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Abstract

When using antenna arrays at the base station of a cellular system, one critical aspect is the transmission strategy. An optimal choice of beamformers for simultaneous transmission to several co-channel users must be solved jointly for all users and base stations in an area. We formulate an optimal transmit strategy and show how the solution can be calculated efficiently using interior point methods for semidefinite optimization. The algorithm minimizes the total transmitted power under certain constraints to guarantee a specific quality of service. The method provides large flexibility in the choice of constraints and can be extended to be robust to channel perturbations.

1 Introduction

The use of antenna arrays brings new possibilities in the design of mobile communications systems. It is well known that the system capacity is more limited in the downlink than in the uplink [7,22]. Yet, the literature on beamforming for transmission is relatively small, compared to the well investigated topic of beamforming for a receiving antenna array. Some results on downlink beamforming can be found in [8, 11, 12, 15, 21, 22].

The simplest transmit strategy is to use standard beamforming, i.e., to point the main lobe of the antenna array in the direction of the specific receiver [13]. Knowing the channel to nearby co-channel users, it is possible to actively suppress the signal to the interfered users. In [15], Rashid-Farrokhi et al. formulate the beamforming design as a constrained optimization problem and present an algorithm that is shown to find the global optimum.

In this paper, we consider the same optimization problem, namely to minimize the total transmitted power while maintaining a certain quality of service for all users, but present an alternative solution using convex optimization. This gives several advantages. First of all, the solution can be efficiently calculated using standard algorithms for semidefinite optimization [16, 19] with guaranteed convergence speed. Secondly, with this technique, it is very easy to introduce modifications, for example adding extra constraints on the dynamic range or adding increased robustness to channel estimation errors. In most cases our method gives a solution in the form of a standard fixed beamformer, in the remaining cases the solution can be implemented using a time-varying beamformer – a space time code [17]. Note however that the problem formulations we consider here will not necessarily give any coding gain.

In [20], Visotsky and Madhow consider the algorithm in [15] for the special case of rank one channels and claim that it does not give the optimal solution unless an extra scaling of the problem is introduced. However, our results confirm that the algorithm in [15] does indeed give the correct solution also without any rescaling. Furthermore, we show that for rank one channels, the problem is essentially constrained by a convex second-order cone [10], Thus, any reasonable iterative minimization procedure should converge to the global optimum.

To our knowledge, convex optimization for beamformer design has previously only been used with traditional filter design-type constraints on pass bands and side-lobe levels [9], but not within the framework of statistically optimal beamforming.

When the antenna array is used as a receiver, i.e. in uplink mode, the instantaneous channel can be estimated directly from the received data, whereas in the downlink, the transmitting beamformer must be based on information collected in the uplink. Several schemes have been proposed for the transformation from uplink to downlink. In a Time Division Duplex (TDD) system with sufficiently short time slots, the downlink channel is virtually identical to the uplink channel, whereas in a Frequency Division Duplex (FDD) system, the channel fades independently at the two duplex frequencies. However, a statistical model of the downlink channel can be obtained from the collected uplink data using a physical model [4,23] or model-free techniques [3, 8]. Throughout this paper, we assume that a stochastic characterization of the downlink transmission channel is known at the base station for all co-channel mobiles within its range (possible located in neighboring cells).

We study the joint design of the beamformers for all co-channel users within a large region which could include several cells. The optimal solution must be solved jointly since every transmitted signal will affect all receivers. In a practical system, a decentralized suboptimal solution may be preferable to reduce the overhead of collecting all channel measurements. The optimal solution does still provide a valuable benchmark for evaluation of other algorithms and for use in system simulations and capacity studies.

The paper is organized as follows. The system model and the basic assumptions are presented in Section 2. In Section 3, we first state the problem and show how it can be solved using a semidefinite relaxation. Then we show how to incorporate additional constraints and give a short overview of some suboptimal solutions. Finally we present a few numerical examples in Section 4.

2 System Model

We consider a system where a number of co-channel mobile users are served by one or more base stations and each base station is equipped with an antenna array. We will design the beamformers assuming a stationary scenario where the fast (Rayleigh) fading is described by its second order properties. We also assume narrow-band signals without any time dispersion, i.e., the channel fading is frequency flat. The model can easily be extended to frequency selective channels, taking both co-channel interference and inter-symbol interference into account, see [15].

In the baseband, the signal received by the ith mobile, $r_i(t)$, is given by

$$r_i(t) = \sum_{k=1}^{K} \mathbf{v}_{i,k}^{*} \mathbf{x}_k(t) + n_i(t) \ , \tag{1}$$

where $(\cdot)^*$ denotes Hermitian vector transpose. Here, $\mathbf{x}_k(t)$ is the complex valued $m \times 1$ vector of the baseband signals transmitted at the antenna elements of base station k, $n_i(t)$ is zero mean white complex noise with variance σ_i^2. The channel from base station k to mobile i is given by the random complex valued vector $\mathbf{v}_{i,k}$ with correlation matrix

$$\mathbf{R}_{i,k} = \mathrm{E}[\mathbf{v}_{i,k}\mathbf{v}_{i,k}^*] \ . \tag{2}$$

In the special case of line of sight transmission, $\mathbf{R}_{i,k} = \mathbf{h}_{i,k}\mathbf{h}_{i,k}^*$, where $\mathbf{h}_{i,k}$ is a deterministic array response vector, but in general, $\mathbf{R}_{i,k}$ can have any rank because of specular or diffuse multi-path propagation.

Each mobile is allocated to one base station and $\kappa(i)$ is used to denote the base station allocated for mobile i. Likewise, $\mathcal{I}(k) = \{i|\kappa(i) = k\}$ denotes the indices of the set of mobiles allocated to base k. We will use the shorthand notation $\mathbf{R}_i$ for $\mathbf{R}_{i,\kappa(i)}$.

The signal transmitted at base k is given by

$$\mathbf{x}_k(t) = \sum_{i \in \mathcal{I}(k)} \mathbf{w}_i s_i(t) \ , \tag{3}$$

where $s_i(t)$ is the scalar data sequence intended for user i and $\mathbf{w}_i$ is the beamforming weight vector for transmission from base $\kappa(i)$ to mobile i. For simplicity we assume that all $s_i(t)$ are uncorrelated and have the same power $\mathrm{E}[|s_i(t)|^2] = 1$.

3 Algorithms

We consider the design of the beamformers $\mathbf{w}_i$ given estimates of all $\mathbf{R}_{i,k}$ and σ_i^2. The goal is to minimize to total transmitted power

$$P = \sum_{i=1}^{d} \|\mathbf{w}_i\|^2 \tag{4}$$

while maintaining an acceptable quality of service for all users. We will first consider an SINR threshold and then show how additional constraints can be used to gain increased robustness to estimation errors in the channel covariance matrices or to handle a limited dynamic range.

3.1 Optimal Beamforming

We consider the same problem formulation as in [15, 20], namely to minimize the total transmitted power under the constraint that the received SINR at each mobile is above a certain threshold, $\mathrm{SINR}_i \geq \gamma_i$. This gives the following optimization problem

$$\begin{aligned} &\min \sum_{i=1}^{d} \|\mathbf{w}_i\|^2 \\ &\text{s.t. } \frac{\mathbf{w}_i^*\mathbf{R}_i\mathbf{w}_i}{\sum_{n \neq i} \mathbf{w}_n^*\mathbf{R}_{i,\kappa(n)}\mathbf{w}_n + \sigma_i^2} \geq \gamma_i, \ i = 1, \dots, d \ , \end{aligned} \tag{5}$$

or equivalently,

$$\begin{aligned} &\min \sum_{i=1}^{d} \mathbf{w}_i^*\mathbf{w}_i \\ &\text{s.t. } \mathbf{w}_i^*\mathbf{R}_i\mathbf{w}_i - \gamma_i \sum_{n \neq i} \mathbf{w}_n^*\mathbf{R}_{i,\kappa(n)}\mathbf{w}_n \geq \gamma_i\sigma_i^2, \ i = 1, \dots, d \ . \end{aligned} \tag{6}$$

It is easy to show that all constraints must be active at the optimum [20], thus the inequality in (6) can be replaced by an equality.

For the special case of rank one channels, $\mathbf{w}_n^* \mathbf{R}_{i,\kappa(n)} \mathbf{w}_n = |\mathbf{w}_m^* \mathbf{h}_{i,\kappa(m)}|^2$ and we can without loss of generality add the constraints $\mathbf{w}_i^* \mathbf{h}_i \geq 0$ which gives the following problem.

$$\begin{aligned} &\min \sum_{i=1}^{d} \mathbf{w}_i^* \mathbf{w}_i \\ &\text{s.t. } (\mathbf{w}_i^* \mathbf{h}_i)^2 \geq \gamma_i \left(\sum_{n \neq i} \mathbf{w}_n^* \mathbf{R}_{i,\kappa(n)} \mathbf{w}_n + \sigma_i^2 \right), \\ &\quad \mathbf{w}_i^* \mathbf{h}_i \geq 0, \; i = 1, \ldots, d\,. \end{aligned} \tag{7}$$

However, these constraints are just an affine transformation of the convex second-order cone $\{\mathbf{x}, y| \; \|\mathbf{x}\|^2 \leq y^2, y \geq 0\}$, which shows that this is a convex problem which can be efficiently solved using standard methods for convex optimization [10].

In the general case, the original constraint set, (6) is not convex, but we will show that the problem still can be efficiently solved using convex optimization.

To this end, introduce the matrices $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^*$ and use the rule $\mathbf{w}^* \mathbf{R} \mathbf{w} = \mathrm{Tr}[\mathbf{R} \mathbf{w} \mathbf{w}^*] = \mathrm{Tr}[\mathbf{R} \mathbf{W}]$ to rewrite the problem into

$$\begin{aligned} &\min \sum_{i=1}^{d} \mathrm{Tr}[\mathbf{W}_i] \\ &\text{s.t. } \mathrm{Tr}[\mathbf{R}_i \mathbf{W}_i] - \gamma_i \sum_{n \neq i} \mathrm{Tr}[\mathbf{R}_{i,\kappa(n)} \mathbf{W}_n] = \gamma_i \sigma_i^2, \\ &\quad \mathbf{W}_i = \mathbf{W}_i^*, \\ &\quad \mathbf{W}_i \succeq 0, \; i = 1, \ldots, d\,. \end{aligned} \tag{8}$$

Here, the notation $\mathbf{W} \succeq 0$ means that $\mathbf{W}$ is positive semidefinite. Note that with the additional constraints $\mathrm{rank}[\mathbf{W}_i] = 1$, (8) is equivalent to (6), thus if the solution of (8) has $\mathrm{rank}[\mathbf{W}_i] = 1$ for all i, then it is also a solution of (5). Relaxing the rank of $\mathbf{W}_i$ gives a semidefinite optimization problem with a solution that always is a lower bound for the original problem. This technique is called a Lagrangian relaxation [2, 19] since it is the Lagrangian dual of the dual of the original problem. For this specific problem, however, we can show a much stronger result, namely.

Theorem 3.1. *If (8) is feasible, than it has at least one solution where* $\mathrm{rank}[\mathbf{W}_i] = 1$*, for all* $i = 1, \ldots, d$*.*

Proof. See [5]. □

If the relaxation has several minima, there is no guarantee that all solutions have rank one. One counterexample is given by $\sigma_i^2 = 1$, $\gamma_i = 1/2$, $\kappa(i) = 1$ and

$$\mathbf{R}_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{R}_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

which has several solutions including

$$\mathbf{W}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{W}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{7}{6} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{W}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

but also

$$\mathbf{W}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{bmatrix} \quad \mathbf{W}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{W}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} \end{bmatrix} .$$

Thus, there is no guarantee that an algorithm for solving semidefinite problems will give the desired rank one solution. However, in practice adding a small perturbation to one of the $\mathbf{R}_i$ matrices will change the problem into having only a rank one solution. An alternative approach to construct a rank one solution from any given solution is given in [5].

3.2 Additional Constraints

With the relaxation technique, it is easy to change the constraints or add more constraints. To illustrate the principle, we will show how to add constraints on the dynamic range of the power amplifier at each antenna element. The total transmitted power of element l of array k is

$$\mathrm{E}[|[\mathbf{x}_k]_l|^2] = \sum_{i \in \mathcal{I}(k)} |[\mathbf{w}_i]_l|^2 = \sum_{i \in \mathcal{I}(k)} [\mathbf{W}_i]_{ll}$$

Thus upper and lower bounds μ_{kl}^U and μ_{kl}^L, respectively, on the average power of an antenna element can be formulated by constraints of the form

$$\mu_{kl}^L \leq \sum_{i \in \mathcal{I}(k)} [\mathbf{W}_i]_{ll} \leq \mu_{kl}^U .$$

Similarly, we could set limits on the relative dynamic range of a single element in comparison to the total power for the whole array by replacing μ_{kl} with $\mu_{kl} \sum_{i \in \mathcal{I}(k)} \mathrm{Tr}[\mathbf{W}_i]$.

Since all these constraints are linear in the elements of $\mathbf{W}_i$, the resulting problem will still be semidefinite. However, we can not in general expect the solutions to have rank one. For system evaluations and simulations, the solution provides a lower bound for the problem, but a high rank solution can actually be implemented also in a practical system. If we allow for time varying beamformers $\mathbf{w}_i(t)$, $\mathbf{W}_i$ can be interpreted as the covariance matrix of the beamformer, $\mathbf{W}_i = \mathrm{E}[\mathbf{w}_i \mathbf{w}_i^*]$. Thus, one possible implementation is to use a random sequence of vectors with covariance $\mathbf{W}_i$, as a time varying beamformer. Compare to [14] where a similar interpretation of $\mathbf{W}_i$ is used to evaluate the usefulness of space-time coding for different scenarios.

3.3 Increased Robustness

A common problem in connection with optimal uplink beamforming is signal cancellation caused by estimation errors in the channel covariances [6]. We could expect similar problems in the downlink, so it is interesting to introduce a design strategy that is robust to small errors in $\mathbf{R}_{ik}$. Assume that the true channel has a covariance matrix $\hat{\mathbf{R}}_{ik} + \Delta_{ik}$, where $\hat{\mathbf{R}}_{ik}$ is the available estimate. The goal is to find $\mathbf{w}_i$ such that the SINR constraints in (5) hold for all choices of Δ_{ik} with $\|\Delta_{ik}\| \leq \epsilon_{ik}$ for some matrix norm $\|\cdot\|$. Since

$$\max_{\|\Delta\| \leq \epsilon} |\mathrm{Tr}[\mathbf{X}\Delta]| = \epsilon \|\mathbf{X}\|_* ,$$

where $\|\cdot\|_*$ is the dual norm of $\|\cdot\|$. The resulting problem can be formulated as

$$\begin{aligned}
&\min \sum_{i=1}^{d} \operatorname{Tr}[\mathbf{W}_i] \\
&\text{s.t. } \operatorname{Tr}[\hat{\mathbf{R}}_i \mathbf{W}_i] - \gamma_i \sum_{n \neq i} \operatorname{Tr}[\hat{\mathbf{R}}_{i,\kappa(n)} \mathbf{W}_n] \geq \gamma_i \sigma_i^2 + \epsilon_i \|\mathbf{W}_i\|_* + \gamma_i \sum_{n \neq i} \epsilon_{i,\kappa(n)} \|\mathbf{W}_n\|_*, \\
&\quad \mathbf{W}_i = \mathbf{W}_i^*, \\
&\quad \mathbf{W}_i \succeq 0, \; i = 1, \ldots, d\ .
\end{aligned} \tag{9}$$

If we use the spectral norm of Δ_{ik}, i.e. put a limit of the maximum eigenvalue $\lambda_{\max}(\Delta_{ik}) \leq \epsilon_{ik}$, then the dual norm is the absolute sum of eigenvalues $\|\mathbf{W}\|_* = \sum |\lambda_k(\mathbf{W})|$ and since all $\mathbf{W}_i$ are positive semidefinite, the optimal robust design is solved by the following semidefinite problem

$$\begin{aligned}
&\min \sum_{i=1}^{d} \operatorname{Tr}[\mathbf{W}_i] \\
&\text{s.t. } \operatorname{Tr}[\mathbf{R}_i \mathbf{W}_i] - \gamma_i \sum_{n \neq i} \operatorname{Tr}[\mathbf{R}_{i,\kappa(n)} \mathbf{W}_n] \geq \gamma_i \sigma_i^2 + \epsilon_i \operatorname{Tr}[\mathbf{W}_i] + \gamma_i \sum_{n \neq i} \epsilon_{i,\kappa(n)} \operatorname{Tr}[\mathbf{W}_n], \\
&\quad \mathbf{W}_i = \mathbf{W}_i^*, \\
&\quad \mathbf{W}_i \succeq 0, \; i = 1, \ldots, d\ .
\end{aligned} \tag{10}$$

3.4 Suboptimal Solutions

Even if an efficient algorithm is available for the joint design of all beamformers, a decentralized algorithm could be preferable in a running network to decrease the overhead communication between different base stations.

A heuristic approach is to determine each beamformer separately, keeping the received SNR at the mobile of interest above the threshold μ_i and the total transmitted power to the interfered users below the threshold ξ_i,

$$\mathbf{w}_i^* \Big(\sum_{n \neq i} \mathbf{R}_{n,\kappa(i)} \Big) \mathbf{w}_i \triangleq \mathbf{w}_i^* \mathbf{Q}_i \mathbf{w}_i \leq \xi_i\ .$$

Minimizing the transmitted power gives the following optimization problem,

$$\begin{aligned}
\mathbf{w}_i = \arg\min P_i &= \|\mathbf{w}_i\|^2 \\
\text{s.t. } \mathbf{w}_i^* \mathbf{R}_i \mathbf{w}_i &\geq \mu_i \sigma_i^2 \\
\mathbf{w}_i^* \mathbf{Q}_i \mathbf{w}_i &\leq \xi_i\ .
\end{aligned} \tag{11}$$

The same relaxation as in the previous sections gives the semidefinite optimization problem,

$$\begin{aligned}
\mathbf{W}_i = \arg\min & \operatorname{Tr}[\mathbf{W}_i] \\
\text{s.t. } & \operatorname{Tr}[\mathbf{W}_i \mathbf{R}_i] \geq \mu_i \sigma_i^2 \\
& \operatorname{Tr}[\mathbf{W}_i \mathbf{Q}_i] \leq \xi_i \\
& \mathbf{W}_i^* = \mathbf{W}_i \\
& \mathbf{W}_i \succeq 0\ .
\end{aligned}$$

Also for this problem it is possible to show that there always exists an optimal solution $\mathbf{W}_i$ of rank one.

A related strategy, which has been suggested in several references including [6, 8, 23], is to use

$$\mathbf{w}_i = \arg\max \frac{\mathbf{w}_i^* \mathbf{R}_i \mathbf{w}_i}{\mathbf{w}_i^* (\alpha \mathbf{I} + \sum_{n \neq k} \mathbf{R}_n) \mathbf{w}_i} , \tag{12}$$

where the optimum is given as the solution of a generalized eigenvalue problem. The parameter $\alpha \geq 0$ can be interpreted as a Lagrange multiplier of (11) and determines the trade-off between interference suppression and a low total transmission power [6].

Finally, we mention traditional beamforming,

$$\mathbf{w}_i = \arg\max \frac{\mathbf{w}_i^* \mathbf{R}_i \mathbf{w}_i}{\mathbf{w}_i^* \mathbf{w}_i} , \tag{13}$$

i.e., to use the principal eigenvector of $\mathbf{R}_i$.

4 Numerical Examples

We illustrate the performance of the suggested downlink algorithms in a simulated scenario with three users served by a single base station. One mobile is located at $\theta_1 = 10°$ relative array broadside and the two others at directions $\theta_{2,3} = 10° \pm \Delta$, where$\Delta$ is varied from 5°to 25°. The transmitting antenna array is linear and has $m = 8$ elements spaced half a wavelength. Each user is surrounded by a large number of local scatterers corresponding to a spread angle of $\sigma_\theta = 2°$, as seen from the base station. Thus, the channel covariance matrix is well approximated by [1, 18],

$$[\mathbf{R}(\theta, \sigma_\theta)]_{kl} = e^{j\pi(k-l)\sin\theta} e^{-\frac{(\pi(k-l)\sigma_\theta \cos\theta)^2}{2}} .$$

We compare the following four transmit strategies.

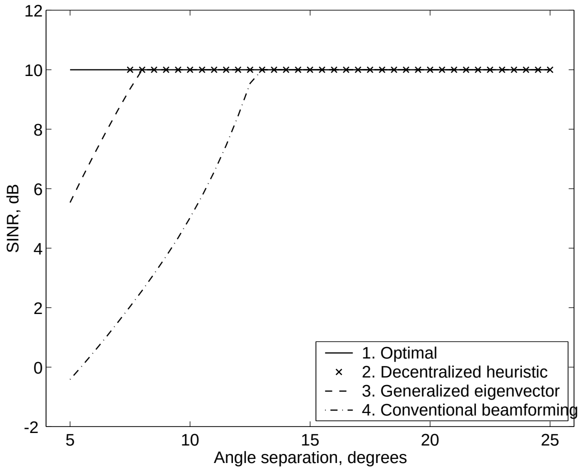
1. The joint optimal design according to Section 3.1.
2. The decentralized heuristic design given by (11).
3. The heuristic generalized eigenvector beamformer given by (12).
4. Traditional beamforming given by (13).

In all cases, the beamformers are scaled, if possible, such that $\text{SINR}_i = \gamma_i$ for all users. In the cases where no such scaling is possible, the scaling is chosen to maximize the worst SINR among the users, thus all users receive the same SINR in all the examples. The semidefinite problems were solved using a primal-dual interior point method [16].

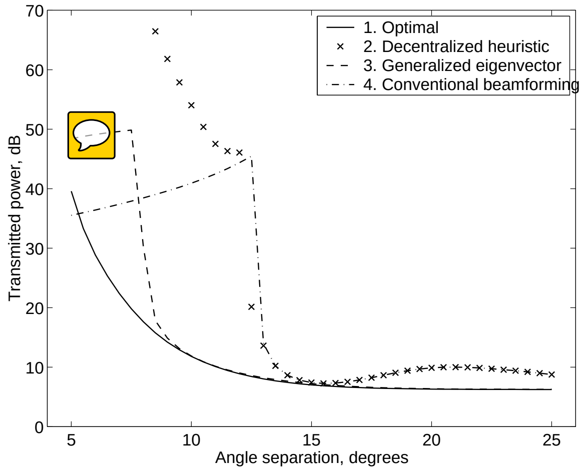
The SINR threshold was set to $\gamma_i = 10$dB and in (11), the thresholds were set to $\mu_i \sigma_i^2 / xi_i = 10$dB. In (12), the parameter α was arbitrarily set to

$$\alpha = \frac{0.1}{m} \operatorname{Tr}\Big[\sum_{n \neq k}^{d} \mathbf{R}_n\Big] , \tag{14}$$

10% of the gain level of the interfered channels.



1. Received signal to interference ratio at the middle user.



2. Total transmitted power, relative the noise level at each receiver.

Figure 1 shows the received SINR level for the users. The total transmitted power needed to achieve this level for all users is shown in Figure 2.

Because of the scaling used for the solutions, Figure 1 mainly shows when it is possible to achieve the desired SINR at all receivers using the different strategies. The decentralized solution in (11) did not produce any feasible solution when the source separation was $< 7°$.

As could be expected, the traditional beamformer performs worst, since it does not take the interfered users into account. However, when the users are sufficiently separated such that the traditional beamformer does give a feasible solution, then it coincides with the heuristic solution in (11), which thus can be seen as a generalization of the traditional beamforming. The performance of the generalized eigenvector solution in (12) depends on the choice of α, but in this example it performs almost as well as the optimal solution for the cases where it gives a feasible solution. In the difficult scenarios where the users are very closely separated, the cost, in terms of total transmitted power, is very large in order to obtain a feasible solution.

5 Conclusions

We have proposed strategies and algorithms for the design of beamformers that keep the total transmitted power at a minimum, still maintaining a certain level of quality. The technique can be applied both on systems with inter-cell channel reuse as well as systems with at most one co-channel user per cell.

The quality constraints are given in terms of the average signal to interference plus noise ratio received at each mobile. We have shown how the resulting optimization problem can be efficiently solved using standard tools for semidefinite optimization. This is a surprising result, since the original problem is non-linear and non-convex. It is also possible to show a similar result for the problem of joint power control and beamforming for the uplink.

Compared to the previously published algorithm [15] for the problem, the solution using convex optimization has guaranteed convergence speed and will quickly detect the infeasible situations where no solution can be found, but the main advantage is the flexibility offered in the choice of constraints. Also robust beamforming is easily incorporated. Even though the method will not always produce a normal time-invariant beamformer when extra constraints are added, the solution does still provide a useful benchmark for e.g. capacity studies.

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