Abstract—Hybrid automatic-repeat-request(HARQ) scheme can be incorporated with the linear precoder in multi-input multi-output(MIMO) transmission to ensure highly reliable communications. To fully utilize Type-I HARQ diversity gain especially in the slow fading channels, we propose the optimal design principle of linear precoders, whose column vectors are orthogonal to each other correspondingly. Simulation results demonstrate the effectiveness of these precoders in reducing the detection bit-error rate and increasing the unified throughput.

I. INTRODUCTION

MIMO technology can efficiently increase the system capacity in rich scattering environments without increasing the bandwidth or transmitted power. However, to exploit the benefits offered by MIMO channels requires choosing a space-time processing scheme such as precoder and receiver algorithm with the tradeoff between the performance and complexity. The optimal linear precoder schemes [1]-[5] under various criteria have been extensively studied for the single transmission, which are assumed to know the full channel state information(CSI) at the transmitter [1][2], the first-order statistics[3], or second-order statistics of the channel [4]. In the practical systems especially in case of frequency-division duplex (FDD), this assumption is not easy to meet because channel coefficients need to be quantized at the receiver and sent back to the transmitter over limited bandwidth control channels regularly and successfully.

In [6], the solutions to the problem of precoder for spatial multiplexing systems with limited feedback overhead have been proposed. The essential idea is that the transmit precoder is chosen from a finite set of precoder matrices, called as the codebook known to both the receiver and the transmitter. The receiver chooses the optimal precoder from the codebook as a function of the current CSI and sends the binary index of this codebook to the transmitter over limited feedback channel.

Furthermore, in many wireless communication systems with appropriate feedback link, an ARQ mechanism can be incorporated in MIMO systems where packet retransmissions are requested if the detection of previous transmissions end in error. The performances of different combining schemes for Type-I hybrid ARQ MIMO retransmission have been analyzed and compared in [7][8]. However, joint detection of the transmitted vector from its multiple transmission is necessary in order to further improve transmission reliability.

In this paper, we propose the design principle for the optimal precoding matrix of MIMO-HARQ retransmission to fully utilize HARQ diversity gain especially in the slow fading channels. The precoding matrix for the first transmission can be selected according to the different optimization criteria. With the proposed principle, the column vectors in each precoding matrix for the (re)transmission should be orthogonal to each other correspondingly. For the practical purpose, the codebook is adopted in our design and only the index of the selected codeword is necessary feedback from the receiver to the transmitter.

This paper is organized as follows. Section II gives the brief description of a MIMO-HARQ system. The design principle for precoder in MIMO-HARQ systems is described in Section III. Section IV gives the practical solution using the proposed design principle in MIMO-HARQ systems. In Section V, the simulation results are presented and discussed. Finally, Section VI gives the conclusion.

II. SYSTEM MODEL

A flat fading MIMO wireless communication system under consideration uses $N_T$ transmit antennas and $N_R$ receive antennas. In general, we assume that $N_T \leq N_R$ to guarantee the symbol recoverability in presence of linear detection and single transmission. In this paper, the first transmission refers to the original transmission before a repeat request. It is assumed that the $N_T$-dimensional signal vector $s_i = [s_{i1}, s_{i2}, \ldots, s_{iN_T}]^H$ will be transmitted during the $i$th transmission. As in [5], we consider a transmitter that utilizes a linear precoding matrix. The baseband received $N_R$-vector signal at the $i$th transmission can be written as follows:

$$r_i = H_{(i)}F_{(i)}s_i + w_i = [r_{(i)1}, r_{(i)2}, \ldots, r_{(i)N_R}]^H$$

where $H_{(i)}$ is the $N_R \times N_T$ channel matrix experienced by the data at the $i$th transmission, $F_{(i)}$ is the $N_T \times N_T$ precoding matrix for the $i$th transmission, and $w_i \sim \mathcal{CN}(0, \sigma^2I_{N_R})$ is the independent identically distributed (i.i.d.) Gaussian noise vector associated with the $i$th transmission.

At the (integrated) receiver, after $M$ ARQ transmission of the symbol vector $s_{(i)}, 1 \leq i \leq M$, the overall received signal can be expressed as

$$r = [r_{(1)}, r_{(2)}, \ldots, r_{(M)}]$$

$$= [H_{(1)}F_{(1)}s_{(1)}, H_{(2)}F_{(2)}s_{(2)}, \ldots, H_{(M)}F_{(M)}s_{(M)}] + [w_{(1)}, w_{(2)}, \ldots, w_{(M)}]$$
Notice that the number $M$ is not predetermined as it depends on the successes of the receiver given previous (re)transmission.

Not only the linear detection including zero-forcing (ZF) and minimum mean-square error (MMSE) but also the optimal maximum likelihood (ML) detection can be used to provide reliable soft symbol decision for $s$ [9]. As discussed in [8], the cumulative combining in MIMO-HARQ system can be performed before or after the linear detection, which is termed as the pre-combining or post-combining scheme.

### III. Optimal Precoder for MIMO-HARQ

The optimal linear precoding schemes under various criteria have been extensively studied for the single transmission including maximization of information rate, maximization of the signal-to-noise ratio (SNR) and minimization of the mean squared error and minimization of the bit error probability for zero-forcing equalization. Similarly, these schemes can be applied for the first transmission in MIMO-HARQ systems.

If the same symbol vector $s_i = s$ is transmitted $M$ times due to ($M$-1) repeat requests, i.e. HARQ Type-I, the symbol-level pre-combining scheme can be utilized at the receiver in order to fully achieve the diversity gain under fast-fading channel. However, in the slow fading or quasi-static channel model, the MIMO channel matrix is assumed to be constant during each individual transmission, i.e. $H_{(i)} = H$, where no diversity gain achieved by combining. Therefore, we have to artificially introduce the diversity in quasi-static channels by using the different precoders during the (re)transmission. Then, the overall received signal in (2) can be rewritten as

$$ r = [r_{(1)}, r_{(2)}, \ldots, r_{(M)}] $$

$$ = H[F_{(1)}s, F_{(2)}s, \ldots, F_{(M)}s] + [w_{(1)}, w_{(2)}, \ldots, w_{(M)}] $$

(3)

In general, the multiple transmission of the same symbol vector can be regarded as the process of space-time coding, i.e. the original $N_T$-dimensional signal vector $s$ is encoded onto the $N_T \times M$ codeword of

$$ c = [F_{(1)}s, F_{(2)}s, \ldots, F_{(M)}s] = 
\begin{bmatrix}
  e_{(1)}^{1} & e_{(1)}^{2} & \cdots & e_{(1)}^{M} \\
  e_{(2)}^{1} & e_{(2)}^{2} & \cdots & e_{(2)}^{M} \\
  \vdots & \vdots & \ddots & \vdots \\
  e_{(N_T)}^{1} & e_{(N_T)}^{2} & \cdots & e_{(N_T)}^{M}
\end{bmatrix} $$

(4)

where $e_{(i)}^{n}$ is the transmitted symbol at the $n$th transmit antenna during the $i$th transmission.

We consider the probability that a maximum-likelihood receiver decides erroneously in the favor of a signal

$$ e = 
\begin{bmatrix}
e_{(1)}^{1} & e_{(2)}^{1} & \cdots & e_{(M)}^{1} \\
e_{(1)}^{2} & e_{(2)}^{2} & \cdots & e_{(M)}^{2} \\
\vdots & \vdots & \ddots & \vdots \\
e_{(N_T)}^{1} & e_{(N_T)}^{2} & \cdots & e_{(N_T)}^{M}
\end{bmatrix} \in \mathbb{C}^{N_T \times M} $$

(5)

assuming that $c$ was transmitted, where $e_{(i)}^{n}$ is the decoded symbol corresponding to the transmitted symbol $c_{(i)}^{n}$.

Then, on the assumption of ideal channel state information, the probability of transmitting $c$ and deciding in favor of $e$ at the decoder is well approximated by

$$ P(c \rightarrow e|H) \leq \exp(-d^2(c, e)E_s/4N_0) $$

(6)

where $N_0/2$ is the noise variance per dimension, $E_s$ is the average symbol energy of the transmitted signal and $d^2(c, e)$ is the distance between $c$ and $e$.

According to the design principle of space-time coding in [10], we need firstly to define the erroneous $N_T \times M$ matrix between $c$ and $e$ as

$$ B(c, e) = c - e $$

$$ = 
\begin{bmatrix}
  c_{(1)}^{1} - e_{(1)}^{1} & c_{(1)}^{2} - e_{(1)}^{2} & \cdots & c_{(1)}^{M} - e_{(1)}^{M} \\
  c_{(2)}^{1} - e_{(2)}^{1} & c_{(2)}^{2} - e_{(2)}^{2} & \cdots & c_{(2)}^{M} - e_{(2)}^{M} \\
  \vdots & \vdots & \ddots & \vdots \\
  c_{(N_T)}^{1} - e_{(N_T)}^{1} & c_{(N_T)}^{2} - e_{(N_T)}^{2} & \cdots & c_{(N_T)}^{M} - e_{(N_T)}^{M}
\end{bmatrix} $$

(7)

which is a square root of $A(c, e)$, i.e. $A(c, e) = B(c, e) \cdot B(c, e)^H$.

Then, the overall diversity gain that can be achieved is [5]

$$ G_d = N_R \min_{e \notin c} \text{rank}(A(c, e)) = N_R \min_{e \notin c} \text{rank}(B(c, e)) $$

(8)

In order to achieve the maximum diversity gain, for any codeword $c$ and $e$, the rank of $A(c, e)$ should be

$$ \text{rank}(A(c, e)) = \min\{N_T, M\} $$

(9)

Also, the coding gain $G_c$ is defined as

$$ G_c = \min_{e \notin c} \prod_{j=1}^{\text{rank}(A(c, e))} \lambda_j $$

(10)

where $\lambda_j$ is the non-zero eigenvalues of $A(c, e)$.

### A. $M \leq N_T$

When minimizing the difference between $c$ and $e$, we first assume that there is only one erroneous symbol between the original signal vector $s$ and the decoded signal vector $s_e$ for the simplification of analysis, i.e.

$$ s - s_e = [0, \ldots, 0, p, \ldots, 0]^H $$

(11)

where $p$ is the position of the erroneous symbol and $\xi_p$ is the difference between the $p$th original and decoded symbol. Then, the $B(c, e)$ can be rewritten as

$$ B(c, e) = c - e $$

$$ = [F_{(1)}s, \ldots, F_{(M)}s] - [F_{(1)}s_e, \ldots, F_{(M)}s_e] $$

$$ = [F_{(1)}(s - s_e), \ldots, F_{(M)}(s - s_e)] $$

$$ = [V_{(1)}^p \xi_p, V_{(2)}^p \xi_p, \ldots, V_{(M)}^p \xi_p] $$

$$ = [V_{(1)}^p, V_{(2)}^p, \ldots, V_{(M)}^p] \xi_p $$

$$ = U_p \xi_p $$

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where $V^p_{(i)} = [F_{(i)}]_p$ represents the $p$th column of the precoding matrix $F_{(i)}$ and $U_p = [V^p_{(1)}, V^p_{(2)}, \ldots, V^p_{(M)}]$ is the overall matrix consisting of the $p$th column of all the precoding matrix $F_{(i)}, 1 \leq i \leq M - 1$.

For the sake of further analysis, the square matrix of $B(c, e)$ is defined as

$$
D(c, e) = B(c, e)^H \cdot B(c, e) = \xi^P U_p^H U_p \xi^P
$$

This matrix $D(c, e)$ has the same rank and non-zero eigenvalues with the matrix $B(c, e)$. Therefore, the maximum diversity gain and coding gain can be achieved when the matrix $D(c, e)$ is full rank and its product of all the non-zero eigenvalues is maximum.

Then, (13) can be rewritten as

$$
D(c, e) = |\xi^P|^2 \begin{bmatrix}
V^p_{(1)} & V^p_{(2)} & \cdots & V^p_{(M)} \\
V^{p*}_{(1)} & V^{p*}_{(2)} & \cdots & V^{p*}_{(M)} \\
\vdots & \vdots & \ddots & \vdots \\
V^{p*}_{(M)} & V^{p*}_{(M)} & \cdots & V^{p*}_{(M)} \\
\end{bmatrix}
$$

where $\gamma^p_{i,j} = V^p_{(i)} V^{p*}_{(j)}, i \neq j, 1 \leq i, j \leq M$.

To let our analysis more general, it should be assumed that there are $K, 1 \leq K \leq N_T$, different symbols between the original signal vector $s$ and the decoded signal vector $s_e$. Then, (11) will be modified as

$$
s - s_e = [0, \cdots, \xi^p_1, \cdots, \xi^p_K, \cdots, 0]^H
$$

where $p_1, \cdots, p_K, 1 \leq K \leq N_T$, are the positions of the different symbols. Accordingly, (15) is also changed to

$$
D(c, e) = \sum_{k=1}^{K} \sum_{l=1}^{K} \xi^p_{k-l} U_p^H U_p \xi^p_l
$$

$$
= \sum_{k=1}^{K} |\xi^p_k|^2 \begin{bmatrix}
1 & \gamma^p_{1,2} & \cdots & \gamma^p_{1,M} \\
\gamma^p_{2,1} & 1 & \cdots & \gamma^p_{2,M} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma^p_{M,1} & \gamma^p_{M,2} & \cdots & 1 \\
\end{bmatrix}
$$

Thus to compute the upper bound on the average probability of error in the fading channel [8][10], we use the mean of $D(c, e)$ for the sake of analysis, which can be expressed by

$$
E\{D(c, e)\} = \sum_{k=1}^{K} E\{|\xi^p_k|^2\} \begin{bmatrix}
1 & \gamma^p_{1,2} & \cdots & \gamma^p_{1,M} \\
\gamma^p_{2,1} & 1 & \cdots & \gamma^p_{2,M} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma^p_{M,1} & \gamma^p_{M,2} & \cdots & 1 \\
\end{bmatrix}
$$

Only in case of $\gamma^p_{i,j} = 0, i \neq j, 1 \leq i, j \leq M$, the $E\{D(c, e)\}$ is full rank and has the maximum product of the eigenvalues, i.e.

$$
E\{D(c, e)\} = \sum_{k=1}^{K} E\{|\xi^p_k|^2\} \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 \\
\end{bmatrix}
$$

Therefore, in order to achieve the maximum diversity gain and coding gain, the corresponding vector columns between the precoding matrix for the different (re)transmission should be orthogonal to each other, i.e.

$$
\gamma^p_{i,j} = 0, i \neq j, 1 \leq i, j \leq M, \forall p_k \in \{1, 2, \cdots, N_T\}
$$

IV. CODEWORD SELECTION IN PRACTICAL HARQ TRANSMISSION

If we do not have constraints on the feedback bandwidth, the optimal choice of precoding matrix is well-known to be the right singular vectors of CSI matrix. However, the cost to feedback these singular vectors is very expensive, especially when fast update is necessary in a system. In [6], a structured closed-loop MIMO precoding method that does not require the actual feedback of the weight matrix is proposed. For each transmit antenna size a set of precoding matrices is constructed first and known at both the transmitter and receiver. Consequently, the receiver only need to feedback to the transmitter the index to a precoding matrix within this set. The set of the matrices (or the codebook) can be constructed to achieve the trade-off between the desired performance and feedback bandwidth. Once the codebook is fixed, the number of feedback bits needed does not grow with the size of the matrix itself, unlike in the existing approaches. It has been shown that the precoding method with limited size of codeword can achieve the near-optimal precoding MIMO performance with reasonably low amount (160f feedback bits. Therefore, in this section, we will apply the proposed optimal precoding scheme for HARQ transmission into this structured closed-loop MIMO with the limited size of codeword.

The matrix proposed in [11] is adopted, where the cross-correlations of the codewords follow a block-circulated structure. In this design, one codebook is fully specified once the first codeword $P_1$ and a diagonal rotation matrix $Q$ is provided. The other codewords in the codebook are given by

$$
P_l = Q^{T} P_1, l = 2, 3, \cdots, L
$$
where \( L \) is the size of the codebook and \( Q \) is a diagonal matrix fully parameterized by an integer vector \( \mathbf{u} = [u_1, u_2, \ldots, u_{N_T}] \).

Furthermore, if the number of spatially multiplexed data streams is no more than the number of the transmitted antennas, i.e. \( N_D \leq N_T \), the first codeword \( \mathbf{P}_1 \) can be chosen to be the \( N_T \times N_D \) DFT matrix whose \((m,n)\) element is specified as \( (P_1)_{m,n} = e^{-j \frac{2\pi}{N_T} (m-1)(n-1)} \) where \( 1 \leq m \leq N_T, 1 \leq n \leq N_D \).

After a codebook is chosen, the receiver observes channel realization and makes its decision on the optimal codeword (precoding matrix) to be used at the transmitter. The index of the optimal codeword is then sent back through the designated feedback channel to the transmitter.

When the first transmission is happened, the MMSE estimates of the transmitted signal with the assumption that the \( l \)th codeword is selected can be written as

\[
\hat{s}_i = (\mathbf{H}(1)^H\mathbf{P}^H_1)(\mathbf{H}(1)^H\mathbf{P}^H_1 + \sigma^2 \mathbf{I})^{-1}\mathbf{r} \quad (21)
\]

Then, the receiver selects the precoding matrix whose maximum MSE is minimum and feedbacks its index to the transmitter [6]:

\[
\mathbf{F}_1 = \min_{l=1,2,\ldots,N_T} \max_{i=1,2,\ldots,L} E[|s_i - \hat{s}_{i,l}|^2] \quad (22)
\]

When the retransmission is happened, the precoding matrix in the codebook whose column vectors are orthogonal to the ones in the precoding matrix of the previous transmission correspondingly should be selected and its index be conveyed to the transmitter:

\[
l_{(i)} = \arg_{i=1,2,\ldots,L} \min_{l=1,2,\ldots,N_T} E[|s_i - \hat{s}_{i,l}|^2] \quad (23)
\]

\[
\mathbf{F}_{(i)} = \mathbf{P}_{l_{(i)}}
\]

V. SIMULATION RESULTS

The performances of the proposed principle for HARQ transmission in the structured closed-loop MIMO with four transmit and receive antennas\((N_T = N_R = 4)\) are evaluated by simulation and discussed in this section. The main system parameters are shown in Table I. The entries of channel matrix \( \mathbf{H} \) are zero-mean i.i.d complex Gaussian random variables with unit variance in each dimension, and generated independently. Assuming that the channel is quasi-static, its state is not changed during the (re)transmission per packet. The maximum of the transmission per packet is set to be four.

To focus on the precoder performance with retransmission, we first assume that every packet will be retransmitted no matter whether it is correctly received or not during the previous transmissions. Since the proposed precoder design specifies the joint decoder after each retransmission, the bit-error-rate (BER) of the overall wireless communication system is collected. The channel encoder/decoder is excluded firstly in order to better show the difference caused by precoder and the combining schemes. And the precoding matrix for the first transmission is optimally selected in the following simulations. Firstly, the pre-combining scheme is adopted due to its better use of the spatial diversity gain in HARQ transmission [8].

Fig.1 compares the performances of precoder with ML or linear MMSE detection, where the precoding matrix for the second transmission is chosen according to the proposed design principle or not. It is clear that the BER performance of ML detection outperforms that of the linear MMSE but with higher computation complexity. Also, it should be pointed that the performance with optimal designed precoding matrix for the 2nd transmission is better than that with non-optimal matrix no matter which detection principle is applied. If the target uncoded BER is assumed to be \( 10^{-2} \), the gain between them is about 1dB which demonstrates the advantage of the proposed design principle.

Then, we compare the performances of MIMO systems with the proposed optimal precoder or the Quasi-Orthogonal Space Time Block Code (QOSTBC) based HARQ [12] in each (re)transmission as shown in Fig.2. The ML detection is used in both of systems. The BER performance is improved much with the increase number of the (re)transmission. And the proposed optimal precoder has almost the same performance as QOSTBC based HARQ transmission but less complexity and more flexibility.

It is well-known that practical systems may be in favor of implementing a Type II hybrid ARQ. Type-II hybrid ARQ

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**TABLE I**

**SYSTEM PARAMETERS**

<table>
<thead>
<tr>
<th>Antenna configuration ((N_T \times N_R))</th>
<th>4 \times 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detection principle</td>
<td>ML/Linear MMSE</td>
</tr>
<tr>
<td>Size of Codebook ((L))</td>
<td>64</td>
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<tr>
<td>Channel coding ((R_c))</td>
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<tr>
<td>/Decoding</td>
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Fig. 1. Uncoded BER performance comparison between ML and MMSE detection with (non)-optimal precoding matrix
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2007 proceedings.

This work is sponsored in part by the National Natural Science Foundation of China under grant No.60572120 and 60602058, and in part by the national high technology research and developing program of China (National 863 Program) under grant No.2006AA012Z257.

ACKNOWLEDGMENT

In this paper, we investigate the design of linear precoders for Type-I Hybrid ARQ transmission in MIMO wireless communication systems. The overall precoded MIMO packet retransmission can be regarded as the process of the space-time coding, where the available diversity gain and coding gain can be fully exploited by well-designed precoding matrix. We propose the optimal design principle of linear HARQ precoders to maximize the diversity gain and coding gain, whose column vectors are orthogonal to each other correspondingly. Simulation results demonstrate the effectiveness of these precoders with the pre-combining scheme at the receiver in reducing the detection bit-error rate and increasing the unified throughput.

REFERENCES


do not repeat the data packet in retransmissions. Instead, it can send the parity bits of a forward-error-coding (FEC) codeword involving the original data packet for better error correction. For further comparison, we also consider a Type-II hybrid ARQ scheme by sending incremental redundancy (IR) during retransmission with the individually optimized precoder, where the post-combining scheme at the bit level and MMSE principle is adopted at the receiver. As shown in Fig.3, the unified throughput, i.e. $TP = (1 - BLER)/M$, $M$ is average (re)transmission times, is used instead of BLER to show the performance difference. And every packet will be retransmitted only when it is not correctly received during the previous transmissions. It can be seen that Type-I hybrid ARQ with the proposed precoder outperforms Type-II hybrid ARQ with individually optimized precoder since more diversity gain is achieved. For instance, at SNR=4dB, Type-I hybrid ARQ with the proposed optimized precoder achieves about 0.3 unified throughput gain compared with the Type-II hybrid ARQ with individually optimized precoder.

Since the proposed precoder in this paper is more suitable to be used under the chase-combining (CC) transmission, it is necessary to compare its performance with that of the common precoder under Increase Residue transmission.

VI. CONCLUSION

Fig. 2. Uncoded BER performance comparison between precoding and QOSTBC with ML detection

Fig. 3. Unified throughput comparison between Type-I HARQ with the proposed precoder and Type-II HARQ with individually optimized precoder

In this paper, we investigate the design of linear precoders for Type-I Hybrid ARQ transmission in MIMO wireless communication systems. The overall precoded MIMO packet retransmission can be regarded as the process of the space-time coding, where the available diversity gain and coding gain can be fully exploited by well-designed precoding matrix. We propose the optimal design principle of linear HARQ precoders to maximize the diversity gain and coding gain, whose column vectors are orthogonal to each other correspondingly. Simulation results demonstrate the effectiveness of these precoders with the pre-combining scheme at the receiver in reducing the detection bit-error rate and increasing the unified throughput.

ACKNOWLEDGMENT

This work is sponsored in part by the National Natural Science Foundation of China under grant No.60572120 and 60602058, and in part by the national high technology research and developing program of China (National 863 Program) under grant No.2006AA01Z2Z57.