Optimal Power Control in Interference-Limited Fading Wireless Channels With Outage-Probability Specifications

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***Abstract*—We propose a new method of power control for interference-limited wireless networks with Rayleigh fading of both the desired and interference signals. Our method explictly takes into account the statistical variation of both the received signal and interference power and optimally allocates power subject to constraints on the probability of fading induced outage for each transmitter/receiver pair. We establish several results for this type of problem. We establish tight bounds that relate the outage probability caused by channel fading to the signal-to-interference margin calculated when the statistical variation of the signal and intereference powers is ignored. This allows us to show that well-known methods for allocating power, based on Perron–Frobenius eigenvalue theory, can be used to determine power allocations that are provably close to achieving optimal (i.e., minimal) outage probability. We show that the problems of minimizing transmitter power subject to constraints on outage probability and minimizing outage probability subject to power constraints can be posed as a geometric program (GP). A GP is a special type of optimization problem that can be transformed to a nonlinear convex optimization problem by a change of variables and therefore solved globally and efficiently by recently developed interior-point methods. We also give a fast iterative method for finding the optimal power allocation to minimize outage probability.**

***Index Terms*—Fading channels, personal communication networks, power control, radio communication.**

I. Introduction

WISE allocation of power is critical in wireless networks for both longer battery life of the mobile devices and for increased utilization of the limited wireless spectrum. Power control provides an intelligent way of determining transmitting power to achieve the quality of service (QoS) goals in wireless channels. Because of these benefits, it has been very well studied [1]–[8]. Traditional power-control schemes whether centralized [9]–[12] or distributed [8], [13], [14] always assume quasi-stationarity of the fading wireless channels and base their power-control schemes on the observed signal-to-interference ratio (SIR) at the receiver or the knowledge of the gains of all the links. Thus, the implicit assumption made is that the power-control updates are made every time the fading state of the channel changes, i.e., whenever the gain of any link changes. In wireless communication channels, which exhibit fast fading where the fades can change within milliseconds (at 900 MHz and mobile traveling at 60 mph), this might not always be practical. Very frequent power updates can also consume a lot of signal processing energy.

In this paper, we propose a power-control scheme in which the power does not need to be updated whenever the channel meanders from one fading state to another. Instead, we explicitly take into account the statistical variation of the SIR of each transmitter/receiver pair and optimally allocate power to minimize probability of fading-induced outage (which occurs when the SIR falls below a threshold $\mathrm{SIR}^{\mathrm{th}}$). Thus, in this paper, we assume that between successive power-control updates, outage occurs because of fast fading (of both signal and interference) and that factors such as log-normal shadowing and distance dependent attenuation remain constant. The outage caused variations in received SIR can be handled by signal processing and coding.

We find a global solution to this minimum outage-probability problem by showing that it can be posed as a nonlinear convex optimization problem. Solution methods for these problems not only produce the global optimum (efficiently), but also unambiguously determine feasibility. This enables us to make QoS guarantees or determine beforehand whether the services requested by the mobile user can be provided or not. Most importantly, our power-control analysis allows power updates to be carried out at a time scale far larger than the Rayleigh fading time scale, which is often the log-normal shadowing time scale. This is a significant contrast[1] compared to the research in the literature [1]–[14], [17].

Clearly, the probability of outage can be reduced by allocating power in such a way that each mobile has an extra margin of SIR, i.e., its SIR is somewhat above the minimum $\mathrm{SIR}^{\mathrm{th}}$ value required for reception. Increasing the margin of SIR reduces the probability of outage, but costs extra power. Our method can be interpreted as an intelligent way to carry out this *ad hoc* method of giving SIR margins to the mobiles. Our method gives each mobile a margin of SIR that is directly based on the required probability of fading-induced outage.

Maximizing the minimum SIR present in the system, an approach known as SIR balancing, is a well-studied topic

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[1]It should be noted that we are referring to *closed-loop power control*, which tracks Rayleigh fading and power control in *ad hoc* networks, which assume quasi-stationarity rather than *open-loop power control*, which is done at the log-normal shadowing time scale [15], [16].

[10]–[12], [17]. This paper derives the relationship between SIR margin (for a power-control scheme designed for quasi-stationary conditions) and the outage probability (for the same power-control scheme operating in fading conditions). We obtain upper and lower bounds on the outage probability for a given SIR margin and show that these bounds are very tight for high SIR margins. We also get analytical results on the outage probability achieved by a power-control scheme designed to maximize SIR margin. While the SIR margin maximizing power-allocation method is very well studied [6], [7], [12], [13], [17], there is no literature that addresses power-allocation methods to minimize outage probability. In this paper, we propose two methods that minimize outage probability: one that finds the global optimum, even if other constraints are included in the problem, and also a fast heuristic based on Perron–Frobenius theory and existing SIR balancing methods.

Using the above results, we compute the outage probability attained by the SIR margin maximizing power-allocation method and find that it is very close to the optimum outage probability. The practical implication of this fact is very important. The power updates can still be done using the efficient SIR margin maximizing power-allocation method, but now the updates can be done at the log-normal shadowing time scale rather than every time the channel changes.

We briefly summarize the contributions and findings of the paper below.

1) This paper offers a unified approach for analyzing power control in interference-limited fading wireless networks by analytically studying the relationship between outage probability and SIR margin.
2) The analytical relationship between the outage probability and SIR margin is used to give bounds on how suboptimal the SIR balancing power allocation is for the problem of achieving minimum outage probability.
3) The problem of power allocation that minimizes outage probability is shown to be a convex problem that can be solved globally and efficiently, even when other constraints (such as on individual powers, total power, etc.) are included in the power-allocation problem. We also give a fast heuristic method for solving the basic outage-probability minimization problem, which is based on Perron–Frobenius methods.
4) The outage probability obtained by the SIR margin maximizing power-allocation method is shown to be very close to the optimal outage probability.

The paper is organized as follows. In Section II, we describe the system and fading model. In Section III, we derive an expression for the probability that a mobile experiences fading-induced outage and also some tight bounds that relate the probability of outage to a margin of SIR, ignoring statistical variation of the interference and signal powers. In Section IV, we formulate different power-allocation problems to minimize outage probability, minimize transmit powers, maximize SIR margin with different constraints, and give different methods for solving them. In Section V, we give a simple illustrative example.

II. Rayleigh–Rayleigh Fading Environment

We consider the following setup. We have n transmitters, labeled $1, \ldots, n$, which transmit at power level $\mathrm{P}_1, \ldots, \mathrm{P}_n$, which are the variables in our optimization problem. We also have n receivers, labeled $1, \ldots, n$; receiver i is meant to receive the signal from transmitter i. (By transmitter and receiver, we do not necessarily mean different physical transmitters and receivers; different receivers, for example, might refer to the same physical receiver with different frequency channels, codes, or antenna beams in an antenna array.) The power received from transmitter j at receiver i, is given by

$$\mathrm{G}_{ij}\mathrm{F}_{ij}\mathrm{P}_j. \tag{1}$$

The number G_{ij}, which is positive, represents the path gain (not including fading) from the jth transmitter to the ith receiver. This gain term can be interpreted in many ways: it can represent distance dependent power attenuation, log-normal shadowing, cross correlations between codes in a code division multiple access (CDMA) system, as well as gain dependency on antenna direction. In the analysis below, we assume that G_{ij} are *constant*, i.e., do not change (much) with time. Therefore, the analysis holds for a time scale over which the factors that determine G_{ij} are approximately constant: the distance between transmitters and receivers does not change much, the log-normal shadowing does not change much, direction dependent antenna gains do not change much.

The numbers F_{ij} model *Rayleigh fading*. They are assumed to be independent exponentially distributed random variables with unit mean. (In a Rayleigh fading environment, the received signal envelope has a Rayleigh distribution; the received signal power has an exponential distribution [19].) In other words, the power received at receiver i from transmitter j is an exponentially distributed random variable with mean value

$$\mathrm{E}[\mathrm{G}_{ij}\mathrm{F}_{ij}\mathrm{P}_j] = \mathrm{G}_{ij}\mathrm{P}_j.$$

We refer to this situation in which both desired signals and interference signals are subject to Rayleigh fading, as a Rayleigh/Rayleigh fading environment. The assumption behind the Rayleigh/Rayleigh fading environment is that the receiver gets no direct line-of-sight signal component, either from its own transmitter or from the interfering transmitters.

We will also assume that the interference from other transmitters is much larger than the white noise in the receivers and, therefore, ignore receiver noise in our analysis. Both the Rayleigh/Rayleigh fading environment and this assumption of interference-limited communication are very realistic in urban wireless networking environments. Sometimes, the noise component in the SIR may arise not from just receiver white noise, but also from cochannel users that are not included in the power-control problem formulation. This may mean that the noise is not negligible. One possible way of accounting for this fact is to assume that there are more than n transmitters and that the $(n+1)$th transmitter transmits power P_{n+1}.

III. Outage Probability and Certainty-Equivalent Margin

A. SIR and Outage Probability

The signal power at the ith receiver is given by $\mathrm{G}_{ii}\mathrm{F}_{ii}\mathrm{P}_i$ and the total interference power is given by

$$\sum_{k\neq i} \mathrm{G}_{ik}\mathrm{F}_{ik}\mathrm{P}_k.$$

The SIR of the ith receiver (or transmitter) is given by

$$\mathrm{SIR}_i = \frac{\mathrm{G}_{ii}\mathrm{F}_{ii}\mathrm{P}_i}{\sum_{k\neq i} \mathrm{G}_{ik}\mathrm{F}_{ik}\mathrm{P}_k}.$$

Note that in a Rayleigh/Rayleigh fading environment, SIR_i is a random variable with what would appear to be a very complex distribution, since it is the ratio of an exponential random variable to a sum of exponential random variables (with different means). (We will see later, however, that there is an analytical expression for its density.)

We assume that the QoS requested is provided when the SIR exceeds a given threshold $\mathrm{SIR}^{\mathrm{th}}$. The *outage probability* of the ith receiver/transmitter pair is given by

$$\begin{aligned}\mathrm{O}_i =& \mathrm{Prob}\left(\mathrm{SIR}_i \leq \mathrm{SIR}^{\mathrm{th}}\right) \\ =& \mathrm{Prob}\left(\mathrm{G}_{ii}\mathrm{F}_{ii}\mathrm{P}_i \leq \mathrm{SIR}^{\mathrm{th}} \sum_{k\neq i} \mathrm{G}_{ik}\mathrm{F}_{ik}\mathrm{P}_k\right). \end{aligned} \quad (2)$$

The outage probability O_i can be interpreted as the fraction of time the ith transmitter/receiver pair experiences an outage due to fading. Note that in our expression for O_i, we take into account statistical variation of both received signal power and received interference power.

Surprisingly, the outage probability can be expressed in analytical form; it was derived in [18] (see also [19] and [22]), although we will use an equivalent form that has not appeared in the literature, as far as we know. The analytic expression for O_i is derived from the following result. Suppose $z_1, \ldots, z_n$ are independent exponentially distributed random variables with means $\mathrm{E}z_i = 1/\lambda_i$. Then, we have

$$\mathrm{Prob}\left(z_1 \leq \sum_{i=2}^n z_i\right) = 1 - \prod_{i=2}^n \left(\frac{1}{1+\frac{\lambda_1}{\lambda_i}}\right).$$

We give a self-contained derivation of this result in Appendix I.

We can include the effect of an additive white Gaussian noise (AWGN) in the receivers, which introduces a constant c in the interference term, so the expression above becomes

$$\mathrm{Prob}\left(z_1 > \sum_{i=2}^n z_i + c\right) = e^{-\lambda_1 c} \prod_{i=2}^n \left(\frac{1}{1+\frac{\lambda_1}{\lambda_i}}\right).$$

This greatly complicates the analysis and resulting optimization problems, so we assume in the sequel that AWGN is not present or, more accurately, insignificant compared to the interference powers.

Applying the result above to (2), we find that the outage probability for the ith transmitter/receiver pair can be expressed as

$$\mathrm{O}_i = 1 - \prod_{k\neq i} \frac{1}{1+\frac{\mathrm{SIR}^{\mathrm{th}}\mathrm{G}_{ik}\mathrm{P}_k}{\mathrm{G}_{ii}\mathrm{P}_i}}. \quad (3)$$

We define the worst outage probability over all transmitter/receiver pairs as

$$\mathrm{O} = \max_i \mathrm{O}_i$$

and simply refer to O as the *outage probability of the system*. (More accurately, it is the maximum of the outage probabilities of the transmitter/receiver pairs.) The outage probability O serves as a simple figure of merit for the system and power allocation.

B. Certainty-Equivalent Margin

We now consider the *certainty-equivalent system* in which we ignore all statistical variation of both signal and noise power by replacing these random variables with their expected values. The certainty-equivalent signal power at the ith receiver is then $\mathrm{G}_{ii}\mathrm{P}_i$ and the certainty-equivalent interference power at the ith receiver is given by $\sum_{k\neq i} \mathrm{G}_{ik}\mathrm{P}_k$. We define the certainty-equivalent SIR at the ith receiver as

$$\mathrm{SIR}_i^{\mathrm{ce}} = \frac{\mathrm{G}_{ii}\mathrm{P}_i}{\sum_{k\neq i} \mathrm{G}_{ik}\mathrm{P}_k}. \quad (4)$$

We can interpret $\mathrm{SIR}_i^{\mathrm{ce}}$ as follows: it is what the signal-to-interference of the ith transmitter/receiver pair would be, if the fading state of the system were $\mathrm{F}_1 = \cdots = \mathrm{F}_n = 1$.

We also define

$$\mathrm{SIR}^{\mathrm{ce}} = \min_i \mathrm{SIR}_i^{\mathrm{ce}} = \min_i \frac{\mathrm{G}_{ii}\mathrm{P}_i}{\sum_{k\neq i} \mathrm{G}_{ik}\mathrm{P}_k}$$

which is the minimum certainty-equivalent SIR of the system over all transmitter/receiver pairs. We refer to $\mathrm{SIR}^{\mathrm{ce}}$ as, simply, the certainty-equivalent SIR. Like the outage probability O, $\mathrm{SIR}^{\mathrm{ce}}$ gives a figure of merit for the system and power allocation.

We define the certainty-equivalent margin (CEM) of the system and power allocation as the ratio of the certainty-equivalent SIR to the signal-to-interference reception threshold

$$\mathrm{CEM} = \frac{\mathrm{SIR}^{\mathrm{ce}}}{\mathrm{SIR}^{\mathrm{th}}} = \min_i \frac{\mathrm{G}_{ii}\mathrm{P}_i}{\mathrm{SIR}^{\mathrm{th}} \sum_{k\neq i} \mathrm{G}_{ik}\mathrm{P}_k}. \quad (5)$$

Clearly, there is a relation between CEM and O: when CEM is large (which means that the SIR, ignoring statistical variation, is well above the minimum required for reception), we should have small O. The relation between CEM and O is the topic of Section III-C.

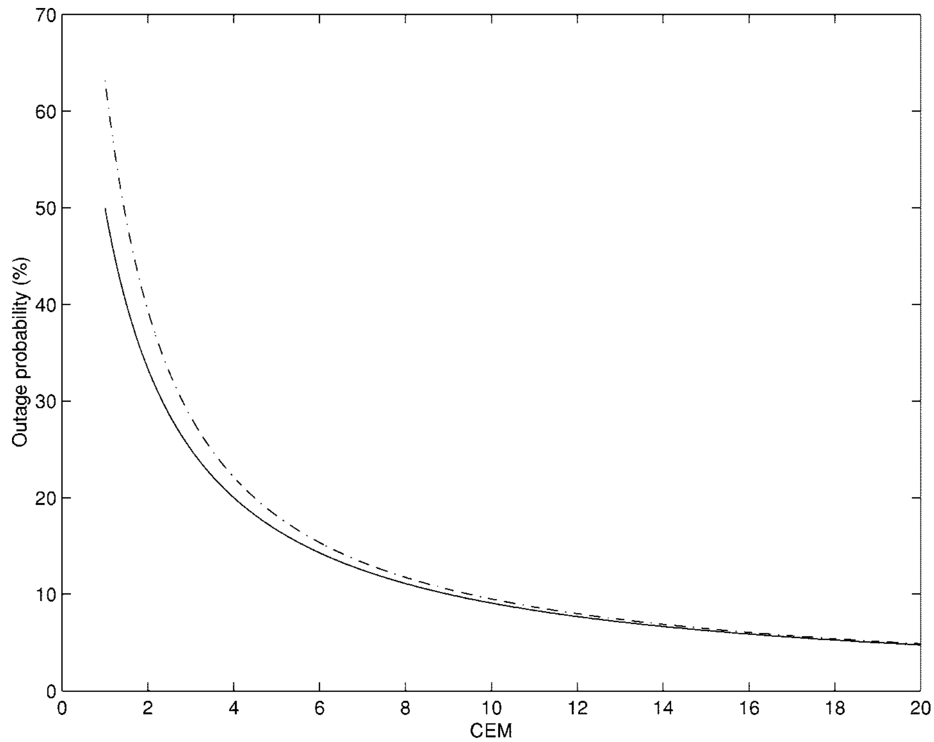

Fig. 1. Upper and lower bound on outage probability O as a function of CEM.

C. Relation Between the CEM and Outage Probability

In this section, we derive some bounds between the CEM and the outage probability. We use the following result (derived in Appendix II): if $z_1, \ldots, z_n \geq 0$, then

$$1+\sum_{k=1}^{n} z_k \leq \prod_{k=1}^{n}(1+z_k) \leq \exp\sum_{k=1}^{n} z_k. \tag{6}$$

By definition, we have

$$\begin{aligned}\mathrm{O} &= \max_i \left(1-\prod_{k\neq i}\frac{1}{1+\frac{\mathrm{SIR}^{\mathrm{th}}\mathrm{G}_{ik}\mathrm{P}_k}{\mathrm{G}_{ii}\mathrm{P}_i}}\right)\\ &= 1-\frac{1}{\max_i \prod_{k\neq i}\left(1+\frac{\mathrm{SIR}^{\mathrm{th}}\mathrm{G}_{ik}\mathrm{P}_k}{\mathrm{G}_{ii}\mathrm{P}_i}\right)}.\end{aligned}$$

Using the right-hand inequality in (6), we get

$$\begin{aligned}\mathrm{O} &\leq 1-\frac{1}{e^{\max_i \sum_{k\neq i}\frac{\mathrm{SIR}^{\mathrm{th}}\mathrm{G}_{ik}\mathrm{P}_k}{\mathrm{G}_{ii}\mathrm{P}_i}}}\\ &= 1-e^{-1/\mathrm{CEM}}.\end{aligned}$$

In a similar way, using the left-hand inequality in (6), we have

$$\begin{aligned}\mathrm{O} &\geq \frac{1}{1+\max_i \sum_{k\neq i}\frac{\mathrm{SIR}^{\mathrm{th}}\mathrm{G}_{ik}\mathrm{P}_k}{\mathrm{G}_{ii}\mathrm{P}_i}}\\ &= \frac{1}{1+\mathrm{CEM}}.\end{aligned}$$

Putting these two inequalities together, we have the bounds

$$\frac{1}{1+\mathrm{CEM}} \leq \mathrm{O} \leq 1-e^{-1/\mathrm{CEM}}. \tag{7}$$

A plot of these bounds is given in Fig. 1. From the plot, it is clear that for outage probabilities of interest, i.e., those smaller than 20% or so, the lower and upper bounds are very close, within about 5%. For larger CEM (and smaller outage probability), the bounds are much closer, confirming our intuition that the CEM and outage probability are closely related. Fig. 2 shows the ratio of the upper to the lower bound as a function of CEM. This plot shows that the bounds are very close for outage probabilities smaller than 10% or so and not far from each other even for small CEM (and large O). For example, with CEM equal to one, the probability of outage is at least 50%, but no more than 63.3%.

IV. Optimal Power Allocation

In this section we consider the problems of finding the power allocations that maximize the CEM, minimize transmit powers, and minimize the outage probability, respectively. The problem of minimizing outage probability can be expressed as

$$\begin{array}{ll}\text{minimize} & \max_i \left(1-\prod_{k\neq i}\frac{1}{1+\frac{\mathrm{SIR}^{\mathrm{th}}\mathrm{G}_{ik}\mathrm{P}_k}{\mathrm{G}_{ii}\mathrm{P}_i}}\right)\\ \text{subject to} & \mathrm{P}_i > 0, \quad i=1,\ldots,n\end{array} \tag{8}$$

and the problem of maximizing the CEM can be expressed as the optimization problem

$$\begin{array}{ll}\text{maximize} & \min_i \frac{\mathrm{G}_{ii}\mathrm{P}_i}{\mathrm{SIR}^{\mathrm{th}}\sum_{k\neq i}\mathrm{G}_{ik}\mathrm{P}_k}\\ \text{subject to} & \mathrm{P}_i > 0, \quad i=1,\ldots,n.\end{array} \tag{9}$$

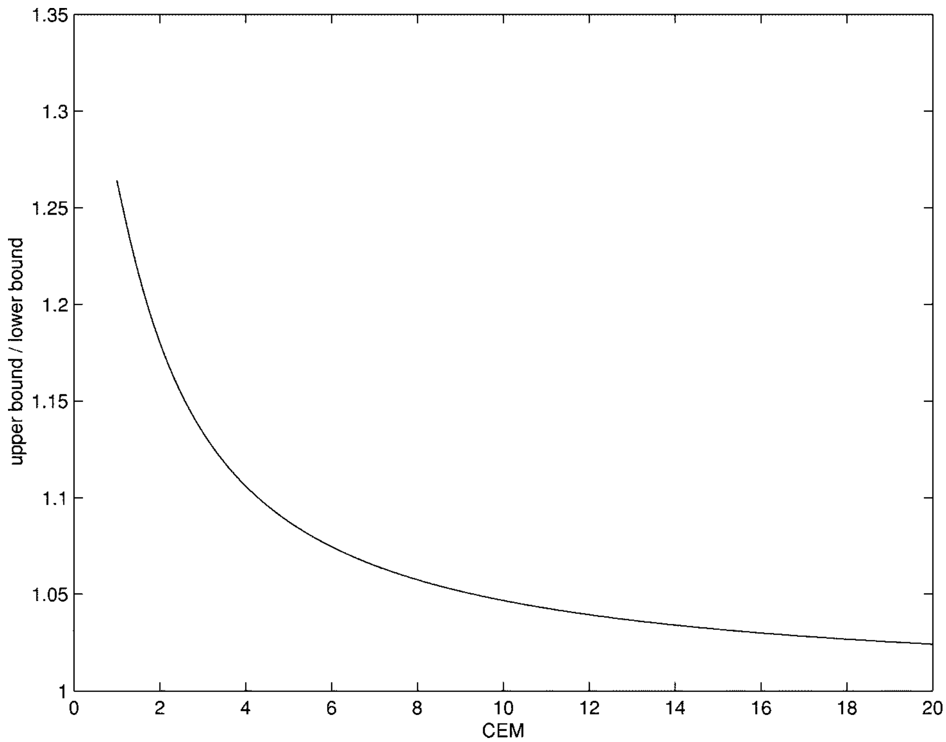

Fig. 2. Ratio of upper to lower bound on outage probability O as function of CEM.

In these problems, the variables are the powers $\mathrm{P}_1, \ldots, \mathrm{P}_n$. The constants $\mathrm{SIR}^{\mathrm{th}}$ and G_{ik}, i, $k = 1, \ldots, n$ are problem parameters. We will assume that $\mathrm{G}_{ik} > 0$.

We observe that the objective functions are homogeneous, i.e., if we scale all powers by any (positive) scale factor, O and CEM remain the same. In other words, outage probability and CEM depend only on the ratios of the powers. Since the constraints $\mathrm{P}_i > 0$ are also homogeneous, it follows that if P is an optimal power-allocation vector (for either problem), then so is $\alpha\mathrm{P}$, for any $\alpha > 0$.

We will let $\mathrm{P}^{\mathrm{out}}$ denote a power-allocation vector that is optimal for the problem (8), i.e., that minimizes the outage probability. Similarly, we will let $\mathrm{P}^{\mathrm{cem}}$ denote a power-allocation vector that is optimal for the problem (9), i.e., that maximizes the CEM.

Our next observation is that in each problem, the optimum is acheived with the values of the maximum (for minimizing O) or minimum (for maximizing CEM) all equal. Let us first consider the problem (8) of minimizing the outage probability. We claim that at an optimal power allocation $\mathrm{P}^{\mathrm{out}}$, the outage probabilities of each transmitter/receiver pair must be equal. In other words, we have

$$\begin{aligned}\mathrm{O}_i(\mathrm{P}^{\mathrm{out}}) =& 1 - \prod_{k \neq i} \frac{1}{1 + \frac{\mathrm{SIR}^{\mathrm{th}}\mathrm{G}_{ik}{\mathrm{P}^{\mathrm{out}}}_k}{\mathrm{G}_{ii}{\mathrm{P}^{\mathrm{out}}}_i}} \\ =& \mathrm{O}(\mathrm{P}^{\mathrm{out}}) = \mathrm{O}^*, \qquad i = 1, \ldots, n,\end{aligned}$$

where O^* denotes the minimal value of outage probability.

To establish the result, we first observe that O_i is monotone increasing in P_k for $k \neq i$ and monotone decreasing in P_i. Now suppose that not all $\mathrm{O}_i(\mathrm{P}^{\mathrm{out}})$ are equal. Choose an index k for which $\mathrm{O}_k < \mathrm{O}^* = \max_i \mathrm{O}_i$. Now, if we decrease ${\mathrm{P}^{\mathrm{out}}}_k$, O_k increases and all other O_i decrease. It follows that if we decrease ${\mathrm{P}^{\mathrm{out}}}_k$ by a small amount, $\mathrm{O} = \max_i \mathrm{O}_i$ will decrease. However, this contradicts the assumption that $\mathrm{P}^{\mathrm{out}}$ minimizes O.

The analogous result holds for the problem (9) of maximizing CEM. In this problem, we observe that each CEM_i is monotonically increasing in P_i and monotonically decreasing in P_k for $k \neq i$. Arguing exactly as above, we conclude that we must have

$$\begin{aligned}\mathrm{CEM}_i(\mathrm{P}^{\mathrm{cem}}) =& \frac{\mathrm{G}_{ii}\mathrm{P}_i}{\mathrm{SIR}^{\mathrm{th}} \sum_{k \neq i} \mathrm{G}_{ik}{\mathrm{P}^{\mathrm{cem}}}_k} \\ =& \mathrm{CEM}(\mathrm{P}^{\mathrm{cem}}) = \mathrm{CEM}^*, \qquad i = 1, \ldots, n\end{aligned}$$

where CEM^* is the maximal value of CEM.

A. Maximizing CEM

In the field of wireless networks, power control by maximizing CEM has been well studied and understood [2], [7], [10]–[12], [17], [23]–[25]. It is based on the Perron–Frobenius theorem for the maximum eigenvalue of a matrix that has nonnegative elements [2].

Using our observation that at the optimum, CEM_i are all equal, we can reformulate the problem (9) as

$$\begin{array}{ll}\text{maximize} & t \\ \text{subject to} & \dfrac{\mathrm{G}_{ii}\mathrm{P}_i}{\mathrm{SIR}^{\mathrm{th}} \sum_{k \neq i} \mathrm{G}_{ik}\mathrm{P}_k} = t, \quad i = 1, \ldots, n, \\ & \mathrm{P}_i > 0, \quad i = 1, \ldots, n\end{array}$$

where t is another variable, whose optimal value is the optimal value of CEM. Substituting the variable $\tau = 1/t$, we can express this problem as

$$\begin{array}{ll} \text{minimize} & \tau \\ \text{subject to} & A\mathrm{P} = \tau \mathrm{P} \\ & \mathrm{P}_i > 0, \quad i = 1, \ldots, n \end{array}$$

where the matrix A is defined as

$$A_{ij} = \frac{\mathrm{G}_{ij}}{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ii}}, \quad i \neq j, \qquad A_{ii} = 0.$$

We recognize the problem above as an eigenvalue problem in which the matrix has all entries nonnegative. According to Perron–Frobenius theory, the eigenvalue λ of A that is largest in magnitude is real and positive and has an associated eigenvector v all of whose components are positive. (Here, we use the fact that A is not cyclic or reducible, which follows from $\mathrm{G}_{ij} > 0$.) The eigenvector v (and associated eigenvalue λ) are called the Perron–Frobenius eigenvector (eigenvalue) of A. The Perron–Frobenius eigenvector v gives an optimal power allocation, i.e., $\mathrm{P}_i = v_i$ maximizes CEM. The optimal CEM is exactly $\mathrm{CEM}^* = 1/\lambda$.

Even though the above optimal solution assumes a centralized controller, there exist distributed methods to achieve the same solution [7], [13], [14] (actually, the specified SIRs on each link are achieved rather than the SIRs that maximize CEM). As mentioned before, these distributed algorithms assume that link gains remain constant. However, it can easily be modified for the Rayleigh fading channel with the accompanying penalty of having outage as below. The distributed algorithm, in a time slotted system, is iterative and uses the value of its link gain and the received interference value in the previous time slot for computing the power to be transmitted in the current time slot. For a Rayleigh fading channel with gain changing from slot to slot instead of using just the values in the previous time slot, we could average the values over the previous few slots to obtain the values of its link gain G_{ii} and the interference at the receiver. This effectively averages out the Rayleigh fading components F_{ij} (approximately). The number of time slots over which the average is taken depends determines the tradeoff between getting an accurate estimate of the parameters G_{ij} and the rate of convergence of the algorithm. Using this method, we update the power only once in a few slots rather than every slot.

1) Relation Between CEM *Optimal and* O *Optimal Allocations:* Using the bounds of Section III, we can show that a power allocation $\mathrm{P}^{\mathrm{cem}}$ that maximizes CEM (which can be found by computing the Perron–Frobenius eigenvector of an $n \times n$ matrix) is not too far from minimizing outage probability.

Let P denote an arbitrary power allocation (with $\mathrm{P}_i > 0$). Then, we have

$$\mathrm{CEM}(\mathrm{P}) \leq \mathrm{CEM}(\mathrm{P}^{\mathrm{cem}})$$

since, by definition, $\mathrm{P}^{\mathrm{cem}}$ maximizes CEM. It follows that

$$\frac{1}{1 + \mathrm{CEM}(\mathrm{P})} \geq \frac{1}{1 + \mathrm{CEM}(\mathrm{P}^{\mathrm{cem}})}$$

[since the function mapping x into $1/(1+x)$ is monotone decreasing for $x \geq 0$]. Combining this inequality with the left-hand bound in (7), we have

$$\mathrm{O}(\mathrm{P}) \geq \frac{1}{1 + \mathrm{CEM}(\mathrm{P})} \geq \frac{1}{1 + \mathrm{CEM}(\mathrm{P}^{\mathrm{cem}})}.$$

This inequality holds for all P, so we have

$$\mathrm{O}^* \geq \frac{1}{1 + \mathrm{CEM}(\mathrm{P}^{\mathrm{cem}})}$$

where O^* denotes the minimum possible outage probability, i.e., the optimal value of the problem (8).

From this inequality, we can make several conclusions. First of all, if we compute $\mathrm{P}^{\mathrm{cem}}$ (by solving a Perron–Frobenius eigenvalue problem), then we can bracket O^*: it is certainly between the lower bound $1/(1 + \mathrm{CEM}(\mathrm{P}^{\mathrm{cem}}))$ and the upper bound $\mathrm{O}(\mathrm{P}^{\mathrm{cem}})$. These bounds are often extremely close and in any case never far apart. Indeed, since

$$\mathrm{O}(\mathrm{P}^{\mathrm{cem}}) \leq 1 - e^{-1/\mathrm{CEM}}$$

we always have

$$\frac{1}{1 + \mathrm{CEM}(\mathrm{P}^{\mathrm{cem}})} \leq \mathrm{O}^* \leq 1 - e^{-1/\mathrm{CEM}(\mathrm{P}^{\mathrm{cem}})}.$$

Since the ratio of these bounds is often near one and never far from one, it follows that maximizing CEM is often very nearly the same as minimizing outage probability and provably never very suboptimal.

B. Minimizing Transmitter Powers

In this section, we consider the problem of minimizing total transmitter power subject to either outage (or CEM) constraints and bounds on individual powers. We show that the problem of power allocation with constraints on the outage probability, as well as other constraints such as limits on the individual powers, can be expressed as a special type of optimization problem called geometric programming (see Appendix III).

To minimize the total transmit power, subject to the constraint that each transmitter/receiver attain a maximum allowed outage probability (i.e., a minimum allowed QoS) and subject to limits on the individual transmitter powers, we form the problem

$$\begin{array}{ll} \text{minimize} & \mathrm{P}_1 + \cdots + \mathrm{P}_n \\ \text{subject to} & \mathrm{P}_i^{\min} \leq \mathrm{P}_i \leq \mathrm{P}^{\max}, \quad i = 1, \ldots, n, \\ & \mathrm{O}_i \leq \mathrm{O}_i^{\max}, \quad i = 1, \ldots, n. \end{array} \tag{10}$$

Here, $\mathrm{P}_i^{\min}$ and $\mathrm{P}_i^{\max}$ are the minimum and maximum transmitter power for transmitter i; the maximum might be dependent on the transmitter hardware and the minimum value guarantees that the white noise at receiver is overcome. The number $\mathrm{O}_i^{\max}$ is the maximum allowed outage probability for the ith transmitter/receiver. Note that these can be the same or different for each pair, allowing different QoS to be assigned to different users.

Evidently, the outage-probability constraints $\mathrm{O}_i \leq \mathrm{O}_i^{\max}$ are the challenging ones, since O_i is a highly nonlinear function of the powers. Using (3), we can express the outage-probability constraint $\mathrm{O}_i \leq \mathrm{O}_i^{\max}$ as

$$1 - \mathrm{O}_i^{\max} \leq \prod_{k \neq i} \left(\frac{1}{1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i}} \right)$$

which, in turn, we can express as

$$(1 - \mathrm{O}_i^{\max}) \prod_{k \neq i} \left(1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i} \right) \leq 1. \tag{11}$$

Since each of the terms $1 + \mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k / \mathrm{G}_{ii} \mathrm{P}_i$ is a posynomial function (see Appendix III) of the powers, we conclude that the left-hand side of the inequality (11) is, in fact, a posynomial function of the powers $\mathrm{P}_1, \ldots, \mathrm{P}_n$.

Using this result, we can express the problem (10) as

$$\begin{aligned} &\text{minimize} \quad \mathrm{P}_1 + \cdots + \mathrm{P}_n \\ &\text{subject to} \quad \frac{\mathrm{P}_i^{\min}}{\mathrm{P}_i} \leq 1, \quad i = 1, \ldots, n, \\ &\qquad \frac{\mathrm{P}_i}{\mathrm{P}_i^{\max}} \leq 1, \quad i = 1, \ldots, n, \\ &(1 - \mathrm{O}_i^{\max}) \prod_{k \neq i} \left(1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i} \right) \leq 1. \end{aligned} \tag{12}$$

This is a geometric program (GP) in the variables $\mathrm{P}_1, \ldots, \mathrm{P}_n$. Therefore, we can solve the power-allocation problem (10) globally and efficiently using interior-point methods for geometric programming. Note that any other constraints that can be handled by geometric programming can be added to the power-allocation problem.

C. Minimum Outage Probability

As mentioned previously, there has not been any work on how to optimize transmit powers to minimize outage probability. In this section, we explore how we can minimize the outage probability efficiently. The problem formulation above (12) can be slightly modified to minimize the outage probability O by solving the GP

$$\begin{aligned} &\text{minimize} \quad \alpha \\ &\text{subject to} \quad \frac{\mathrm{P}_i^{\min}}{\mathrm{P}_i} \leq 1, \quad i = 1, \ldots, n, \\ &\qquad \frac{\mathrm{P}_i}{\mathrm{P}_i^{\max}} \leq 1, \quad i = 1, \ldots, n, \\ &\left(\frac{1}{\alpha} \right) \prod_{k \neq i} \left(1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i} \right) \leq 1 \end{aligned} \tag{13}$$

with optimization variables $\mathrm{P}_1, \ldots, \mathrm{P}_n$ and α. Here, α is an upper bound on $1/(1 - \mathrm{O}_i^{\max})$, so when we solve the GP (13), the optimal value of α is $1/(1 - \mathrm{O}^*)$, where O^* is the minimal value of the maximum outage probability. Even though GPs can be solved efficiently, they still take considerable time compared to the extremely fast requirement of signal processing at the transceiver. So, we now propose a faster heuristic to minimize outage probability (without any minimum/maximum constraints on the powers).

According to our observation made before that at the optimum, all outage probabilities are equal, the problem can be expressed as

$$\begin{aligned} &\text{minimize} \quad t \\ &\text{subject to} \quad 1 - \prod_{k \neq i} \frac{1}{1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i}} = t, \quad i = 1, \ldots, n \\ &\qquad \mathrm{P}_i > 0, \quad i = 1, \ldots, n \end{aligned} \tag{14}$$

where t is another variable. (In fact, the condition that all of the outage probabilities be equal is not only necessary for optimality; it is also sufficient. This follows by examining the convex form of the problem.) In the remainder of this section we describe a simple iterative algorithm that, in our experience, computes $\mathrm{P}^{\mathrm{out}}$ within a few iterations, where each iteration consists of solving a Perron–Frobenius eigenvector problem. We do not have a proof that the method always converges, but we have never observed a case where it fails to converge in, at most, four or five iterations.

To motivate our iterative method, we start with the equality constraints

$$1 - \prod_{k \neq i} \frac{1}{1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i}} = t, \qquad i = 1, \ldots, n$$

where t is the variable to be minimized. This can be rewritten as

$$\prod_{k \neq i} \left(1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i} \right) = \beta, \qquad i = 1, \ldots, n$$

where $\beta = 1/(1 - t)$. Here, the objective is to minimize β.

We rewrite these equations in the form

$$\sum_{k \neq i} \log \left(1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i} \right) = \gamma, \qquad i = 1, \ldots, n$$

where $\gamma = \log \beta$ is to be minimized. This is equivalent to

$$\sum_{k \neq i} \left[\frac{\mathrm{P}_i}{\mathrm{P}_k} \log \left(1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i} \right) \right] \mathrm{P}_k = \gamma \mathrm{P}_i$$

which we express as $\mathrm{B}(\mathrm{P})\mathrm{P} = \gamma \mathrm{P}$, where B is the matrix given by

$$\mathrm{B}_{ik} = \frac{\mathrm{P}_i}{\mathrm{P}_k} \log \left(1 + \frac{\mathrm{SIR}^{\mathrm{th}} \mathrm{G}_{ik} \mathrm{P}_k}{\mathrm{G}_{ii} \mathrm{P}_i} \right), \quad i \neq k$$

and $\mathrm{B}_{ii} = 0$.

Now our problem can be stated as finding P (with positive entries) and γ, which minimize γ and satisfy the condition $\mathrm{B}(\mathrm{P})\mathrm{P} = \gamma \mathrm{P}$. If we ignore the fact that B depends on P, this problem can be solved as a Perron–Frobenius eigenvector problem.

We can now describe our iterative method. We start with $\mathrm{P} = \mathrm{P}^{\mathrm{cem}}$, then fix $\mathrm{B} = \mathrm{B}(\mathrm{P})$ and update P by solving the Perron–Frobenius eigenvector problem $\mathrm{BP} = \gamma \mathrm{P}$. This is repeated until P does not change, so we have $\mathrm{B}(\mathrm{P})\mathrm{P} = \gamma \mathrm{P}$, which solves the problem of minimizing outage probability. In our experience, the algorithm always converges in fewer than five or

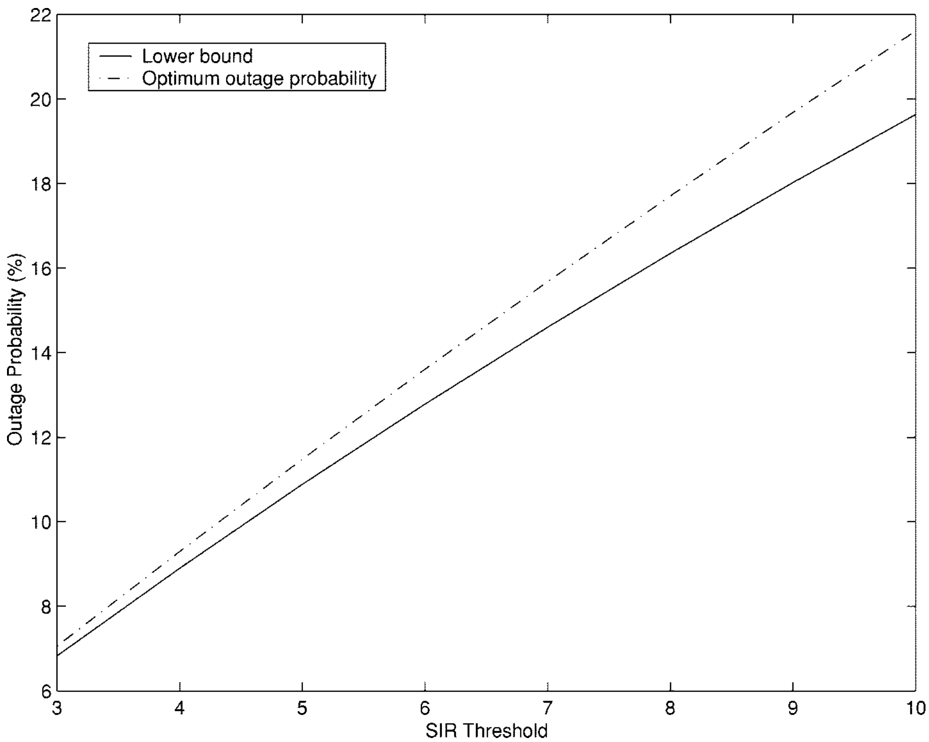

Fig. 3. Outage probability versus $\mathrm{SIR}^{\mathrm{th}}$ for a system with 50 wireless links. Dotted curve shows the outage probability achieved both by the exact optimal power allocation and the outage probability achieved by the power allocation that maximizes CEM (the difference is negligible). Solid curve shows the lower bound on optimal outage probability based on CEM.

so steps to an accuracy far exceeding any significance for the engineering problem (i.e., to ten significant figures).

This iterative algorithm requires all the link gains G_{ij}, i.e., it is not distributed. So, if one does not have access to the link gains, one has to fall back on the CEM maximizing (SIR balancing) method which can be implemented in a distributed way.

V. Example

In this section, we give a simple numerical example demonstrating the results of this paper. We consider a system with 50 transmitters and receivers with Rayleigh/Rayleigh fading and ambient white-noise power that is insignificant compared to interference power.

Before we proceed with the description of the G matrix, it is useful to recall what the entries represent and what the optimization problem parameters are. The path gain G_{ij} takes into account distance dependent attenuation, log-normal shadowing, antenna gains, and cross correlations between CDMA codes. To solve the optimization problem of minimizing powers subject to outage constraints (or the problem of minimizing outage probability) at any time instant, one requires the values of G_{ij} at that time instant as input parameters (i.e., one instance of link gains, not expected value of link gains). Since, G_{ij} is the product of different time-varying random quantities, we simplify the example as described below.

We take all the gains G_{ii} (from ith transmitter to ith receiver) to be one and we generate the cross gains $\mathrm{G}_{ij},\ i \neq j$ as independent random variables uniformly distributed between between zero and 0.001. We only assume that the signal path is "stronger" than the interference path, since it is only the ratios of $\mathrm{G}_{ii}/\mathrm{G}_{ij}$ that matter in the absence of white-noise power.

We varied $\mathrm{SIR}^{\mathrm{th}}$ from three to ten and, for each value, computed $\mathrm{P}^{\mathrm{cem}}$ and $\mathrm{P}^{\mathrm{out}}$ using the CEM maximizing method and the iterative method, respectively. For each value of $\mathrm{SIR}^{\mathrm{th}}$, we also computed $\mathrm{O}(\mathrm{P}^{\mathrm{cem}})$, the outage probability achieved by $\mathrm{P}^{\mathrm{cem}}$, as well as O^* (which is $\mathrm{O}(\mathrm{P}^{\mathrm{out}})$) and the lower bound $1/(1+\mathrm{O}(\mathrm{P}^{\mathrm{cem}}))$. $\mathrm{O}(\mathrm{P}^{\mathrm{cem}})$, O^* and $1/(1+\mathrm{O}(\mathrm{P}^{\mathrm{cem}}))$ are plotted in Fig. 3 for different values of $\mathrm{SIR}^{\mathrm{th}}$. Since each instance of link gains corresponds to a maximum achievable $\mathrm{SIR}^{\mathrm{ce}}$ as the $\mathrm{SIR}^{\mathrm{th}}$ keep increasing, the CEM [see (5)] keeps decreasing and the outage probability increases (see Fig. 1) as shown. The results were found to be similar for different instances of link gains.

We observe that $\mathrm{P}^{\mathrm{cem}}$, the power allocation that maximizes CEM, also minimizes outage probability for any practical purpose. The differences in outage probabilities obtained by the power allocation that maximizes CEM and those obtained by the power allocation that exactly minimize outage probability were insignificantly small.

VI. Conclusion

We have considered the problem of allocating power in a wireless system, taking into account the statistical fluctuation in SIR induced by Rayleigh fading. In the general case, we establish that this problem can be cast as a geometric programming problem. We show that the problem of minimizing probability of outage is for all practical purposes solved by maximizing the CEM, which can be done using Perron–Frobenius eigenvalue methods or other iterative distributed methods developed for this problem. While maximizing this margin is certainly a natural heuristic for minimizing outage probability, we prove a rigorous bound on how suboptimal this heuristic can be.

The benefit of the outage-probability minimizing method of allocating power is that it allows power allocation to be done on the far longer time scale of log-normal shadowing instead of the time scale of Rayleigh fading. The disadvantage is a positive probability of fading-induced outage. (Of course, this disadvantage is also present in a power-allocation method that attempts to track fading state: for some fading states, allocating power to guarantee reception for all transmitter/receiver pairs is impossible.)

The important conclusion of the paper is that the SIR maximizing method has an outage probability that is almost same as the optimal outage probability. This means that the current power-control algorithm for *ad hoc* wireless networks [7], [14], [25] can be implemented at the log-normal shadowing time scale rather than the Rayleigh fading time scale as outlined in Section IV-A.

Appendix I
Derivation of Probability Expression

In this section, we give a self-contained derivation of the following result. Suppose $z_1, \ldots, z_n$ are independent exponentially distributed random variables with means $\mathrm{E} z_i = 1/\lambda_i$. Then, we have

$$\mathrm{Prob}\left(z_1 \le \sum_{i=2}^{n} z_i\right) = 1 - \prod_{i=2}^{n}\left(\frac{1}{1+\frac{\lambda_1}{\lambda_i}}\right). \tag{15}$$

To prove this, we note that

$$\begin{aligned}\mathrm{Prob}\left(z_1 > \sum_{i=2}^{n} z_i\right) &= \int_{t_2=0}^{\infty}\cdots\int_{t_n=0}^{\infty}\mathrm{Prob}\left(z_1 > \sum_{i=2}^{n} t_i\right)\\ &\qquad \prod_{i=2}^{n}\lambda_i e^{-\lambda_i t_i}\,dt_2\cdots dt_n\\ &= \int_{t_2=0}^{\infty}\cdots\int_{t_n=0}^{\infty} e^{-\lambda_1(t_2+\cdots+t_n)}\\ &\qquad \prod_{i=2}^{n}\lambda_i e^{-\lambda_i t_i}\,dt_2\cdots dt_n\\ &= \prod_{i=2}^{n}\int_{t_i=0}^{\infty}\lambda_i e^{-(\lambda_1+\lambda_i)t_i}\,dt_i\\ &= \prod_{i=2}^{n}\frac{\lambda_1}{\lambda_1+\lambda_i}.\end{aligned} \tag{16}$$

Subtracting this expression from one yields (15).

Appendix II
Derivation of Bounds on $\prod_{k=1}(1+z_k)$

In this section, we derive the following inequalities. If $z_1, \ldots, z_n \ge 0$, then

$$1 + \sum_{k=1}^{n} z_k \le \prod_{k=1}^{n}(1+z_k) \le \exp\sum_{k=1}^{n} z_k.$$

To establish the left-hand inequality, we expand the middle expression as

$$\prod_{k=1}^{n}(1+z_k) = 1 + \sum_{k=1}^{n} z_k + \sum_{k=1}^{n}\sum_{j>k} z_k z_j + \cdots.$$

The first and second terms are the left-hand side of the inequality we wish to establish; the third and other remaining terms are nonnegative, since they consist of sums of products of z_i, which are nonnegative.

To establish the right-hand inequality, we will derive the equivalent inequality

$$\sum_{k=1}^{n}\log(1+z_k) \le \sum_{k=1}^{n} z_k.$$

This follows from the simple inequality $\log(1+z) \le z$ for $z \ge 0$.

Appendix III
Geometric Programming

Let $x_1, \ldots, x_n$ be n real positive variables and x denote the vector of these n variables. A function $f : \mathbf{R}_+^n \to \mathbf{R}$ is called a *posynomial* function if it has the form

$$f(x_1, \ldots, x_n) = \sum_{k=1}^{t} c_k x_1^{\alpha_{1k}} x_2^{\alpha_{2k}} \ldots x_n^{\alpha_{nk}}$$

where $c_k \ge 0$ and $\alpha_{ij} \in \mathcal{R}$. Note that the coefficients c_k must be nonnegative, but the exponents α_{ij} can be any negative (or fractional) number. The function f is called a *monomial* function if $t = 1$ and $c_1 > 0$, i.e., it consists of one nonzero term. Posynomials are closed under addition and multiplication.

A GP is an optimization problem of the form

$$\begin{array}{ll}\text{minimize} & f_0(x)\\ \text{subject to} & f_i(x) \le 1, \quad i = 1, \ldots, m\\ & g_i(x) = 1, \quad i = 1, \ldots, p\\ & x_i > 0, \quad i = 1, \ldots, n\end{array} \tag{17}$$

where $f_1, \ldots, f_m$ are posynomial functions and $g_1, \ldots, g_p$ are monomial functions. Geometric programs were introduced by Duffin [26]; recent applications include wire and transistor sizing for digital circuits [27] and op-amp design [28]; see [29]. Using interior-point methods for nonlinear convex programming, originally developed by Nesterov and Nemirovsky [30], GPs can be solved with great efficiency. Indeed, very large GPs can be solved using primal-dual interior-point methods; see [31]–[33].

A GP can be reformulated as a *convex optimization problem*, i.e., the problem of minimizing a convex function subject to convex inequality constraints and linear equality constraints, by a change of variables. Suppose that f is a posynomial and define $y_i = \log x_i$ so that $x_i = e^{y_i}$ (which automatically enforces the positivity constraint on x_i). We define the function

$$h(y) = \log f(e^{y_1}, \ldots, e^{y_n}) = \log\left(\sum_{k+1}^{t} e^{a_k^T y + b_k}\right)$$

where $a_k = (\alpha_{1k}, \ldots, \alpha_{nk})$ and $b_k = \log c_k$. It can be shown that h is a *convex* function of the new variable y; if the original function f were a monomial, then the function h is affine (i.e., linear plus a constant). Applying this change of variable to the GP (17), we obtain

$$\begin{array}{ll} \text{minimize} & \log f_0(e^{y_1}, \ldots, e^{y_n}) \\ \text{subject to} & \log f_i(e^{y_1}, \ldots, e^{y_n}) \leq 0, \quad i = 1, \ldots, m \\ & \log g_i(e^{y_1}, \ldots, e^{y_n}) = 0, \quad i = 1, \ldots, p. \end{array} \tag{18}$$

This is called the *convex form* of the GP. It is a (nonlinear) convex optimization problem, since the objective and inequality constraint functions are all convex and the equality constraint functions are affine.

One important consequence is that we can solve GPs globally with great efficiency using recently developed interior-point methods (see, e.g., [29] and [30]).

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