DFT-BASED CHANNEL ESTIMATION IN COMB-TYPE PILOT-AIDED OFDM SYSTEMS WITH VIRTUAL CARRIERS

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ABSTRACT
In Orthogonal Frequency Division Multiplexing (OFDM) systems, conventional channel estimation techniques using comb-type preambles and interpolation between pilot subcarriers give relatively large mean square errors (MSEs) at the edge subcarriers. Furthermore, the spacing between pilot subcarriers in the frequency domain has to be close enough according to sampling theorem. To solve these problems, an iterative discrete Fourier transform (DFT)-based channel estimation with comb-type pilot-aided is proposed in this paper. The complete frequency channel responses (CFRs) including virtual subcarrier positions are estimated by recursive method with the small size of DFT/Inverse DFT. Simulation results show that the proposed algorithm outperforms the conventional Linear Minimum Mean Square Error (LMMSE) algorithm using comb-type preamble.

I INTRODUCTION
OFDM has been successfully applied to a wide variety of wireless communication systems such as Wireless LAN and 3G long-term evolution system due to its capability to effectively combat inter-symbol interference (ISI), and its high spectral efficiency achieved by spectrum overlapping [1][2].

In an OFDM system, the receiver should know the frequency response of the fading channel to achieve the coherent signal detection. Usually the channel estimation can be performed by either inserting pilot tones into all of the subcarriers of OFDM symbols with a specific period or inserting pilot tones into each OFDM symbol [3]. The first one, the block type pilot channel estimation, has been developed under the assumption of slow fading channel. The estimation of the channel for this block type pilot arrangement can be based on Least Square (LS) or Linear Minimum Mean-Square (LMMSE). Also, DFT-based channel estimation algorithms have been widely studied because of their low complexity and good performance [4][5]. The later, the comb-type pilot channel estimation, has been introduced to satisfy the need for equalizing when the channel rapid changes even in one OFDM block. The conventional comb-type pilot channel estimation consists of algorithms to estimate the channel at pilot subcarriers and to interpolate the channel between pilot subcarriers. The estimation of the channel at the pilot subcarriers for comb-type based channel estimation can be also based on LS or LMMSE. However, due to the interpolation in the frequency domain, this method may cause relatively large mean squared error (MSE) at the edge subcarriers [4] and the spacing between pilot subcarriers has to be close enough according to sampling theorem [6]. To avoid these problems, DFT-based interpolation can be applied in comb-type pilot-aided OFDM systems.

Furthermore, in practical OFDM systems, direct current (DC) subcarrier is not used due to DC offset problem and subcarriers at high frequencies keep empty to avoid adjacent channel interference as virtual subcarriers. It means that the channel estimate on the frequency domain cannot be determined for all subcarriers. Thus, when the channel estimation algorithm based on DFT is applied, the channel energy will spread in the time domain if this incomplete CFR estimate is transformed to the time domain by IDFT. So the iterative procedure with block-type pilot is proposed to solve this problem [7].

In this paper, a novel DFT-based channel estimation method with comb-type pilot is proposed with the iterative procedure. In this method, the noisy channel frequency-domain estimates at pilot subcarriers are transformed to the time domain by IDFT, and then the time-domain estimates are transformed back to the frequency domain by a DFT after appropriately being processed. This procedure will be performed iteratively in order to eliminate the energy spread in the time domain, where the size of DFT/IDFT only equals to the number of pilot positions including the corresponding virtual subcarriers positions. After getting the convergent channel impulse response (CIR) estimates in the time domain, the complete CFR at all the subcarriers is obtained by transforming interpolated CIR into the frequency domain with the full size of DFT/IDFT. The simulation results demonstrate the good performance of the proposed method.

The reminder of this paper is organized as follows. After a brief description of the system model in Section II, the conventional channel estimator using comb-type pilot is described in Section III and an iterative DFT-based channel estimator presented in Section IV respectively. The simulation results are given in Section V. Finally our conclusion is drawn.

II SYSTEM MODEL
Firstly, the binary information data are grouped and mapped into multi-amplitude-multi-phase signals according to the modulation method. After inserting pilot either to all subcarriers with a specific period or uniformly between the information data sequence, the data sequence \( \{X_k\} \) is divided into \( N \) parallel substreams first. Then, these substreams are modulated by IDFT with \( N_c \geq N \) points onto \( N \) subcarriers with the follow-
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\[
x(n) = IDFT\{X_k\} \quad n = 0, 1, \ldots, N - 1
\]

\[
x(n) = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c-1} X_ke^{j2\pi kn/N_c}
\]

Subsequently a cyclic prefix (CP) of length \(N_g\) is added to avoid ISI and inter-carrier-interference (ICI) before the signal is transmitted over a multipath fading channel. The resultant OFDM symbol is given as follows:

\[
x_g(n) = \begin{cases} x(N_c + n) & n = -N_g, -N_g + 1, \ldots, -1 \\ x(n) & n = 0, 1, \ldots, N_c - 1 \end{cases}
\]

A frequency-selective Rayleigh fading channel with \(L\) non-zero taps is considered. The channel impulse response remains unchanged during at least one OFDM symbol and it can be expressed as

\[
h(n) = \sum_{l=0}^{L-1} h_l \delta(\lambda - \tau_l)
\]

where the \(l\)th tap gain \(h_l\) with propagation delay \(\tau_l\) is a complex Gaussian random variable with zero mean and variance of \(\sigma^2\), \(\lambda\) is the delay spread index.

The received signal can be represented by

\[
y_g(n) = x_g(n) \bigotimes h(n) + w(n)
\]

where \(w(n)\) is the additive white Gaussian noise (AWGN) with zero mean and variance of \(\sigma^2\). The cyclic prefix is removed first and the resultant signal \(y(n)\) is transformed to the frequency domain by the DFT. Thus, the received signal on the \(k\)th subcarrier can be expressed as

\[
Y_k = DFT\{y(n)\}
\]

\[
= \frac{1}{\sqrt{N_c}} \sum_{n=0}^{N_c-1} y(n)e^{-j2\pi kn/N_c}
\]

Assuming that the length of CP is longer than that of channel impulse response, that is, there is no inter-symbol-interference between OFDM symbols. Therefore, we can rewrite (5) as

\[
Y_k = X_k H_k + W_k
\]

where

\[
H_k = \sum_{l=0}^{L-1} h_le^{-j2\pi \tau_l k/N_c}
\]

\[
W_k = \frac{1}{\sqrt{N_c}} \sum_{n=0}^{N_c-1} w(n)e^{-j2\pi kn/N_c}
\]

III CONVENTIONAL CHANNEL ESTIMATION IN COMB-TYPE PILOT-AIDED OFDM

A Structure of Comb-type Pilot

In an OFDM system, the comb-type pilot is a well-known training structure for channel estimation. In this comb-type training structure, only part of the subcarriers in one OFDM symbol is allocated for channel estimation. For the sake of description, it is assumed that \(N = N_c\), i.e., no virtual subcarrier is considered firstly. Specifically, the pilot signals are mapped at subcarriers following the equi-spaced position set:

\[
\Omega_{eq} \equiv \{0, N_f, 2N_f, \ldots, (M_c - 1)N_f\}
\]

where \(M_c = N_c/N_f\) is the number of the pilot subcarriers and the pilot spacing \(N_f\) in the frequency domain has to satisfy the limitation[6]:

\[
N_f \leq \frac{1}{2\tau_{max}}
\]

where \(\tau_{max}\) is the maximum excess delay of the channel and \(\Delta f = \frac{1}{T_c}\) the subcarrier bandwidth. Here \(T_c\) is the sample period. So on the other words, the number of pilot subcarriers should meet the following equation:

\[
M_c \geq \frac{2\tau_{max}}{T_c}
\]

In practical, the virtual subcarriers at the high frequency of the spectrum are not used to avoid aliasing problems at the receiver and the DC subcarrier is not used to avoid intermodulation effects and difficulties in D/A and A/D conversion. In other words, the subcarriers that are the components of the position set \(\Omega_{VC}\) are not used for transmission where \(\Omega_{VC}\) is defined as follows:

\[
\Omega_{VC} = \{\overbrace{0, N/2 + 1, N/2 + 2, \ldots, N_c - 1 - N/2}^{\text{High Frequency}}\}
\]

Thus the number of used subcarriers is less than the DFT size, i.e. \(N < N_c\), due to existence of virtual subcarriers. Therefore, we have to allocate the pilot signal to the subcarriers which belong to \(\Omega_{eq}\) but are not either DC or high frequencies. In other words, the allocated subcarriers which belongs to the position set \((\Omega_{eq} \cap \Omega_{VC})\) with the size of \(M = N/N_f\) are employed for transmitting the pilot signal. Here the position set \((\Omega_{eq} \cap \Omega_{VC})\) means the set in which the subcarriers belong to \(\Omega_{eq}\) but not to \(\Omega_{VC}\).

B LMMSE algorithm with interpolation

Following the DFT block, the received pilot signals are extracted from \(\{Y_k\}\) and can be conveniently expressed in vector notation as

\[
\hat{\mathbf{Y}} = \mathbf{X}\hat{\mathbf{H}} + \mathbf{W}
\]

where the transmitted pilot signal vector, the channel frequency response vector, and AWGN term are given by

\[
\hat{\mathbf{X}} = \text{diag}\{X_1 X_{N_f} \cdots X_k \cdots X_{(M_c-1)N_f}\} \in \mathbb{C}^{M_c \times M_c}
\]

\[
\hat{\mathbf{H}} = [H_1 H_{N_f} \cdots H_{k} H_{(M_c-1)N_f}]^T \in \mathbb{C}^{M_c \times 1}
\]

\[
\mathbf{W} = [W_1 W_{N_f} \cdots W_k \cdots W_{(M_c-1)N_f}]^T \in \mathbb{C}^{M_c \times 1}
\]

with \(k \in (\Omega_{eq} \cap \Omega_{VC})\)
Then, the channel estimate at its pilot position set, based on Least Squares (LS) criterion, is given by

\[ \hat{H}_{ls} = \hat{X}^{-1} \hat{Y} \in \mathbb{C}^{M_c \times 1} \]

\[ = [Y_1 Y_{N_f} Y_{2N_f} \ldots Y_{(M_c-1)N_f}]^T \]  \hspace{1cm} (13)

The LS channel estimates are obtained by using only the knowledge of the pilot signals, which can be further improved by making use of frequency domain correlation of the multipath channel. We then obtain the LMMSE channel estimates as follows [8]:

\[ \hat{H}_{mmse} = R_{\hat{H}\hat{H}}(R_{\hat{H}\hat{H}} + \frac{\beta}{\gamma} I)^{-1} \hat{H}_{ls} \]  \hspace{1cm} (14)

where \( \gamma \) is the signal to noise ratio of the pilot sequences, and \( \beta \) is a constant depending on the pilot sequence’s constellation. For MPSK transmission, \( \beta = 1 \). \( R_{\hat{H}\hat{H}} = E(\hat{H}\hat{H}^H) \) is the channel autocorrelation matrix and its elements can be given by

\[ r_{k_1,k_2} = \sum_{l=0}^{L-1} \sigma^2 e^{-j2\pi \tau_l (k_1-k_2)/N_c} \]  \hspace{1cm} (15)

After the estimate of CFR at pilot position set, the CFR at other positions can be interpolated according to adjacent pilot positions. In this paper, we only consider a piecewise second-order polynomial interpolation method which due to its inherent simplicity is easy to be implemented. The system performance is highly dependent on the rigorosity of estimate at pilot position set and the accuracy of interpolation.

IV DFT-BASED CHANNEL ESTIMATION FOR COMB-TYPE PILOT-AIDED OFDM

The DFT-based channel estimation approaches are promising because of their low complexity and good performance. The noisy channel frequency-domain estimates are transformed to the time domain by an IDFT, and then the time-domain estimates are transformed back to the frequency domain by a DFT after appropriately being processed in order to reduce noise effects. When the pilot signals are uniformly inserted, the number of pilot subcarriers in comb-type pilot structure for DFT-based algorithms has to be no less than the sampled maximum excess delay of the channel:

\[ M_c \geq \frac{\tau_{\text{max}}}{T_c} \]  \hspace{1cm} (16)

It can be seen that the number of pilot subcarriers required by using DFT-based algorithms is less than that shown in (9). Usually \( M_c \) is chosen to be the power of two for the feasibility of DFT/IDFT implementation.

Due to the implementation issue, the subcarriers that are the components of the position set \( \Omega_{VC} \) are not used for transmission. It means that the channel estimate on the frequency domain cannot be determined for all \( M_c \) subcarriers. If we simply assign arbitrary values to the channel estimates at virtual subcarriers (e.g. make them "0"), it means to create a significant channel energy spread in the time domain, which makes it impossible to select the significant taps only once without distorting original CIR. So an iterative algorithm is proposed to solve this energy spread problem and makes the DFT-based channel estimation more practical[7]. However, Belotserkovsky’s algorithm is only based on the block-type pilot, which needs much overhead and computation complexity. Our proposed algorithms is adopted with comb-type pilot of less overhead and smaller size of DFT/FFT transform, which means less computation complexity is necessary.

The DFT-based channel estimation with comb-type pilot proceeds as following:

Step 1: The LS estimates with size of \( M_c \) are expanded by adding \( (M_c-M) \) zeros in the corresponding virtual subcarrier position set to generate the initial length-\( M_c \) channel estimate \( \hat{G}_k \) in the frequency domain, which can be expressed as

\[ \hat{G}_k = \begin{cases} \frac{Y_k}{X_k}, & \forall k \in (\Omega_{eq} \cap \Omega_{VC}) \\ 0, & \forall k \in (\Omega_{eq} \cap \Omega_{VC}) \end{cases} \]  \hspace{1cm} (17)

Step 2: Transform the LS estimates into the time domain by \( M_c \)-point IDFT and get the noisy CIR estimate in the time domain:

\[ \hat{g}(n) = \frac{1}{\sqrt{M_c}} \sum_{k=0}^{M_c-1} \hat{G}_k e^{j2\pi kn/M_c}, \forall n \in \{0, 1, \ldots, M_c - 1\} \]  \hspace{1cm} (18)

Step 3: Post-processing in the time domain is applied to reduce the noise effect. The most channel power concentrates on only small parts of the estimated time-domain samples that are transformed by IDFT. A straightforward way is to ignore the coefficients, called as the non-significant taps, which contain more noise than channel power and only transform the remaining coefficients, called as the significant taps, back to the frequency domain. Neglecting those non-significant channel taps will lead to some performance degradation since the channel power in those taps is lost. However, the noise perturbation to channel estimation can be also eliminated well at the same time. Usually the noise perturbation from those neglected channel estimated taps is much higher than the multipath energy contained in them, especially for low SNR values. Therefore, the positive effects of neglecting the non-significant channel estimate taps are dominant and the performance of channel estimation will be improved significantly. Firstly, the noisy estimated CIR \( \hat{g}(n) \) will be truncated by the "brick wall" window in the time domain,

\[ \hat{g}(n) = \begin{cases} \hat{g}(n), & \forall n \in \{0, 1, \ldots, Q\} \\ 0, & \forall n \in \{Q + 1, \ldots, M_c - 1\} \end{cases} \]  \hspace{1cm} (19)

Usually the window length \( Q \geq L \) can be set as the sampled maximum excess delay of the channel. Secondly, the estimated CIR after post-processing, \( \hat{g}(n), 0 \leq n \leq M_c - 1 \), is generated by selecting only \( L \) significant taps with maximum power from \( \hat{g}(n) \) and the rest \( (M_c - L) \) samples are zeroed,

\[ \hat{g}'(n) = \begin{cases} \hat{g}(n), & \forall n \in \Psi_L \\ 0, & \forall n \notin \Psi_L \end{cases} \]  \hspace{1cm} (20)
where $\Psi_L$ is the position set of taps with $L$ maximum power. If at the last iteration, the estimated new CIR will be used to generate the complete CFR.

Step 4: Take $M_c$-point DFT of the estimated CIR after post-processing to obtain the CFR estimates of length-$M_c$.

$$\hat{G}_k = \frac{1}{\sqrt{M_c}} \sum_{n=0}^{M_c-1} \hat{g}(n)e^{-j2\pi kn/M_c}, \forall k \in \{0, 1, \ldots, M_c-1\}$$

Step 5: The estimated CFR of length-$M_c$ is reconstructed by keeping the value for $k \in (\Omega_{eq} \cap \Omega_{VC})$ and replacing the values for $k \in (\Omega_{eq} \cap \Omega_{VC})$ with those obtained in Step 1. Then, (17) is modified as:

$$\tilde{G}_k = \begin{cases} Y_k/X_k, & \forall k \in (\Omega_{eq} \cap \Omega_{VC}) \\ \hat{G}_k, & \forall k \in (\Omega_{eq} \cap \Omega_{VC}) \end{cases}$$

Step 2 through Step 5 are repeated as necessary to eliminate the energy spread in the time domain. Since only the length-$M_c$ signal are processed during the iterative period, the computation complexity is low.

Step 6: When the iterative procedure is finished, the CIR estimates in Step 3 is interpolated by transforming the $M_c$-point into $N_c$-point with the following method according to the basic multi-rate signal processing properties:

$$\hat{g}'_{N_c}(n) = \begin{cases} \hat{g}(n), & 0 \leq n < \frac{M_c}{2} - 1 \\ 0, & \frac{M_c}{2} \leq n < N - \frac{M_c}{2} - 1 \\ \hat{g}(n - N_c + M_c), & N_c - \frac{M_c}{2} - 1 \leq n < N_c - 1 \end{cases}$$

The estimate of the CFR at all the subcarriers is obtained by:

$$\tilde{H}'_k = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c-1} \hat{g}'_{N_c}(n)e^{-j2\pi kn/N_c}, \forall k \in \{0, 1, \ldots, N_c-1\}$$

V RESULTS AND DISCUSSIONS

The performance of the proposed iterative algorithm is evaluated by simulation and discussed in this section. The main system parameters are shown in Table I [2]. A radio frame with duration of 0.5ms includes 7 OFDM symbols. It has 75 (i.e. $M = 75$) pilot subcarriers in the first OFDM symbol and 600 data subcarriers in the later OFDM symbols of each frame. The size of DFT/IDFT for iterative channel estimation is set to be 128, i.e. $M_c = 128$. The delay profiles of the multipath fading channels are according to ITU document [9], i.e. PB channel with the speed of 3km per hour. The independent fading paths are generated using Jakes Model having a U-shape Doppler power spectrum.

Fig.1 shows the performance in term of MSE versus the number of iteration at a fixed SNR of 10dB under the PB3 channel environment. It can be observed that the MSE performance is improved when the number of iteration increases. However, the slope of the curves becomes much smaller when the number of iteration is larger than 10. Furthermore, a MSE error floor will be reached if the number is higher than 20. Therefore, the number of iteration can be selected to be around 20 due to the tradeoff between the complexity and performance and kept in the later simulations.

In Fig.2, we compare the MSE performances with different channel estimation algorithms under PB3 channel. The results of LS and LMMSE algorithms based on comb-type pilot and interpolation are also given for comparison. It is clear that the MSE performance of the proposed DFT-based algorithms is much better than that of LS algorithm. The edge effect of interpolation is obvious and causes the MSE performance floor in case of LMMSE algorithm with interpolation when the SNR increases. On the other hand, the MSE performance of the proposed DFT-based algorithm is little worse than that of LMMSE when the operating SNR is less than 12dB and becomes better after the operating SNR becomes larger. It is because the negative effect of noise with DFT-based algorithm in case of lower SNR is more obvious than that of interpolation with LMMSE.

In Fig.3, the Bit Error Rate (BER) performances with different channel estimation algorithms under PB3 channel are compared. When SNR is lower than 12dB, the performances with LMMSE and DFT channel estimation algorithms are almost
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Figure 2: Averaged MSE of channel estimation using the conventional LS/LMMSE and iterative DFT-based algorithms under PB3 Channel

same because the effect of addition noise to BER performance is more serious than that of the channel estimation error. With the increased SNR, the performance gain of the proposed DFT-based algorithm is about 0.5 dB compared with the results of LS and LMMSE algorithms if the target PER is assumed to be $10^{-4}$.

VI  CONCLUSION

In this paper, we propose an iterative DFT-based channel estimation algorithm for OFDM systems. In this method, the comb-type pilot is used for channel estimation due to its efficiency. The iterative procedure with small size of DFT/IDFT is applied in order to get all the completed channel frequency responses when the virtual subcarriers exist in the systems. From simulation results, we can find that the performance of the proposed iterative DFT-based channel estimator is better than the conventional LS or LMMSE algorithm using comb-type pilot and less constraint.

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