supported by a single base station (BS).
- In the forward link (downlink): BS transmits to multiple MSs, i.e. one-to-many transmission. This mode of transmission is referred to as “broadcasting”.
- In the reverse link (uplink): Multiple MSs transmit to the BS, i.e. many-to-one transmission. This mode of transmission is referred to as “multiple access”.
- If two or more user signals arrive at the BS at the same time, there will be interference, unless the signals are orthogonal.
- $x_i(t)$ and $x_j(t)$, $t \in T$, are said to be orthogonal if their inner product is zero, i.e.
  \[ \int_{0}^{T} x_i(t)x_j(t)dt = 0, \text{ for } i \neq j \]
- The question is how to maintain orthogonality among the transmitted signals from different users: FDMA, TDMA and CDMA
Frequency Division Multiple Access (FDMA)

- The total system bandwidth is divided into non-overlapping frequency subbands, i.e. channels.

- Orthogonality among transmitted signals from different mobile users is achieved by bandpass filtering. (This brings strict requirements on RF filters)

- Each user occupies a channel for the duration of the connection. (Not an efficient use of resources particularly for multimedia transmission with various transmission rates)

- Frequency division duplexing (FDD) is used to prevent interference between uplink and downlink channels. There should be enough separation for isolation among the corresponding uplink/downlink channels.
Time Division Multiple Access (TDMA)

- The channel time is partitioned into “frames”, which is partitioned into “slots”. Each frame contains data slots plus additional slots for preamble and trailer.

TDMA frame structure

- Users have to transmit in their assigned slots from frame to frame. The slot assignment can be “fixed” or “dynamic”. Based on the type of slot assignment, TDMA is classified as “synchronous” or “asynchronous”.

  - In Synchronous TDMA (STDMA), a user is assigned a fixed time slot for the duration of its connection (whether it is active or not). The frame length is fixed by the number of users.

  - In Asynchronous TDMA (ATDMA), a user is assigned a time slot when it has a packet to send. The frame length varies from frame to frame, depending on the number of active users in the frame.
TDMA (cont’d)

- Another TDMA classification depends on how the total bandwidth is assigned:
  - In Wideband TDMA, transmission in each slot uses the entire bandwidth.
  - In Narrowband TDMA, transmission in each slot uses only a subband since the whole frequency band is divided into subbands. This can be regarded as a FDMA/TDMA system.

- TDMA can use time division duplex (TDD) or frequency division duplex (FDD). FDD provides two simplex channels at the same time while TDD provides two simplex time slots on the same frequency band.
Code Division Multiple Access (CDMA)

- CDMA is a spread spectrum multiple access technique.
- In spread system communications, the spectrum of the baseband message signal is spread by a significant order of magnitude larger than the minimum required bandwidth. Originally proposed and used for military communications.
- Direct sequence (DS) CDMA: The spread spectrum is achieved by directly multiplying the baseband signal with a high-rate pseudorandom noise (PN) sequence, called the “spreading sequence”.

- For simplicity, we will assume BPSK in the following.
CDMA (cont’d)

The information-carrying baseband signal for the $k^{th}$ user is

$$d_k(t) = A \sum_i s_{k,i} P_{T_s} \left( \frac{t - iT_b}{T_b} \right)$$

$s_{k,i} \in \{-1, +1\}$: $i^{th}$ information bit
$T_b$: Information bit interval
$P_{T_s}(\cdot)$: Pulse shape

The spreading signal of the $k^{th}$ user is

$$a_k(t) = \sum_l a_{k,l} P_{T_c} \left( t - lT_c \right)$$

$a_{k,l} \in \{-1, +1\}$: $l^{th}$ “chip” of the PN sequence
$T_c$: Chip interval
$T_b/T_c = L >> 1$
$P_{T_c}(\cdot)$: Chip pulse shape

Assuming rectangular pulse shaping, the PSD of transmitted signal before/after spreading
Assuming that all the $K$ users in the cell are synchronized in time and have the same received signal power (achieved through power control in practice), the received signal is

\[ r(t) = \sum_{k=1}^{K} x_k(t) + I(t) + w(t) \]

\[ = Aa_1(t)d_1(t)\cos(2\pi f_c t) + \sum_{k=2}^{K} Aa_k(t)d_k(t)\cos(2\pi f_c t) + I(t) + w(t) \]

- Intended user, $k=1$
- Intracell interference
- Intercell
CDMA (cont’ d)

The output of the demodulator is

\[ y(t) = \frac{1}{T_c} \int_{lT_c}^{(l+1)T_c} r(t) \cos(2\pi f_c t) dt \]

Noting \( a_k(t)d_k(t) \) is a constant over each chip interval

\[ y(t) = \frac{A}{2} a_1(t)d_1(t) + \sum_{k=2}^{K} \frac{A}{2} a_k(t)d_k(t) + n(t) \]

where \( n(t) \) is due to intercell interference and additive background noise.

\[ n(t) = \frac{1}{T_c} \sum_{l} \left\{ \int_{lT_c}^{(l+1)T_c} [I(t) + w(t)] \cos(2\pi f_c t) dt \right\} \prod \left( \frac{t-lT_c}{T_c} \right) \]
CDMA (cont’ d)

- Despreading is achieved by first multiplying the demodulator output with the spreading sequence of the intended user and then integrate over bit interval

\[
\frac{1}{T_b} \int_{iT_b}^{(i+1)T_b} a_1(t)y(t)dt = \frac{AT_b}{2} \alpha_1 d_{1,i} + \frac{AT_b}{2} \sum_{k=2}^{K} \alpha_k d_{k,i} + n_i \\

n_i = \frac{1}{T_b} \int_{iT_b}^{(i+1)T_b} a_1(t)n(t)dt
\]

where \( \alpha_1 \) and \( \alpha_k, k = 2,3...K \) denote autocorrelation and crosscorrelation

\[
\alpha_1 = \frac{1}{T_b} \int_{iT_b}^{(i+1)T_b} [a_1(t)]^2 dt = 1 \\
\alpha_k = \frac{1}{T_b} \int_{iT_b}^{(i+1)T_b} a_1(t)a_k(t)dt, \ k = 2,3...K
\]

- If all the spreading signals \( a_k(t) \) are orthogonal, there will be no intracell interference in the recovered baseband signal. If they are not orthogonal, they will create “multiple access interference (MAI)” which can be approximated as Gaussian for large number of users.

- For BPSK, the error rate performance is given as \( P_b = Q\left(\sqrt{2E_b/(I_0 + N_0)}\right) \)
where \( I_0 \) is the two-sided PSD of MAI.
Advantages of CDMA

- **Universal frequency use:** The total frequency bandwidth can be reused from cell to cell, achieving minimum cell cluster size (=1) and maximum frequency reuse.

- **Soft capacity:** The maximum number of users that can be supported in each cell depends on the required quality of service (QoS) and is limited by MAI. As a result, unlike TDMA and FDMA, there is no hard limit on the number of users in each cell.

- **Flexibility:** If a user does not transmit, it does not generate any interference with other active users and, therefore, does not use the system resources. This feature translates to a high resource utilization via statistical multiplexing for “on-off voice traffic” and “bursty data traffic”. It also provides flexibility in supporting multimedia services which require various time-varying traffic rates.
Disadvantages of CDMA

- **Near-far problem:** Even if the different users transmit at the same power level, the power levels received from different users at the BS will differ due to the different location (distance) of users within the cell. This phenomenon is called “near-far” problem and can be avoided through “power control” which is implemented to keep the same received signal powers from different users, independent of their location.

- **Implementation complexity:** CDMA systems operate at a high chip rate and require accurate PN synchronization. The complexity of transmitter/receiver is typically higher than that of TDMA and FDMA systems.