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Advances in Network Information Theory

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March 17–19, 2003
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Wireless Network Information Theory

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ABSTRACT. We address the problem of forming a wireless network out of a set of geographically dispersed nodes, each equipped with a radio transceiver. The two questions that we address are: (i) What should be the strategy for transporting information over the wireless network?, and (ii) How much information can possibly be transported?

We present a model of such systems which lends itself to a tractable information theory that is able to shed light on these issues. A key role is played in our model by the distances between nodes, and the attenuation as a function of distance suffered by radio signals. The main quantity we study is the transport capacity which is the supremal distance weighted sum of routes that the network can support. We obtain scaling laws for the growth of the transport capacity as the number of nodes in the network increases, and bound the pre-constant involved. There is an interesting dichotomy. When the medium has any absorption or the path loss exponent is greater than 3, then the transport capacity grows linearly, and an order optimal strategy is multihop transport where nodes relay packets after fully decoding them treating all interference as noise. This is in fact the strategy which current protocol development efforts are targeted towards, which is reassuring. However, when there is no absorption at all and the path loss exponent is small, then one can obtain unbounded transport capacity for finite total transmitted power, and even superlinear scaling. A strategy which emerges as of interest is for upstream nodes to coherently cooperate in sending a packet to the next downstream node, and for each node to decode a packet based on all its observations and subtracting out the known interference generated by downstream transmissions.

1. The architecture for information transfer in wireless networks

We consider wireless networks which are to be formed out of groups of geographically dispersed set of nodes each equipped with a radio transceiver.

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How should information be transferred from sources to destinations in such networks? One possibility is to choose for each pair of source and destination nodes a sequence of nodes forming the path, and then to relay packets from node to node along the path until they reach the destination. At each node along the path, packets can be fully decoded by treating all interference as noise. The digitally regenerated packets are then retransmitted to the next node along the path. This mode of information transfer is the one around which there is much protocol development activity today; let us call it the "multihop node."

The choice of the multihop mode as the method of information transfer creates the need for several protocols to realize it. For example, since one is treating all interfering transmissions as noise, one would like to regulate the number of such interferers in the vicinity of the receiver. This gives rise to the medium access control problem. The currently popular IEEE 802.11 [**1**] MAC protocol addresses (among other things) this issue. As another example, since interference is greater when the power of the interfering transmission is large, there is a social need to regulate the powers of all transmissions and thus be able to spatially reuse the spectrum. This gives rise to the power control problem for ad hoc networks. The protocols COMPOW [**2**] and Cluster POW [**3**] are aimed at this problem. As a third example, since the goal is to relay packets along a string of nodes from source node to destination node, one needs to find such a path from source to destination, and that too when nodes are possibly mobile. This gives rise to the routing problem. The routing protocols DSDV [**4**], AODV [**5**], DSR [**6**], TBRPF, OLSR, ZRP [**7**], STARA [**8**], etc., address this problem.

Thus, we see that the problem of designing wireless networks can be roughly divided into two phases. First one needs to choose the strategy by which nodes equipped with wireless transceivers should cooperate to achieve information transfer. This problem of choosing the overall strategy to adopt is one of choosing the architecture for information transport. Subsequent to the choice of strategy/architecture, there arises the problem of protocol development, where the goal is to realize the architecture that has been adopted.

In this paper, we consider the first question: How should wireless nodes cooperate to achieve information transfer? The multihop mode is only one possibility. There are countless other possibilities, since cooperation over the ether can be done in many strange ways. Just to stretch one's imagination, let us consider the following possibility. Suppose node A is receiving a transmission from node B in the presence of an interfering transmission from node C. Then a node D can help A by transmitting a signal which cancels the interference created by node C at node A's location. This is a form of cooperation which is not possible in wireline networks, but which may be possible (if one knows the channel, etc.) for wireless networks. One can think of this cooperation in the following way. Consider the signal to interference plus noise ratio (SINR) at node A:

$$\text{SINR at node } A = \frac{\gamma_{BA} P_B}{N + \gamma_{CA} P_C}$$

where P_B is the power of node B, and P_c the power of node C, while γ_{BA} and γ_{CA} are the attenuations from B to A, and C to A, respectively. Here, node D attempts to cancel the contribution $\gamma_{CA} P_C$ from the denominator, and thus boost the SINR at node A to $\frac{\gamma_{BA} P_B}{N}$. This is a strange form of cooperation (akin to noise cancelling). Should node D indeed help node A in this way?

As another possibility, suppose (A, B, C) is a route from source A to destination C, and node B is a relay node along the path. Then node B could serve as a relay not by fully decoding packets from node A and then retransmitting the digitally regenerated packets to node C, but simply by amplifying the signal plus noise that is received from node A; see [**9**]. Thus the relaying could be done by "amplifying and forward" rather than "decode and forward." Which is it to be?

As yet another possibility consider a network consisting of nodes $\{A, B, C, D, E, F\}$. Suppose node A is a source and node F is a destination for a certain stream of information. This could be transported in two phases. In the first phase, node A could broadcasts a packet simultaneously to $\{B, C, D, E\}$. In the second phase, nodes $\{B, C, D, E\}$ simultaneously transmit this packet to F. Thus, the node of information transfer is a fan-out, followed by a fan-in; see [**10**]. Should this be how nodes cooperate?

In fact since one is in the wireless would, one need not devote all of the power to any one mode. A node A could spend 30% of its power cancelling the interference created by node B at node C, 20% of its power to relay packets from node D to node F by "amplifying and forwarding," 10% of its power to relay packets from node F to node G, 20% of its power in broadcasting packets of node H to nodes $\{I, J, K, L\}$, etc.

To quote Shakespeare's Hamlet:

> "There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy."

Multihop is clearly not the only strategy for information transfer over wireless networks. What indeed should be the architecture for information transfer in wireless networks?

2. The amount of information that wireless networks can transport

Another utilitarian issue of importance for wireless networks is to determine how much information can be carried over wireless networks. That is, what is the capacity of wireless networks? Since there are many possible source-destination pairs, one needs to talk of a rate vector. For a network with n nodes, there are $n(n-1)$ possible source-destination pairs, and the rate vector is an $n(n-1)$-dimensional vector $R = (R_{12}, R_{13}, \ldots, R_{1n}, R_{21}, R_{23}, \ldots, R_{1n}, \ldots, R_{n1}, R_{n2}, \ldots, R_{n(n-1)})$. The capacity is (the closure of) the set of such feasible rate vectors. This a clearly a subset of a very dimensional space when n is large.

Another issue needs to be kept in mind, and that is that we are not simply interested in the (already complicated) capacity of a single wireless network, but in the class of wireless networks. That is, we are not interested in having an architecture for wireless network 1, another architecture for wireless network 2, etc. What one wants is to say something about a large class of wireless networks of interest to us. That is, we seek some sort of a uniformity result.

So the problem is to deal with the sets of feasible regions for the class of wireless networks of interest to us, where the number of nodes, then locations, channel conditions, etc., may differ. This raises the issue of how one is to measure the capacity of a class wireless networks.

3. The need for an information theory for wireless networks

The two problems noted above, that of determining the appropriate architecture for information transfer among a group of wireless nodes, and that of determining the amount of information that they can transport, both belong to the realm of network information theory.

Indeed, Shannon's contributions in [**11**] are at (at least) two fold. Not only does his theory provide the architecture for information transfer, but also the capacity. For the first issue, the determination that the problems of source coding and channel coding can be separated is of paramount importance, and, in fact, is at the heart of the digital communication revolution. Source coding today is performed by programs such as zip, while channel coding is preformed by network interface cards. For the second problem, he provided, amazingly, a beautifully elegant (what we call today a single letter) characterization of capacity.

Ever since Shannon's work, there has been great interest in generalizing information theory from a single link to a network, and Shannon himself was interested in this. However, while there have been some singular amazing triumphs, by and large the problem of developing a general network information theory has defied a solution. Among the triumphs are the characterization of the capacities of the scalar Gaussian broadcast channel [**12, 13, 14, 15**], the general multiple access channel [**16, 17**], and certain generalizations of source coding to take advantage of dependencies and side information [**18**].

However, almost all other problems have defied a general solution. One such is the three node relay channel, consisting of two nodes a source and a destination with the only complication being that there is one other node, a relay, which is there to assist in the information transfer. In spite of at least three decades of sustained attention, the general case is unsolved, though special cases have been most elegantly characterized [**19**]. So also for the interference channel which features two source-destination pairs (A, B) and (C, D) with the only complication being that A's transmission interferes with D's reception, and C's transmission interferes with B's reception. When the power levels of A and C are moderate, this problem is unsolved.

One can perhaps regard the three node relay problem as the simplest generalization of Shannon's original problem consisting of a source and a destination. When even this problem appears formidable, what possible guidance can information theory provide concerning the architecture for information transport? Indeed, this has been the reason for the "unconsummated union" [**20**] between information theory and the world of networking.

4. The Model

To address this problem, we consider the following model. It should be noted that the distances between nodes will play a key and explicit role in this model.

Consider a two-dimensional plane (or a one-dimensional line) on which are located n nodes. We will suppose that the (Euclidean) distance between any two pair of nodes i and j, denoted ρ_{ij}, is at least a minimum distance ρ_{min}. If $x_i(t)$ is broadcast by node i at time t, then

$$y_j(t) \quad = \quad \sum_{i \neq j} \frac{c e^{-\gamma \rho_{ij}}}{\rho_{ij} \delta} x_i(t) + n_j(t),$$

is received at node j at time t, where $n_j(t)$ is AWGN of variance σ^2. The factor $\frac{ce^{-\gamma\rho_{ij}}}{\rho_{ij}^{\delta}}$ is the attenuation of a signal from node i to node j. The quantity δ will be called the path loss exponent, and $\delta = 1$ (since δ needs to be doubled if one is interested in power) corresponds to the familiar inverse square law in free space. The quantity $\gamma \geq 0$ will be called the absorption constant, and it is generally strictly positive if there is absorption; see [**21, 22**]. Each node is also assumed to have a power constraint P.

It should be noted that at this point, distance has already come in explicitly in two ways. First, we have modeled the distances between nodes, and second, we have modeled attenuation as a function of distance.

The third way distance will explicitly enter our model is through the choice of the performance measure — the transport capacity. Denote by $R = (R_{12}, R_{13}, \ldots, R_{1n}, R_{21}, R_{23}, \ldots, R_{2n}, \ldots, R_{n1}, R_{n2}, \ldots, R_{n(n-1)})$, a feasible rate vector (which is defined in the standard information theoretic manner; see Chapter 14 of [**23**], for instance), and let $\Re$ be the set of feasible rate vectors. Then we study

$$C_T \quad := \quad \sup_{R \in \Re} \sum_{i=1} \sum_{\substack{j=1 \\ j \neq i}}^{n} R_{ij} \rho_{ij},$$

the supremal distance weighted sum of rates. We call this the "transport capacity." It is analogous to the man-miles/year metric used to measure the size of an airline.

We note that rather than studying the entire feasible region $\Re$ which is a subset of a very high dimensional space, we have chosen a much simpler scalar quantity to study.

5. The Results

It turns out, interestingly, that there is a dichotomy. The cases where there is relatively high attenuation and relatively small attenuation differ, and require different architectures for information transport.

When the attenuation is relatively large, then the transport capacity scales linearly in the number of nodes: (Expressions for constants, proofs of the results, as well as other results and more details, can all be found in the full paper [**24**]).

THEOREM 5.1. *When* $\gamma > 0$ *or* $\delta > 3$,

$$C_T \quad \leq \quad c_1 n.$$

What is interesting about this result is that in many cases one can obtain this order of transport capacity, or nearly this order, by simply using the multihop strategy. It is shown in [**25**] that for n nodes in an area A, the transport capacity is $O(\sqrt{An})$. In fact, when nodes are regularly arranged, say at the sites of an integer lattice, then the order can, in fact, be achieved by multihop transport. Even when nodes are randomly located, a transport capacity of $\Theta\left(\sqrt{\frac{An}{\log n}}\right)$ can be obtained. Keeping in mind that in our model in this paper the area A grows like $\Omega(n)$ since nodes are mutually separated by a minimum distance ρ_{min}, we see that $\Theta(n)$ can be realized by multihop transport when nodes are regularly arranged, while $\Theta\left(\frac{n}{\sqrt{\log n}}\right)$ can be realized when nodes are randomly located. Since $\sqrt{\log n}$ grows slowly in comparison to n we see that the multihop mode of operating a wireless network is nearly order optimal.

Thus, we determine that multihop is an order optimal architecture for information transport in wireless networks when there is relatively large attenuation. This is gratifying since it is, in fact, the architecture towards which current protocol efforts are targeted.

Now let us turn to the relatively low attenuation case. Here a different strategy emerges as interesting, one which we call Coherent Relaying with Interference Subtraction (CRIS). To illustrate CRIS, consider a set of nodes numbered 1, 2, ... , K with 1 being the source node and K the destination node. Node 1 begins by sending packet 1 to node 2 in slot 1. In slot 2 nodes 1 and 2 coherently cooperate to send the packet 1 to node 3. (In slot 2, node 1 also spends a portion of its power to send the next packet, packet 2, to node 2). In slot 3, nodes 1, 2, and 3 coherently cooperate to send the packet 1 to node 4. (Also node 1 spends a portion of its power to send packet 3 to node 2, and nodes 1 and 2 both spend certain portions of their power to send packet 2 to node 3). The scheme continues with upstream nodes coherently cooperating to send the next packet to a downstream node in each slot.

For decoding, assuming correct decoding has occurred in the past (and that the channel state information is known), each node can simply subtract that portion of its received signal which is due to downstream nodes transmitting packets that it has already decoded. Also, each node hears a packet many times when each of its upstream nodes was receiving it. Each node uses all this cumulative information, and subtracts already decoded signals, to decode the current packet of interest.

It turns out that one can actually achieve unbounded transport capacity, and that it can in fact grow superlinearly.

THEOREM 5.2. : *(i) When $\gamma = 0$ and $\delta < \frac{3}{2}$ then even if the entire network's transmission power budget is limited to P_{total}, there are networks where nodes can allocate this power among themselves to achieve unbounded transport capacity, and the order optimal strategy is CRIS.*

: *(ii) When $\gamma = 0$ and $\delta < 1$, then there are networks of nodes arranged along a straight line, i.e., a linear world, where $C_T = \Theta(n^\sigma)$ for $\frac{1}{2} < \sigma < \frac{1}{\delta}$, i.e., superlinear growth is possible, and CRIS is an order optimal strategy.*

We thus see that when attenuation is low, nodes can profitably cooperate among themselves to achieve superlinear growth, and multi-user estimation and coherence can be used to advantage.

The result of Theorem 5.2(i) may also be interesting when viewed in the context of [**26**] which studies the power requirements of communication across large distances.

6. Concluding Remarks

We have sketched the elements of an information theory for wireless networks which sheds light on important issues such as how information ought to be transported over wireless networks, and how much information can be so transported; see [**24**] for full details. A key role is played in our model by distances between nodes, and the attenuation as a function of distance. Another aspect of our studies is that rather than considering the set of feasible rate vectors, we consider the transport capacity, which is simply a scalar. Further, rather than characterize the transport capacity precisely, we study how it grows as the number of nodes increases, i.e., its scaling laws. We characterize the exponent in the scaling law, and since it is also important, we bound the preconstant involved. Through this approach we are able to get to the all important issue of the architecture of information transport in wireless networks. We see that there is a dichotomy. When the medium has absorption (generically always the case when not in free space) or has large enough path loss exponent, the transport capacity grows linearly, and multihop transport with packets fully decoded at each hop, treating interference as noise, is order optimal. This is gratifying since it is the scheme to which current protocol development efforts are targeted. However, when the medium has no absorption and low enough path loss exponent, then relaying by coherent cooperation among upstream nodes to send a packet to the next downstream node, along with decoding by subtracting the known interference from downstream nodes, a strategy we dub CRIS, emerges as of interest. Unlimited transport capacity for fixed finite total transmission power and superlinear scaling are both possible. How to exploit this in the future is an interesting question.

Needless to say, we have only scratched the surface, and much remains to be done.

References

[1] IEEE Protocol 802.11, "Draft standard for wireless LAN: Medium access control (MAC) and physical layer (PHY) specifications." IEEE, July 1996.

[2] S. Narayanaswamy, V. Kawadia, R. S. Sreenivas, and P. R. Kumar, "The compow protocol for power control in ad hoc networks: Theory, architecture, algorithm, implementation, and experimentation." http://black1.csl.uiuc.edu/ prkumar/postscript_files.html#Wireless Networks, May 16 2001.

[3] V. Kawadia and P. R. Kumar, "Power control and clustering in ad hoc networks," in *INFOCOM 2003*, (San Francisco, CA), March 30 - April 3 2003.

[4] C. E. Perkins and P. Bhagwat, "Highly dynamic destination-sequenced distance-vector routing (DSDV) for mobile computers," *Computer Communications Review*, vol. 24, no. 4, pp. 234–244, 1994. (ACM SIGCOMM '94).

[5] C. Perkins, E. Royer, and S. Das, "Ad hoc on demand distance vector (AODV) routing." http://www.ietf.org/internet-drafts/draft-ietf-manet-aodv-03.txt. IETF Internet Draft (work in progress), June 1999.

[6] J. Broch, D. Johnson, and D. Maltz, "The dynamic source routing protocol for mobile ad hoc networks." http://www.ietf.org/internetdrafts/draft-ietf-manet-dsr-01.txt, December 1998. IETF Internet Draft (work in progress).

[7] M. IETF, 2003. http://www.ieft.org/html.charters/manet-charter.html.

[8] P. Gupta and P. R. Kumar, "A system and traffic dependent adaptive routing algorithm for ad hoc networks," in *Proceedings of the 36th IEEE Conference on Decision and Control*, (San Diego, CA), pp. 2375–2380, December 1997.

[9] B. Schein and R. Gallager, "The Gaussian parallel relay network," in *Proc. IEEE Int'l Symp. Information Theory*, Sorrento, Italy, June 25–30, 2000.

[10] M. Gastpar and M. Vetterli, " On the asymptotic capacity of Gaussian relay networks," *Proc. IEEE Int'l Symp. Information Theory*, Lausanne, Switzerland, July 2002.

[11] C. E. Shannon, "A mathematical theory of communication," *Bell Syst. Tech. J.*, vol. 27, pp. 379–423, 1948.

[12] T. Cover, "Broadcast channels," *IEEE Trans. Inform. Theory*, vol. 18, pp. 2–14, 1972.

[13] P. Bergmans, "Random coding theorem for broadcast channels with degraded components," *IEEE Trans. Inform. Theory*, vol. 19, pp. 197–207, 1973.

[14] P. Bergmans, "A simple converse for broadcast channels with additive white Gaussian noise," *IEEE Trans. Inform. Theory*, vol. 20, pp. 279–280, 1974.

[15] T. Cover, "Comments on broadcast channels," *IEEE Trans. Inform. Theory*, vol. 44, pp. 2524–2530, 1998.

[16] R. Ahlswede, "Multi-way communication channels," in *Proceedings of the 2nd Int. Symp. Inform. Theory (Tsahkadsor, Armenian S.S.R.)*, (Prague), pp. 23–52, Publishing House of the Hungarian Academy of Sciences, 1971.

[17] H. Liao, *Multiple access channels.* PhD thesis, University of Hawaii, Honolulu, HA, 1972. Dept of Electrical Engineering.

[18] D. Slepian and J. K. Wolf, "Noiseless coding of correlated information sources," *IEEE Trans. Inform. Theory*, vol. IT-19, pp. 471–480, 1973.

[19] A. El Gamal and T. Cover, "Multiple user information theory," *Proceedings of the IEEE*, vol. 68, pp. 1466–1483, 1980.

[20] A. Ephremides and B. Hajek, "information theory and communication networks: An unconsummated union," *IEEE Transactions on Information Theory*, vol. IT-44, pp. 2416–2434, 1998.

[21] M. Franceschetti, J. Bruck, and L. Schulman, "Microcellular systems, random walks and wave propagation." To appear in *Proceedings of the IEEE Symposium on Antennas and Propagation Society* (IEEE AP-S 2002, San Antonio, TX), June 2002.

[22] M. Franceschetti, J. Bruck, and L. Schulman, "The wandering photon model of wave propagation," technical report, California Institute of Technology, 2003.

[23] T. Cover and J. Thomas, *Elements of Information Theory.* New York: Wiley and Sons, 1991.

[24] L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation", submitted to *IEEE Trans. Inform. Theory*, Apr. 2002.

[25] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. IT-46, pp. 388–404, Mar. 2000.

[26] C. Rose and G. Wright, "Write or radiate? Inscribed Mass and Electromagnetic Channels", Aug. 13, 2003. Preprint. Rutgers University.

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