Analysis of Frequency Diversity in MC-CDMA Systems
under Correlated Fading Channels
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Abstract
MC-CDMA can obtain a good frequency diversity effect through spreading and despreading operation in frequency domain. In this paper, the impact of channel parameters to inherent frequency diversity is analyzed at first. Then, three spreading methods to achieve the frequency diversity are investigated. Not only frequency diversity but also multiple user interference (MUI) are taken into account in our analysis. The achieved frequency diversity gains with different spreading methods are also affected by the combining schemes at the receivers when there are multiple users in the systems.

Keywords
OFDM; MC-CDMA; frequency diversity; MUI

I. INTRODUCTION
The amazing increase of Internet services and electrics commercial is a strong demand for higher data rate communication services not only in fixed network but also in the mobile communication systems. MC-CDMA, which is based on orthogonal frequency division multiplexing (OFDM), can not only solve the difficult multipath environment problem but also have good frequency efficiency and easy implementation [1]-[3].

In MC-CDMA systems, the original data stream is spread by using a given spreading code, and then modulated to a different sub-carrier with each chip, i.e., spreading in the frequency domain. Using such transmission scheme, MC-CDMA can achieve frequency Rake diversity effect. In order to obtain good spectral efficiency, the spacing between the subcarriers is kept as close as possible while maintaining the orthogonality between them. However, such close subcarrier spacing can’t guarantee the frequency diversity conceived in [3] because the subcarriers that carry on the same information are correlated, not independent. So how much diversity gain can be achieved highly depends on the channel correlation properties. Then, the spreading methods and the length of spreading sequence need to be selected carefully to exploit the frequency diversity order inherent in the channel. On the other hand, orthogonality between different users will be destroyed by the transmission through the fading channels. Multiple user interference (MUI) will deteriorate the system performance when multiple active users. The more frequency diversity obtained, the more sensitivity the systems are to MUI.

Keywords
OFDM; MC-CDMA; frequency diversity; MUI

II. SYSTEM MODEL
In MC-CDMA systems, the original signal of each user is spread by a special spreading code in frequency domain at the transmitter. In the downlink Walsh-Hadamard codes are used for the frequency spreading to achieve the orthogonality among users. The binary information data sequence of kth user is QPSK-modulated. And then the resulting symbol sequence is $P$ converted to $N/SF$ parallel sequences $\{a_{k0}[l], a_{k1}[l], \ldots, a_{kP}[l]\}$. Here $N$ is the number of the subcarriers and $SF$ is the value of the spreading factor, i.e., the length of the spreading sequence. Consequently, $P$ is the number of symbols transmitted at the same time by one user. Each branch of the parallel complex sequences is copied into $SF$ parallel sequences and is multiplied by a corresponding chip of the special spreading sequence. As shown in Fig.1 (a), in order to obtain the better frequency diversity effect, the interval of every chip of one spread sequence can be $P$ subcarriers. And this spreading method is called as Spreading A . Usually the data can also be spread onto the continuous subcarriers by the chips of one spread sequence showed in Fig.1 (b), which is called as Spreading B. Then, the parallel data sequences are modu-
lated by the inverse discrete Fourier transform (IDFT) and guard intervals are inserted between the OFDM symbols to avoid the ISI caused by multipath fading. Finally the parallel sequences are converted back into a serial data stream. Thus, the transmitted signal of the kth user can be expressed as

$$s_k(t) = \sum_{j=0}^{K-1} a_{k,p}[j] \sum_{n=0}^{N-1} c_k[n] p_i(t-jT_s)e^{j2\pi f_0(n,p)t}$$

where $$\Delta f^r = 1/(T_s' - \Delta)$$, $$T_s' + \Delta = PT_s$$, $$c_k[n]$$ is the nth chip of the k-th user's spreading code and $$f_0$$ is the lowest subcarrier frequency. $$T_s'$$, $$\Delta f^r$$ and $$\Delta$$ are the symbol duration at each subcarrier, the minimum subcarrier separation and the guard interval (GI), respectively. $$p_i(t)$$ is the rectangular pulse response defined on the interval $$(0, T_s')$$. For downlink transmissions, since a terminal receives its interfering signal designated for other users ($$k = 1, 2, \ldots, K-1$$) through the same channel as the desired signal, the user index for the channel can be omitted. The signal from the 0th user is assumed the desired without the loss of generality. The Rayleigh fading channel with L paths can be expressed as

$$h(t; \tau) = \sum_{j=0}^{L-1} h_j(t) \delta(\tau - \tau_j)$$

where $$h_j(t)$$ is the jth path gain which is independently complex Gaussian random process with zero mean and variance $$\sigma_j^2$$ for different s, and $$\tau_j$$ is the propagation delay for the jth path.

Transmitted through the Rayleigh fading channel with additive white Gaussian noise (AWGN), the received signal can be written as

$$r(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} H_{n,p}(t) a_{k,p}[i] c_k[n] p_i(t-jT_s)e^{j2\pi f_0(n,p)t} + n(t)$$

where K is the number of active users, $$H_{n,p}(t)$$ represents the complex envelop at the $$f(n,p)$$-th sub-channel. At the receiver, the inverse operation, which comprises S/P conversion, removal of guard interval and DFT (or FFT) are performed. After that, each received symbol of each subcarrier is multiplied by the gain factor, which is the combined signal of the corresponding chip of the spread sequence and the estimated channel gain associated with each subcarrier. They are then accumulated in the frequency domain to get

<table>
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<th>TABLE 1. System Parameters</th>
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<td><strong>Channel Model</strong></td>
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![Figure 2. Distribution of cumulated eigenvalues](image)

The decision variable. Without the loss of generality, the decision variable for the 0th user can be written as

$$D_0(iT_s') = \sum_{n=0}^{N-1} G_0(n) H_{n,p}(iT_s') a_{0,p}[i] c_0[n] + \sum_{n=0}^{N-1} G_0(n) H_{n,p}(iT_s')$$

Two different combining techniques are discussed: Maximal Ratio Combining (MRC) and Minimum Mean Square Error Combining (MMSEC). The gains are given by (6) and (7) respectively [2],

$$G_0^{\text{MRC}}(n) = c_0[n] H_{n,p}^*$$

$$G_0^{\text{MMSEC}}(n) = c_0[n] H_{n,p}^* [H_{n,p}^2 + \sigma_n^2]$$

**III. IMPACT OF CHANNEL FREQUENCY DIVERSITY**

The system parameters for our analysis and simulation [4] are shown in Table 1. The effect of frequency diversity depends on frequency correlation of the subcarriers on which the same information is transmitted. The path delay of the channel is assumed to be uniform distributed over $$(0, T_{\text{max}})$$ and the delay profile assumed to be exponential.
delay, i.e. $\theta(\tau) = Ce^{-\tau/\tau_{\text{m}}}$, The channel frequency correlation matrix $R_{hh} = E\{ HH^H \} = \{ \rho_{nn} \}$ can be expressed as

$$\rho_{nn} = \sum_{n=0}^{L-1} \sigma_n^2 e^{-j2\pi n r/n}$$

Since the eigenvalue $\lambda_j$ of $R_{hh}$ can represent the way by which the power is shared and spread among the subcarriers, their cumulated sum is a good criterion to show the power distribution among the subcarriers

$$P(i) = \sum_{j=1}^{i} \lambda_j$$

To highlight the different frequency diversity gain in the different channel, the channels with different parameters are analyzed by mean of SVD. As shown in Fig.2, the cumulated eigenvalue sum in Channel A is increased very rapidly with the eigenvalue index, which means most of transmitted signal power concentrated on few subcarriers and not much frequency diversity gain can be achieved. On the other hand, the cumulated eigenvalue sum in Channel D or E becomes close to 1 when the eigenvalue index increases more slowly, and more frequency diversity gain is inherent.

IV. FREQUENCY DIVERSITY VS SPREADING

Single User (SU)

Frequency diversity order inherent in the channel can't be exploited easily. It depends not only on the spreading methods but also on the length of spreading sequence. In order to further exploit inherent high diversity available in Channel D and Channel E, a frequency hopping (FH) scheme, which enables to map chips related to the same copy of one symbol onto randomly spaced subcarriers, is used together with Spreading A and Spreading B mentioned above.

Since MRC scheme is optimal for the single-user receiver, the single-user systems with MRC, i.e. no MUI are considered at first and Fig.3 shows the system performance applying FH spreading method with various spreading factors at target BER=10$^{-3}$ under different channels. As spreading factor increases, the required mean SNR is decreased significantly. However, after spreading factor becomes larger than 32, the system performance change slightly. It indicates that with spreading factor increasing, the system can achieve certain diversity, but the diversity will eventually saturate depending on the channel environments. And with the same spreading factor, the required SNR in Channel D and Channel E is smallest, which is consistent with the fact that Channel D and Channel E have the highest frequency diversity order.

Since the frequency diversity of Channel D can be achieved more easily, the analysis followed will be based on it. The performance applying FH is only regarded as reference because FH scheme is lack of practicability. Then, the system performance with practical Spreading A and Spreading B in Channel D have to be further investigated and shown in Fig.4. The performance applying Spreading A is much better than that applying Spreading B, and almost as same as that applying FH which can achieve largest frequency diversity gain. The performance gain between Spreading A and Spreading B is increased slightly after the spreading factor becomes larger than 16 since the frequency diversity has been close to its saturation point. Furthermore, the length of spreading factor won’t be too large and usually is selected to be 16 or 32. Obviously, in order to make good use of frequency diversity, Spreading A will be a good choice when no MUI.

Multi-User (MU)

From the analysis and simulation results above, Spreading A seems to be a good choice when single active user in the systems. However, in MC-CDMA systems, MUI usually exists and should be considered when system design. More
active users, more serious MUI is. So the system applying Spreading A won't be always better than that applying Spreading B. In the other words, the negative effect of MUI will become more dominant with more active users. Multi-user interference (MUI) is taken into account with the simulation of different number of active users at a fixed SNR=8dB. The spreading factor of 4, 8 or 16 is adopted in the systems respectively. On the other hand, the system performance of multiple active users also highly depends on the combining schemes at the receiver.

Fig.5 shows the performance in term of BER versus the system load, in which MRC is applied. From the second term of the equation (5), i.e. the multi-user interference, it can be seen that the system will suffer from the less multi-user interference when the correlation of the subcarriers related to the same copy of one symbol is higher if MRC is applied. The continuous subcarriers related to the same copy of one symbol in the system applying Spreading B have relatively stronger correlation than the discontinuous subcarriers in the system applying Spreading A. So the performance of system applying Spreading B is less sensitivity to MUI. If Spreading B is adopted, the BER performance is degraded slightly with the number of active users increases. On the other hand, the BER performance with Spreading A is deteriorated rapidly with more active users since it will introduce more multi-user interference than Spreading B. When spreading factor (SF) is small (4 or 8), the BER curves with Spreading A and Spreading B are crossed at the point with about less than half system load. The BER performance with Spreading A is even worse than that with Spreading B when full system load. Although the BER performance with Spreading A is much better than that with Spreading B in single-user systems, it is deteriorated too rapidly and become worse than with Spreading B in the systems that have heavier load. If MUI is considered, Spreading A is no longer an appropriate spreading method for the system with MRC.

Not only the noise power but also the multi-user interference is taken into account when MMSEC is applied at the receiver. Since it can restore the orthogonality of the different users signals approximately, MMSEC shows its robustness to MUI. Meanwhile, when Spreading A is applied in the system with MMSEC, frequency diversity can be exploited well even in multi-user systems. Fig.6 shows the performance in term of BER versus the system load, in which MMSEC is applied. The BER performance with Spreading A is always better that with Spreading B no matter how many active users is in the systems.

V. CONCLUSION

In this paper, three different spreading methods (Spreading A, Spreading B and FH) in MC-CDMA system are investigated. Considering the practicability of the system, Spreading A is an appropriate spreading method in a single user case. When the number of active users increases, the system performance will be deteriorated due to the increasing

MUI, the dominant factor in CDMA systems. For the purpose of simplification and practicability, MRC with Spreading B may be a good choice when system load is not heavy. If more active users were, MMSEC with Spreading A outperforms the others.

REFERENCES


