Precoder Design in Cooperative Systems with Amplify-and-Forward Relaying

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Abstract—Cooperative communication systems using various relay protocols can achieve spatial diversity gains and enlarge the coverage. For the practical amplify-and-forward (AF) protocol, an optimal precoder design for cooperative systems with single-antenna nodes is proposed in this paper. Through iterative computation, we can obtain the exact precoder vector for the transmitter, by which spatial diversity can be efficiently exploited. The numerical results and simulations demonstrate the effectiveness of the proposed technique in improving the data detection error.

I. INTRODUCTION

Cooperative communication provides a new way of introducing spatial diversity in wireless systems where the mobile stations may not be able to support multiple antennas due to size or other constraints [1]. Cooperative diversity has emerged as a special form of spatial diversity. The basic idea behind cooperative diversity rests on the observation that in a wireless environment, the signal transmitted by the source nodes is overheard by other nodes, which can be defined as partners. The source and its partners can jointly process and transmit their information, creating a virtual antenna array (VAA) although each of them is equipped with only one antenna [2].

Several repetition-based cooperative diversity algorithms such as amplify-and-forward (AF), decode-and-forward (DF) are developed to exploit the spectral diversity [3]. Furthermore, the conventional space-time codes (STCs) schemes, originally proposed for coding across collocated antennas, can be used in a distributed fashion for practical implementation of user cooperation in cooperative systems [4]. The distributed STC operating in AF mode is analyzed in [5] and the optimal designs are also developed in [6][7]. In these schemes, there is no need for the transmitter to know the channel. However, the realizable benefits of multiple-input multiple-output (MIMO) can be further exploited if the communication channel knowledge is known for use at the transmitter as well as the receiver.

Precoder is a processing technique that exploits channel state information (CSI) by operating on the signal before transmission to improve link performance. It has been an active research area during the last decade, fueled by applications in commercial wireless technology. When perfect CSI is available at both the transmitter and the receiver, optimal precoder designs have been proposed to minimize the trace of the mean-squared error matrix, the trace of the weighted mean squared error matrix or to maximize the average mutual information [8] [9]. This technique can be extended into the cooperative systems.

In this paper, the full instantaneous CSIs of all the involved links between the source, relay and destination are assumed to be available at the transmitter for linear precoder design in a cooperative system with AF relaying, where the relay amplifies and forwards the signal received from the source with gain factor to the destination. Under this assumption, we derive the optimum linear precoder at the source by maximizing the signal-to-noise ratio (SNR). It will be shown that the proposed optimal precoder provides a considerable performance gain compared with the systems without efficient use of CSI. In most of research work done before, usually either more than one antenna at one node or more than one relay node is assumed for the precoder designs in the relaying cooperative systems. Our novelty is to propose the idea of distributed precoder in the single-antenna relay network, which can also be extended to more complex scenarios in the next steps.

This paper is organized as follows. Section II gives the brief description of a system model. The precoder design in cooperative systems with AF relaying on assumption of the idealistic CSI is proposed in Section III. In Section IV, the numerical and simulation results are presented and discussed. Finally, Section V gives the conclusion.

Notations: Throughout this paper, matrices and vectors are set in boldface. (·)T, (·)H, (·)∗ and (·)+ denote transpose, Hermitian transpose, conjugate, and Moore-Penrose pseudo-inverse of a vector, respectively. I_N denotes N × N identity matrix. A circularly symmetric complex Gaussian vector x with mean m and covariance matrix R is denoted x ∼ CN(m, R).

II. SYSTEM MODEL

As shown in Fig.1, let us consider a simple wireless cooperative communication system in which a source (S) node cooperates with a relay (R) node to transmit information to a destination (D) node. All of the nodes are equipped with...
single transmit and receive antenna. We assume the half-duplex relay’s deployment which is motivated from practical concern on the large difference between transmit and receive powers in many applications. The source communicates with the relay and destination during the first time slot. In the second time slot, both the relay and source communicate with the destination. Then perfect channel state information (CSI) knowledge is assumed to be known at the source, relay and destination.

The signals transmitted by the source during the first and second time slots are denoted as $x_1(n)$ and $x_2(n)$, respectively, where $n$ is the time index. We assume that $E\{x_1(n)\} = 0$ and $E\{|x_i(n)|^2\} = P_z$ for $i \in \{0, 1\}$, where $P_z$ is the average transmitted power. In the following parts, we consider symbol-by-symbol transmission so that the time index $n$ can be omitted and $x_1$ and $x_2$ are simply written for representing the symbols transmitted in the first and second time slots, respectively. The signals received at $R$ and $D$ in the first time slot are given by

\begin{align}
y_{R,1} &= h_{SR}x_1 + z_{R,1} \\
y_{D,1} &= h_{SD}x_1 + z_{D,1}
\end{align}

where $h_{SR}$ and $h_{SD}$ denote the independent complex fading channel gain from $S$ to $R$ and $D$, modeled as $h_{SR} \sim \mathcal{CN}(0, \sigma_{SR}^2)$ and $h_{SD} \sim \mathcal{CN}(0, \sigma_{SD}^2)$ with $\sigma_{SR}^2 := E\{|h_{SR}|^2\}$ and $\sigma_{SD}^2 := E\{|h_{SD}|^2\}$ respectively. Without loss of generality, we assume that the noise terms $z_{R,1}$ and $z_{D,1}$ have equal variances $N_0$ and are modeled as $z_{R,1} \sim \mathcal{CN}(0, N_0)$, $z_{D,1} \sim \mathcal{CN}(0, N_0)$.

The relay normalizes the received signal by a factor of $\beta$ (so that the average energy is unity) and retransmits the signal during the second time slot. The destination receives a superposition of the relay transmission and the source transmission during the second time slot according to

\begin{align}
y_{D,2} &= h_{SD}x_2 + \beta h_{RD}y_{R,1} + z_{D,2}
\end{align}

where $h_{RD}$ denotes the fading channel gain from $R$ to $D$, $h_{RD} \sim \mathcal{CN}(0, \sigma_{RD}^2)$ with $\sigma_{RD}^2 := E\{|h_{RD}|^2\}$, and $z_{D,2} \sim \mathcal{CN}(0, N_0)$ is additive noise at $D$. It is noted that (2) contains the additional assumption of constant $h_{SD}$ over the two time slots in slow fading channel. Define the instantaneous SNR at link $S-R$, $R-D$ and $S-D$ as $\gamma_{SR} := |h_{SR}|^2 \tilde{\gamma}$, $\gamma_{RD} := |h_{RD}|^2 \tilde{\gamma}$, and $\gamma_{SD} := |h_{SD}|^2 \tilde{\gamma}$, respectively, with $\tilde{\gamma} = P_z/N_0$. Using $\beta = 1/\sqrt{|h_{SR}|^2 E\{|x_1|^2\} + N_0}$, we can rewrite (2) as

\begin{align}
y_{D,2} &= h_{SD}x_2 + \frac{h_{SR}h_{RD}}{\sqrt{|h_{SR}|^2 E\{|x_1|^2\} + N_0}} x_1 + \tilde{z}
\end{align}

where the effective noise term $\tilde{z} \sim \mathcal{CN}(0, N'_0)$ with $N'_0 = N_0(1 + \beta^2|h_{RD}|^2)$. Finally, we assume that the receiver normalizes $y_{D,2}$ by factor of $\alpha = 1/\sqrt{1 + \beta^2|h_{RD}|^2}$. The effective input-output relation in the AF protocol can be summarized in the matrix form as

\begin{align}
y &= Hx + z
\end{align}

where $y = [y_{D,1} \ y_{D,2}]^T$ is the received signal vector, $x = [x_1 \ x_2]^T$ is the transmitted signal vector, and $z = [z_{D,1} \ z_{D,2}]^T$ is the complex Gaussian noise vector with $E[x \cdot z^H] = N_0 I$. $H$ is the effective channel matrix given by

\begin{align}
H = \begin{bmatrix} h_{SD} & 0 \\ \alpha h_{SR}h_{RD} & \alpha h_{SD} \end{bmatrix}
\end{align}

In the notation of matrices, the matrix $H$ has a singular value decomposition (SVD):

\begin{align}
H = U \Sigma V^H
\end{align}

where $U$ and $V$ are $2 \times 2$ left and right unitary matrix, $\Sigma$ is a diagonal $2 \times 2$ matrix with the ordered singular values $\lambda_1$ and $\lambda_2$. $\lambda_1 \geq \lambda_2$, starting in the top left corner.

Note that this AF-based protocol converts the spatially distributed antenna system into MIMO channels allowing the fundamental gains of multiple-antenna systems such as diversity gain and array gain to be exploited in a distributed fashion.

### III. Precoder Design with Amplify-and-Forward Relaying

In this section, we consider a transmitter that utilizes a linear precoding matrix for the fading relay channel assuming AF-based protocol. The source transmits the precoded signals serially over two time slots with spatial rate one, i.e.,

\begin{align}
x = \sqrt{2} F s
\end{align}

where $F = [F_1 \ F_2]^T$ with $|F_1|^2 + |F_2|^2 = 1$ is the linear precoder vector and $s$ is the original information symbol. Stacking the signals received at the destination to form a vector, we rewrite (4) and obtain the input-output relation as follows:

\begin{align}
y = \sqrt{2} H F s + z
\end{align}

Accordingly, the normalized factor at the relay can be rewritten as

\begin{align}
\beta = 1/\sqrt{2|h_{SR}|^2 |F_1|^2 + N_0}
\end{align}

On the other hand, the optimal precoder matrix usually can be calculated as a function of the CSI. For the design of

![Fig. 1. Block model of a simple cooperative system](image-url)
precoder with rate one, the optimal precoder vector is given
by the first column vector of matrix V [10], i.e.

\[ F = V \begin{bmatrix} 1 \\ 0 \end{bmatrix} \] (10)

Integrating (5) and (6) into (10), \( F \) can be written as

\[ F_1 = \frac{1}{2\sqrt{\rho + 1}} + \frac{1}{2} \] (11)
\[ F_2 = \left( \frac{1}{2\sqrt{\rho + 1}} - \frac{1}{2} \right) \cdot e^{i\theta} \] (12)

where

\[ \rho = \frac{\alpha^2|h_{SD}|^2 + \alpha^2\beta^2|h_{SR}h_{RD}|^2}{[(1-\alpha)^2|h_{SD}|^2 + \alpha^2\beta^2|h_{SR}h_{RD}|^2]^2} \] (13)
\[ \theta = -\text{angle}(h_{SR}h_{RD}h_{SD}) \] (14)

From the description above, it can be seen that \( F_1 \) is
dependent on \( \beta \) while \( \beta \) is also the function of \( F_1 \). Next,
we can integrate (11) and (13) into (9) and get the form
like \( f(\beta) \), where \( \beta \) can be expressed by an equivalent
function of itself. Then, the Aitken iterative method [11] with
fast converge is used to calculate the root of this equation,
which is summarized as follows:

\begin{itemize}
  \item **Initialization**
    \[ \beta_0 = \frac{1}{\sqrt{|h_{SR}|^2 + N_0}} \text{ and } k = 0, \varepsilon = 0.0001 \]
  \item **Iteration**
    \begin{enumerate}
      \item \( u = f(\beta^{(k)}) \)
      \item \( v = f(u) \)
      \item \( \beta^{(k+1)} = v - (v - u)^2 / (v - 2u + \beta^{(k)}), k = k + 1 \)
    \end{enumerate}
    Repeat (1)-(3) until \( |v - u| < \varepsilon \)

Once \( \beta \) is determined, the optimal precoder vector \( F \) can
also be calculated by (11) and (12). After combining the
received signals in two time slots, the equivalent SNR at the
destination is given by

\[ \gamma_{eq} = \frac{2|h_{SD}|^2 + 2\alpha^2|\beta h_{SR}h_{RD}F_1 + h_{SD}F_2|^2}{N_0} \] (15)

Then, on the assumption of QPSK transmission, the average
bit error rate (BER) at the destination can be computed by

\[ P_e(\gamma_{eq}) = \frac{1}{2} \text{erfc}(\sqrt{\frac{\gamma_{eq}}{2}}) \] (16)

IV. NUMERICAL AND SIMULATION RESULTS

The performances of the proposed precoder for a coopera-
tive system are evaluated by numerical results and simu-
lations, then discussed in this section. The performances
of direct transmission (DT) and AF without a precoder, i.e.
\( F = [1/\sqrt{2}] \begin{bmatrix} 1/\sqrt{2} \end{bmatrix} \), are also given as reference. QPSK
modulation and the coherent detections both at the relay and
destination are assumed. The nodes transmit with the same
power \( P_x \), resulting in an average input SNR \( \bar{\gamma} = P_x / N_0 \). With
the reference to Fig.1, we consider representative attenuation
levels that corresponding to those in which \( R \) is located either
close to \( S \), \( D \) or equidistant from both; the corresponding

average output SNRs (\( \bar{\gamma}_{SR}, \bar{\gamma}_{RD}, \bar{\gamma}_{SD} \)) in logarithmic-scale are
(\( \bar{\gamma} + 10\text{dB}, \bar{\gamma} \), \( \bar{\gamma} + 10\text{dB}, \bar{\gamma} \) and \( \bar{\gamma}, \bar{\gamma}, \bar{\gamma} \), respectively.

Firstly let us examine the SNRs after combining the received
\( y_{SD} \) and \( y_{RD} \) at the destination. Fig.2 shows the SNR
probability density function of the systems with/without precoder
respectively, where (\( \bar{\gamma}_{SR}, \bar{\gamma}_{RD}, \bar{\gamma}_{SD} \))=(\( \bar{\gamma}, \bar{\gamma}, \bar{\gamma} \)) with \( \bar{\gamma} = 10\text{dB} \) is
assumed. The probability of the event that SNR is large
value of AF relaying system with precoder is a little bigger
than that without precoder or direct transmission, which means
that the average SNR of the system with the proposed precoder
will be more likely to be larger than that without precoder or
direct transmission. The similar results are happened in other
scenarios.

Consequently, the better BER performances of the system
with precoder are expected as shown in Fig.3. If the target
BER is assumed to $10^{-3}$, the SNR gain with precoder is about 3dB compared with AF relaying system without precoder in each scenario since more spatial diversity gain can be exploited efficiently by precoder. The similar results can also be found in Fig.4 and Fig.5. Moreover, the performances when $R$ is located either close to $S$, $D$ is better than that when equidistant from both because of the better link quality between $R$ and $S$ or $D$.

V. CONCLUSION

In this paper, we have proposed an optimal linear precoder design in a cooperative system with AF relaying. The exact precoding vector can be calculated by iterative method. Numerical and simulation results show that the AF relaying system with the proposed precoder performs better than that without the precoder by making full use of spatial diversity. The work done in the simple scenario is only our first step, which can be as fundamental theory for further research. The more complex scenarios are considered in the next steps.

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