Multisource, Multidestination, Multirelay Wireless Networks

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***Abstract*—Networks with multiple source–destination pairs, involving possibly multicast, and where there are multiple nodes that can serve as potential relay nodes, are considered. A multisource, multirelay coding scheme is developed. In this scheme, each source's information is sent to its destination nodes via a multirelay route, with the multiple multirelay routes operating concurrently even when they intersect with each other, in the same spirit as code-division multiple access (CDMA). It is found that in the generalization to multiple sources, backward decoding achieves higher rates than sliding-window decoding. The routing structure where a joint backward decoding can be performed is characterized. The achievable rate region is found to combine aspects of both multiple relay and multiple access. Potential applications of this coding scheme to sensor networks are discussed. In particular, the exact capacity for the data downloading problem in sensor networks, where there are multiple sensor sources and one sink or collector node, is established for certain geometries when there is phase fading that is unknown to the transmitter.**

***Index Terms*—Backward decoding, multiple-access channel, multiple-relay channel, network information theory, sensor networks, wireless networks.**

I. Introduction

In current protocols for wireless networks, interference is often regarded as undesirable, and a common tactic is to avoid it at least locally by silencing other transmitters in the neighborhood. An example is the RTS-CTS handshake in IEEE 802.11 [1]. However, this practice limits the number of concurrent transmissions in a network, so that the total throughput is reduced [2].

Viewed more fundamentally though, even "interference" is a signal that can potentially be exploited. This motivates the challenge of exploiting interference rather than succumbing to it, which necessitates an information-theoretic treatment.

A. The Relay Channel

Perhaps the simplest context in which this arises is the three-node network depicted in Fig. 1. Suppose node 1 is the source, which wants to send information to the destination node 3. In many situations, the destination node 3 may be at a great distance from node 1 so that any signal transmitted directly from node 1 to node 3 suffers such a considerable attenuation that it precludes any direct reliable communication at a high rate. In such a situation, one wants to exploit the presence of the intermediate node 2, so that node 1 can first transmit to node 2, with node 2 then transmitting to node 3. This results in two shorter-range communications, both reliably feasible at a high enough rate. The fundamental question that arises at this point is whether the signal transmitted by node 1 necessarily causes "interference" to node 3? The fact is that although this signal is intended for node 2, it carries exactly the same information that node 3 wants to decode eventually.

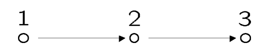

Fig. 1. A one-relay network.

Motivated by this problem, the "relay channel" first proposed in [3], [4] almost forty years ago, has become one of the basic topics of multi-user information theory, where the interest centers on developing coding schemes such that node 3 can effectively exploit both the signals transmitted by node 1 and node 2. Two fundamentally different coding strategies, called decode-and-forward and compress-and-forward, differing in whether the relay node 2 decodes the information or not, were developed in [5].

Denoting the signals transmitted by node 1 and node 2 as $x_1(t)$ and $x_2(t)$ respectively, and the signals received by node 2 and node 3 as $y_2(t)$ and $y_3(t)$, respectively, we assume that they are related by the probability transition function

$$p\left(y_2(t), y_3(t)|x_1(t), x_2(t)\right), \quad \text{for any time } t$$

which describes the discrete memoryless channel involved. It has been proved in [5] that the following rate is achievable with a decode-and-forward strategy:

$$R < \max_{p(x_1,x_2)} \min\{I(X_1;Y_2|X_2), I(X_1,X_2;Y_3)\}. \tag{1}$$

Examining more closely the two constraints on R

$$R < I(X_1;Y_2|X_2) \tag{2}$$

$$R < I(X_1,X_2;Y_3) \tag{3}$$

we observe that (2) is what is needed for the relay node 2 to indeed be able to decode the information based on the signal transmitted by node 1. The second constraint (3) applies irrespective of the particular scheme used, since it represents the limit that at best node 3 can only make use of the signals transmitted by nodes 1 and 2.

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Fig. 2. A multiple-relay network.

At first sight, both (2) and (3) look quite understandable and even straightforward. This is however misleading. If one looks closely at the formula (1), it is actually very surprising that the rate satisfying only these two constraints is feasible, since the maximization is over $p(x_1, x_2)$ rather than over $p(x_1)p(x_2)$, which can only be achieved by node 1 and node 2 cooperating with each other when transmitting signals. That this is feasible is rather surprising since there is always a positive delay before node 2 can decode the information concerning the intention of node 1. But, by that time, node 1 would have moved on to transmit new information. Hence, node 2 can never catch up with node 1, which raises the issue of how they can cooperate together to transmit to node 3.

Indeed, the coding scheme developed in [5] to achieve (1) is nontrivial. The essential technique used is what is called block Markov encoding, which also has profound applications in other areas of multi-user information theory, including the multiple-access channel with feedback [6].

B. Multiple Relays

A natural extension of the one-relay network in Fig. 1 is to the case of multiple relays, depicted in Fig. 2, where it takes multiple hops to send information from the source node 1 to the destination node n. A natural question is whether the formula (1) can be extended. Surprisingly, such an extension studied in [7]–[13] turned out to be not trivial at all. It was not until [9] that it was finally realized that the following rate is achievable:

$$R < \max_{p(x_1,\ldots,x_{n-1})} \min_{2\le k\le n} I(X_1,\ldots,X_{k-1};Y_k|X_k,\ldots,X_{n-1}). \tag{4}$$

Although (4) is still achievable with a decode-and-forward strategy, it is not achievable with the specific "irregular" encoding/successive decoding scheme developed in [5]. Instead, in [9], a "regular" encoding/"sliding-window" decoding scheme was shown to achieve (4). The sliding-window decoding had been used in [14] in the context of the multiple-access channel with generalized feedback. Later on, it was discovered in [11] that (4) can also be achieved with the "backward" decoding scheme, which was invented in [15] and has since been used for the one-relay channel in [16]. Recently, a modified successive decoding scheme (having the flavor of sliding-window decoding) was developed in [17] to achieve (4). Among all these schemes, the sliding-window decoding scheme is the simplest, while the backward decoding gives rises to large decoding delay and is also the most involved.

The formula (4) has a similar interpretation as (1). For each node $k \in \{2,\ldots,n\}$, the corresponding constraint is

$$R < I(X_1,\ldots,X_{k-1};Y_k|X_k,\ldots,X_{n-1}). \tag{5}$$

The conditional mutual information on the RHS above implicitly presumes that for the decoding at node k, the signals transmitted by nodes $k,\ldots,n-1$ are known *a priori*, and that the signals transmitted by nodes $1,\ldots,k-1$ are cooperating in providing the information. Let us examine the first issue of why node k should know what will be transmitted by nodes $k+1,\ldots,n-1$. The reason is that in this system, there is only one source-destination pair, for which information is passed along the route $1\to 2\to\cdots\to n$. Thus, any information obtained by nodes $k+1,\ldots,n-1$ has already been obtained by node k. Hence node k does indeed already know completely what nodes $k+1,\ldots,n-1$ know, and therefore knows what they will transmit.

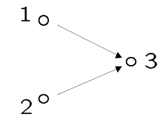

Fig. 3. A multiple access network.

The preceding interpretation illustrates the remarkable feature of formula (4): There is no interference at all in the whole network! To any node, the signal transmitted by any other node is either a "real" signal that can be used for decoding, or a *priori* known signal that can be subtracted completely.

Compared to the simple practice of regarding other transmitters as interferers, the relay schemes therefore seem to be obsessed with exploiting interference rather than succumbing to it. But is it worth it? How much can be gained by using such "smarter" schemes? A study in [9] shows that in wireless networks with low signal attenuation, the multirelay scheme can achieve higher-order (super-linear) scaling laws compared to those obtained in [2]. However, when signal attenuation is high enough, the scaling laws cannot be improved as proved in [9], but it is still possible to achieve substantially higher rates, especially for small-scale networks.

So far, we have been focused on the decode-and-forward strategy, which fundamentally relies on the relay nodes being able to decode the information they are transmitting. However, the relay nodes can help even without decoding the information themselves. This is the strategy employed in schemes such as compress-and-forward, amplify-and-forward, etc. Unfortunately, there is no single known relay strategy which is superior to all others in all scenarios. Which relay strategy is better really depends on the network topology, power distribution, etc. (see [18], [11]). For our study in this paper, we prefer and concentrate on the decode-and-forward strategy, since it is the only one where interference completely disappears (as in (4)). In either compress-and-forward or amplify-and-forward, interference is inevitably present, since without decoding, noise cannot be filtered out and will always be forwarded. But it should be noted that the decode-and-forward strategy may be severely suboptimal for some network topologies.

C. Multiple Sources

When there is only one source in the network and all transmitted signals are devoted to it, interference can be completely avoided as (4) demonstrates. But what if there are multiple sources? Will signals devoted to different sources always interfere with each other?

Consider the simple two-source network in Fig. 3, where nodes 1 and 2 are two separate sources. Let us consider the case where both source node 1 and source node 2 want to send information to the same destination node 3. Can they transmit at the same time and be both successful? The answer actually is yes, and is now well known. From the characterization of the capacity region for the multiple-access channel in [19] and [20], we know that not only can both transmissions simultaneously be successful, but also that simultaneous transmission is indeed a way to achieve the maximum rates, and can be realized using the coding scheme of code-division multiple access (CDMA).

Denoting the signals transmitted by node 1 and node 2 by $x_1(t)$ and $x_2(t)$, respectively, and the signal received by node 3 by $y_3(t)$, we consider the discrete memoryless channel where they are related by the probability transition function

$$p\left(y_3(t)|x_1(t), x_2(t)\right), \quad \text{for any time } t.$$

The following rate pair (R_1, R_2) is achievable with CDMA:

$$\begin{cases} R_1 < I(X_1; Y_3|X_2) & (6)\\ R_2 < I(X_2; Y_3|X_1) & (7)\\ R_1 + R_2 < I(X_1, X_2; Y_3). & (8)\end{cases}$$

This rate region (6)–(8) has been proved to be maximal in [19], [20]. If only (6) and (7) were present, it would seem that source 1 and source 2 can be decoded separately without interfering with each other. However, the sum-rate constraint (8) makes it impossible to achieve both (6) and (7) at the same time. Therefore, the two sources are indeed affecting each other. Nevertheless, (6)–(8) is already the best one can do.

The main purpose of this paper is to develop a multiple-relay scheme for networks with *multiple sources*. We will try to preserve the spirit of (4), in the sense that all useful signals are used, and all *a priori* known interferences are subtracted. We will also try to achieve the best rate region in a form similar to (6)–(8), for nodes that have multiple sources to decode. The final scheme will accordingly have the flavors of both the multiple-relay and multiple-access channels.

The remainder of this paper is organized as follows. We first consider a simple two-source network, and illustrate the essential ideas of the coding scheme in Section II. Then, in Section III, for general networks, we develop a general multisource multirelay scheme, and characterize the corresponding achievable rate region. Potential applications of this scheme to sensor networks are discussed in Section IV. In Section V, the exact capacity for the data downloading problem in sensor networks is obtained for some geometries, under phase fading unknown at the transmitter.

II. A Two-Source Relay Channel

As a starting point towards a general multisource multirelay coding scheme, consider the network depicted in Fig. 4, where two source nodes 1 and 2 want to send independent information to the same destination node 5, with nodes 3 and 4 acting as the relays.

According to their relative locations, two relay routes are chosen in the network: $1 \to 3 \to 5$ and $2 \to 4 \to 5$. That is, node 3 helps source node 1, and node 4 helps source node 2. For the reasons discussed in the introduction, we only consider the decode-and-forward strategy in the paper. Therefore,

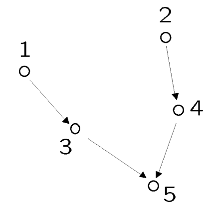

Fig. 4. A two-source relay network.

we consider schemes where node 3 needs to decode the information sent by node 1, and node 4 needs to decode the information sent by node 2.

Before rigorously stating the achievable rates for the network, we need introduce some information-theoretic notation for the network channel model. This network channel is modeled by

$$\left(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3 \times \mathcal{X}_4,\, p\left(y_3, y_4, y_5 |\, x_1, x_2, x_3, x_4\right),\, \mathcal{Y}_3 \times \mathcal{Y}_4 \times \mathcal{Y}_5\right)$$

where $\mathcal{X}_i$ and $\mathcal{Y}_j$ are finite input and output alphabets respectively, and $p\left(y_3, y_4, y_5 |\, x_1, x_2, x_3, x_4\right)$ is a probability distribution on $\mathcal{Y}_3 \times \mathcal{Y}_4 \times \mathcal{Y}_5$ for each $(x_1, x_2, x_3, x_4) \in \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3 \times \mathcal{X}_4$. At any time $t = 1, 2, \ldots$, each node $i \in \{1, 2, 3, 4\}$ sends $x_i(t) \in \mathcal{X}_i$ into the channel, and each node $j \in \{3, 4, 5\}$ receives $y_j(t) \in \mathcal{Y}_j$ from the channel. The distribution of the outputs $\left(y_3(t), y_4(t), y_5(t)\right)$ only depends on the inputs at the time t via

$$p\left(y_3(t), y_4(t), y_5(t) |\, x_1(t), x_2(t), x_3(t), x_4(t)\right).$$

While we consider only the case of finite alphabets, a continuous model can be approximated arbitrarily well by choosing the alphabet size large enough as in [21, Ch. 7].

The source nodes 1 and 2 transmit signals $x_1(t)$ and $x_2(t)$ based on the messages they want to send. The relay nodes 3 and 4 decide what to transmit based only on the signals they have already received

$$x_3(t) = f_{3,t}(y_3(1), \ldots, y_3(t-1))$$
$$x_4(t) = f_{4,t}(y_4(1), \ldots, y_4(t-1))$$

where $f_{3,t}(\cdot)$, $f_{4,t}(\cdot)$, $t \geq 1$ can be any functions. After a time block $1 \leq t \leq T$, the destination node 5 needs to decode both the messages sent by nodes 1 and 2 based on the signals $\{y_5(1), y_5(2), \ldots, y_5(T)\}$ it has received. The strict definitions of codes, encoding functions, decoding functions, probability of error, and achievable rates are as is standard in information theory. See, e.g., [12] for details.

Denote by R_1 and R_2 the transmission rates of source nodes 1 and 2, respectively. A rate pair (R_1, R_2) is said to be achievable if both the messages can be decoded at the destination node 5 with an arbitrarily small probability of error. We have the following theorem characterizing the achievable rate pairs.

Theorem 2.1: For the two-source relay network defined above, any rate pair (R_1, R_2) satisfying the following five inequalities is achievable:

$$R_1 < I(X_1; Y_3|X_3) \quad (9)$$
$$R_2 < I(X_2; Y_4|X_4) \quad (10)$$

and

$$\begin{cases} R_1 < I(X_1, X_3; Y_5 | X_2, X_4) & (11) \\ R_2 < I(X_2, X_4; Y_5 | X_1, X_3) & (12) \\ R_1 + R_2 < I(X_1, X_3, X_2, X_4; Y_5) & (13) \end{cases}$$

for some joint distribution $p(x_1, x_3)p(x_2, x_4)$.

The constraints (9) and (10) can be understood similarly to (2), since in our scheme node 3 needs to decode the information sent by node 1, and node 4 needs to decode the information sent by node 2. The constraints (11)–(13) are for the decoding at node 5, which looks like an extension of (6)–(8), only different in that now with the help of the relays, there are two inputs (X_1, X_3) for source 1, and two inputs (X_2, X_4) for source 2. Therefore, the achievable rate region (9)–(13) is a natural combination of multiple relay and multiple access. Note also the cooperative feature embodied in the optimization over $p(x_1, x_3)p(x_2, x_4)$.

What is the coding scheme to achieve (9)–(13)? Obviously, it should have both the elements of the decode-and-forward scheme as well as the code-division multiple-access scheme. Among the several decode-and-forward schemes mentioned in the introduction, the sliding-window decoding is the simplest, while, as noted above, the backward decoding is the most involved and also induces excessive delays. However, it turns out that for the case of multiple sources, only with the backward decoding can the rate region (9)–(13) be achieved. Neither the sliding-window decoding scheme nor the successive decoding scheme can achieve the same region.

The essential reason for the difference is that backward decoding is a *one-block-decision scheme*, while both the sliding-window decoding and the successive decoding schemes are *multiple-block-decision schemes*. Specifically, for the one-level relay network in Fig. 4, both the sliding-window decoding and the successive decoding need two consecutive blocks to make one decoding decision.

Proof of Theorem 2.1: Consider B blocks of transmission, each of T transmission slots. Two sequences of messages $w_1(b) \in \{1, \ldots, 2^{TR_1}\}$, and $w_2(b) \in \{1, \ldots, 2^{TR_2}\}$, $b = 1, 2, \ldots, B-1$ will be sent over in TB transmission slots. (Note that as $B \to \infty$, the rate $TR_i(B-1)/TB$ is arbitrarily close to R_i for any T, $i = 1, 2$.)

Consider any fixed $p(x_1, x_3)$ and $p(x_2, x_4)$. We use regular block Markov encoding for both the relay routes $1 \to 3 \to 5$ as well as $2 \to 4 \to 5$, according to $p(x_1|x_3)p(x_3)$ and $p(x_2|x_4)p(x_4)$ respectively; see [12] for the details of regular block Markov encoding.

The relay nodes 3 and 4 decode $w_1(b)$ and $w_2(b)$ respectively at the end of each block $b = 1, 2, \ldots, B-1$. The decoding error can be made arbitrarily small by choosing sufficiently large T, if

$$R_1 < I(X_1; Y_3 | X_3),$$
$$R_2 < I(X_2; Y_4 | X_4).$$

According to the regular block Markov encoding, in each block $b = 1, 2, \ldots, B$, depending on the message $w_1(b)$ of this block and the message $w_1(b-1)$ of the previous block, node 1 transmits a vector $\mathbf{x}_1(w_1(b)|w_1(b-1))$ of length T, and similarly, node 2 transmits a vector $\mathbf{x}_2(w_2(b)|w_2(b-1))$; and if without decoding error, the relay nodes 3 and 4 would transmit vectors $\mathbf{x}_3(w_1(b-1))$ and $\mathbf{x}_4(w_2(b-1))$, respectively. Note that a special arrangement is needed at block 1 and block B, for which we set $w_1(0) = w_2(0) = 1$ and $w_1(B) = w_2(B) = 1$.

The destination node 5 does not commence decoding until the end of block B, when it starts to decode backwardly. First, it decodes $w_1(B-1)$ and $w_2(B-1)$, based on the vector $\vec{Y}_5(B)$ it received in block B. It declares $(w_1(B-1), w_2(B-1)) = (w_1, w_2)$ if $w_1 \in \{1, \ldots, 2^{TR_1}\}$ and $w_2 \in \{1, \ldots, 2^{TR_2}\}$ is the unique pair such that

$$\big(\mathbf{x}_1(1|w_1), \mathbf{x}_2(1|w_2), \mathbf{x}_3(w_1), \mathbf{x}_4(w_2), \vec{Y}_5(B)\big)$$

is jointly typical according to

$$p(x_1, x_3)p(x_2, x_4)p(y_5|x_1, x_2, x_3, x_4).$$

This is a two-source decoding process with each source having two inputs. According to the results of the multiple-access channel [22, Sec. 14.3], the decoding would be successful with a high probability, if (11)–(13) hold.

Then recursively for $b = B-1, \ldots, 2$, based on the knowledge of $(w_1(b), w_2(b))$, node 5 decodes $(w_1(b-1), w_2(b-1))$ by checking the joint typicality of

$$\big(\mathbf{x}_1(w_1(b)|w_1(b-1)), \mathbf{x}_2(w_2(b)|w_2(b-1)), \mathbf{x}_3(w_1(b-1)), \mathbf{x}_4(w_2(b-1)), \vec{Y}_5(b)\big).$$

Similarly, this would be successful with a high probability, if (11)–(13) hold. □

However, if instead of backward decoding, node 5 uses sliding-window decoding, then it would need to first decode $(w_1(1), w_2(1))$ based on blocks 1 and 2 as follows: It declares $(w_1(1), w_2(1)) = (w_1, w_2)$, if (w_1, w_2) is the unique pair such that in block 1

$$\big(\mathbf{x}_1(w_1|1), \mathbf{x}_2(w_2|1), \mathbf{x}_3(1), \mathbf{x}_4(1), \vec{Y}_5(1)\big) \tag{14}$$

is jointly typical, and also in block 2

$$\big(\mathbf{x}_3(w_1), \mathbf{x}_4(w_2), \vec{Y}_5(2)\big) \tag{15}$$

is jointly typical. The decoding would be successful with a high probability if and only if

$$\begin{cases} R_1 < I(X_1; Y_5 | X_2, X_3, X_4) + I(X_3; Y_5 | X_4) & (16) \\ R_2 < I(X_2; Y_5 | X_1, X_3, X_4) + I(X_4; Y_5 | X_3) & (17) \\ R_1 + R_2 < I(X_1, X_2, X_3, X_4; Y_5) & (18) \end{cases}$$

where (16) accounts for the error events where w_1 is wrong but w_2 is correct; (17) accounts for the error events where w_2 is wrong but w_1 is correct; and (18) accounts for the error events where both w_1 and w_2 are wrong, noting that $I(X_1, X_2, X_3, X_4; Y_5)$ follows from

$$I(X_1, X_2; Y_5 | X_3, X_4) + I(X_3, X_4; Y_5). \tag{19}$$

Obviously, in all the three bounds (16)–(19), the first mutual information comes from the typicality check (14), and the second mutual information comes from the typicality check (15).

If we insert X_2 and X_1 into the conditional part of the second mutual information in (16) and (17) respectively, we will obtain the same three bounds as (11)–(13). Since (X_1, X_3) are independent of (X_2, X_4) for any joint distribution $p(x_1, x_3)p(x_2, x_4)$, this insertion will generally increase the bounds. Hence, generally, the sliding-window decoding achieves lower rates.

In the relay structure shown in Fig. 4, since node 3 does not decode source 2, the signals transmitted by nodes 2 and 4 cause interference to it. Similarly, nodes 1 and 3 cause interference to node 4. These interferences can also be seen from (9)–(10). It is possible to change the relay structure to avoid such interferences. For example, we could let node 3 also decode source 2, so that nodes 2 and 4 no longer cause interference to node 3. This may not be a wise choice, however, since depending on the network topology, it may be even harder for source 2 to reach node 3 than for it to reach the destination node 5.

A special multisource one-relay network has been considered in [23], [24], where multiple sources try to send to the same destination via the same relay node, and the relay node needs to decode all the sources. An achievable rate region using backward decoding was obtained in [24], where both the constraints for the relay and for the destination are like multiple-access.

It is worth noting that in the context of multiple-access channel with generalized feedback, it has been discovered ([15], [16], [25]) that backward decoding can achieve higher rates than either successive decoding or sliding-window decoding.

III. General Networks

In this section, we develop a general multisource, multidestination, multirelay scheme for general networks.

Consider a network of n nodes $\mathcal{N} = \{1, 2, \ldots, n\}$. We consider the multisource multi-cast problem, where there can be more than one source in the network. Each source originates at a single node and may have multiple destinations. Let $\mathcal{M} = \{1, 2, \ldots, m\}$ denote the set of sources. Any source $k \in \mathcal{M}$ corresponds to a source node $s^{(k)} \in \mathcal{N}$ and a set of destination nodes $\mathcal{D}^{(k)} \subset \mathcal{N}$. The communication task is to send the information of source k from the source node $s^{(k)}$ to all the nodes in $\mathcal{D}^{(k)}$ over the network. Note that the number m can be greater than n, since multiple sources having different destinations can originate from the same node.

Consider a multirelay route $\mathcal{N}^{(k)} \subseteq \mathcal{N}$ for each source $k \in \mathcal{M}$, where, $\mathcal{N}^{(k)}$ is an ordered set of nodes starting with $s^{(k)}$. For any $i, j \in \mathcal{N}^{(k)}$, the order is defined by $i \overset{(k)}{\prec} j$ if node i is upstream of node j along the route. Since all the nodes on the multirelay route will obtain the source information, the multicast task is fulfilled as long as the route is chosen such that $\mathcal{D}^{(k)} \subset \mathcal{N}^{(k)}$.

Consider a discrete memoryless network channel model described by

$$\big(\mathcal{X}_1 \times \cdots \times \mathcal{X}_n,\, p\,(y_1, \ldots, y_n |\, x_1, \ldots, x_n),\, \mathcal{Y}_1 \times \cdots \times \mathcal{Y}_n\big)$$

where $\mathcal{X}_i$ and $\mathcal{Y}_i$, $i = 1, \ldots, n$ are finite input and output alphabets respectively, and $p\,(y_1, \ldots, y_n |\, x_1, \ldots, x_n)$ is a probability distribution on $\mathcal{Y}_1 \times \cdots \times \mathcal{Y}_n$ for each $(x_1, \ldots, x_n)$. At any time $t = 1, 2, \ldots$, each node $i \in \mathcal{N}$ sends $x_i(t) \in \mathcal{X}_i$ into the channel and receives $y_i(t) \in \mathcal{Y}_i$ from the channel. The distribution of the outputs $\big(y_1(t), \ldots, y_n(t)\big)$ depends only on the inputs at the time t via

$$p\,\big(y_1(t), \ldots, y_n(t) |\, x_1(t), \ldots, x_n(t)\big).$$

We choose a multirelay route $\mathcal{N}^{(k)} \subseteq \mathcal{N}$ for each source $k \in \mathcal{M}$, such that $\mathcal{D}^{(k)} \subset \mathcal{N}^{(k)}$. Along each route, we use the scheme of regular block Markov encoding/backward decoding. These m routes can be united into a joint backward decoding scheme, if

A1): It is possible to assign a nonnegative integer to each node in the network, such that along any multirelay route excluding the source node, the integers are strictly increasing.

We note that this assumption rules out two-way communication as well as other non-acyclic unions of routes, as we note in Remark 3.1 below. It should also be noted that the condition (A1) depends on the specific relay routes chosen, and different relay routes may be chosen for the same network topology.

For any source $k \in \mathcal{M}$, introduce an auxiliary random variable $U_i^{(k)}$ with cardinality equal to $|\mathcal{X}_i|$ for each node $i \in \mathcal{N}^{(k)}$. Loosely speaking, $U_i^{(k)}$ stands for the information node i has of source k. Denote

$$\mathcal{U}^{(k)} = \{U_j^{(k)} : j \in \mathcal{N}^{(k)}\}.$$

Since $\mathcal{N}^{(k)}$ is an ordered set of nodes as defined above, $\mathcal{U}^{(k)}$ is an ordered list of random variables. Consequently, define

$$\begin{aligned} \mathcal{U}_{i-}^{(k)} &= \{U_j^{(k)} : j \overset{(k)}{\prec} i,\, j \in \mathcal{N}^{(k)}\} \\ \mathcal{U}_{i+}^{(k)} &= \{U_j^{(k)} : i \overset{(k)}{\prec} j,\, j \in \mathcal{N}^{(k)}\}. \end{aligned}$$

For any node $i \in \mathcal{N}$, denote by $\mathcal{M}_i := \{k : i \in \mathcal{N}^{(k)}\}$ the set of all the sources with the multirelay route passing through node i. For any $\mathcal{S} \subseteq \mathcal{M}_i$, let

$$\begin{aligned} \mathcal{U}_i^{(\mathcal{S})} &= \{U_i^{(k)} : k \in \mathcal{S}\} \\ \mathcal{U}^{(\mathcal{S})} &= \bigcup_{k \in \mathcal{S}} \mathcal{U}^{(k)} \\ \mathcal{U}_{i-}^{(\mathcal{S})} &= \bigcup_{k \in \mathcal{S}} \mathcal{U}_{i-}^{(k)} \\ \mathcal{U}_{i+}^{(\mathcal{S})} &= \bigcup_{k \in \mathcal{S}} \mathcal{U}_{i+}^{(k)}. \end{aligned}$$

Then we have the following characterization of the m rates simultaneously achievable along the m multirelay routes by a joint backward decoding scheme.

Theorem 3.1: Under the assumption (A1), a rate vector $R^{(\mathcal{M})} = (R^{(1)}, R^{(2)}, \ldots, R^{(m)})$ is achievable if there exist some product distribution

$$\prod_{k \in \mathcal{M}} p\,(u_j^{(k)},\, j \in \mathcal{N}^{(k)})$$

and some functions

$$x_i = f_i(u_i^{(k)}, k \in \mathcal{M}_i), \quad i \in \mathcal{N}$$

such that for any node $i \in \mathcal{N}$ and any $\mathcal{S} \subseteq \mathcal{M}_i$

$$\sum_{k \in \mathcal{S}} R^{(k)} < I(\mathcal{U}_{i-}^{(\mathcal{S})}; Y_i | \mathcal{U}_i^{(\mathcal{S})}, \mathcal{U}_{i+}^{(\mathcal{S})}, \mathcal{U}^{(\mathcal{M}_i \setminus \mathcal{S})}). \tag{20}$$

Proof: Consider an extended channel, where for each node $i \in \mathcal{N}$, we set $x_i = f_i(u_i^{(k)}, k \in \mathcal{M}_i)$. That is, instead of x_i, regard $u_i^{(k)}, k \in \mathcal{M}_i$, as the multiple inputs of node i to the channel.

Regular Block Markov Encoding: For each source k, taking $u_i^{(k)}$ as the input of each node $i \in \mathcal{N}^{(k)}$, generate a multirelay codebook along the route $\mathcal{N}^{(k)}$, according to

$$p\,(u_j^{(k)}, j \in \mathcal{N}^{(k)})$$

in the same way as done in [12]. The codebook is generated in the reverse order of the route, so that for any node $i \in \mathcal{N}^{(k)}$, the inputs $\mathcal{U}_{i+}^{(k)}$ of the downstream nodes are predictable, and the inputs $\mathcal{U}_{i-}^{(k)}$ of the upstream nodes provide the information of source k.

Joint Backward Decoding: The idea of backward decoding was introduced in [15]. It was applied to the three-node relay network in [16]. Before we develop a joint backward decoding scheme for multiple sources with multiple routes, we first examine the three-node relay channel to convey the essential idea of this scheme.

Consider a three-node relay network, where node 1 is the source, trying to send information to node 3 via node 2. The route is $1 \to 2 \to 3$.

Consider B blocks of transmission, each of T transmission slots. In each block $b = 1, 2, \ldots, B-1$, node 1 sends new information $w(b) \in \{1, 2, \ldots, 2^{TR}\}$, until in the last block B, $w(B)$ is always set to be 1. At the end of each block $b = 1, 2, \ldots, B-1$, node 2 can decode the information $w(b)$ with an arbitrarily small probability of error if $R < I(X_1; Y_2 | X_2, X_3)$. Then in the next block $b+1 = 2, 3, \ldots, B$, node 2 sends $w(b)$. Hence in each block $b = 2, 3, \ldots, B$, node 1 sends $w(b)$ and node 2 sends $w(b-1)$. Note that node 1 chooses its codebook based on what node 2 is transmitting, according to the regular encoding scheme.

For node 3, the decoding does not happen until the end of block B. During the block B, node 2 sends $w(B-1)$; and node 1 does not send any new information by setting $w(B) = 1$, but it is helping node 2 by choosing its codebook according to $w(B-1)$. Hence node 3 can decode $w(B-1)$ with an arbitrarily small probability of error, if $R < I(X_1, X_2; Y_3 | X_3)$. Now, with $w(B-1)$ known to node 3, it appears as if node 1 is not sending any new information during the block $B-1$, so that $w(B-2)$ can similarly be decoded, based on what node 3 received during the block $B-1$. This process continues until block 2, whence $w(1)$ is finally decoded. Note that the decoding of $w(1), w(2), \ldots, w(B-1)$ at node 3 is done backwardly. This is the reason for calling this scheme "backward decoding."

Note that the actually achieved rate is $\frac{B-1}{B}R$. However, this can be arbitrarily close to R by letting $B \to \infty$.

Now, suppose there is one more node, node 4, down the route: $1 \to 2 \to 3 \to 4$. Then we need add another layer of blocks, for backward decoding.

Consider B^2 blocks of transmission. Sequentially, every B blocks are assigned into a group. So, in total, there are B groups. In each group $g = 1, 2, \ldots, B-1$, since there are B blocks, node 1, node 2 and node 3 can perform the functions described above, so that, at the end of the group, node 3 can decode all the information sent by node 1 at the following rate:

$$R < \min\{I(X_1; Y_2 | X_2, X_3, X_4),\, I(X_1, X_2; Y_3 | X_3, X_4)\}.$$

Then, in the next group $g = 2, 3, \ldots, B$, node 3 sends this piece of information to node 4. In the last group $g = B$, node 1 and node 2 will not send any new information, but just help node 3 in the transmission to node 4. Node 4 waits till the end of all the groups, then it does all the decoding starting from the group B in the same backward manner. The decoding succeeds with an arbitrarily small probability of error if $R < I(X_1, X_2, X_3; Y_4 | X_4)$.

Note that the actually achieved rate is $\left(\frac{B-1}{B}\right)^2 R$. However, it also can also be arbitrarily close to R by letting $B \to \infty$.

Now it is clear how to extend this backward decoding scheme to a multirelay route with an arbitrary number of nodes: $1 \to 2 \to \cdots \to n$. For each additional node, we only need to add another layer of blocks. Node 2 decodes at the end of every block; node 3 decodes at the end of every B blocks; node 4 decodes at the end of every B^2 blocks; and, in general, node n decodes at the end of every B^{n-2} blocks. For each node $i \in \{2, 3, \ldots, n\}$, by using the backward decoding scheme, the upstream nodes $1, \ldots, i-1$ appear to be cooperating in sending information to it, while the inputs by the downstream nodes $i+1, \ldots, n$ are totally predictable. Hence, the following rate is achievable:

$$R < \min_{2 \le i \le n} I(X_1, \ldots, X_{i-1}; Y_i | X_i, \ldots, X_n).$$

Actually, instead of every $1, B, B^2, \ldots, B^{n-2}$ blocks, the decoding time of each node $i \in \{2, \ldots, n\}$ can be scheduled at the end of every B^{j_i} blocks,[1] as long as these integers $\{j_i, i = 2, \ldots, n\}$ satisfy $0 \le j_2 < j_3 < \cdots < j_n$. Of course, to minimize the decoding delay, it is the best to set $j_i = i-2$ as in the above. But this flexibility of choosing decoding times allows the merging of multiple multirelay routes into a joint backward decoding scheme, as explained in the sequel.

Finally, we arrive at the setup of multiple multirelay routes. Under the assumption A1), each node $i \in \mathcal{N}$ is assigned a nonnegative integer j_i, and along any multirelay route excluding its source node, the integers are in a strictly increasing order. Hence, by scheduling the decoding time of each node $i \in \mathcal{N}$ at the end of every B^{j_i} blocks, backward decoding can be performed along each of these multirelay routes, and for any node that lies on multiple routes, it decodes multiple sources jointly at the same time. This constitutes a multiple-access channel, and the achievable rate region is characterized by the CDMA bound (20).

[1]Correspondingly, node $i-1$ schedules its transmission in every $B^{j_i - 1}$ blocks.

Fig. 5. A two-way multirelay network.

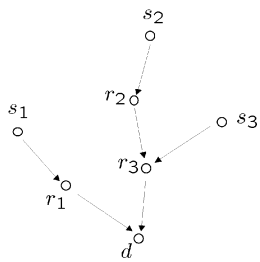

Fig. 6. A multisource single-sink sensor network.

Remark 3.1: The assumption A1) is necessary for us to be able to merge multiple routes into a joint backward decoding scheme. To see this, consider the network in Fig. 5, where there are two routes: $1 \to 2 \to 3 \to 4$ and $4 \to 3 \to 2 \to 1$. Obviously, the assumption (A1) does not hold for this network, since it is impossible to assign integers j_2 to node 2 and j_3 to node 3, so that both $j_2 < j_3$ and $j_2 > j_3$. Since in the backward decoding scheme, a node must decode at a faster frequency than the node after it, it creates a conflict in that node 2 needs to decode more frequently than node 3 (along the route $1 \to 2 \to 3 \to 4$), while node 3 too needs to decode more frequently than node 2 (along the route $4 \to 3 \to 2 \to 1$).

IV. Application to Sensor Networks

Some sensor networks consist of several sensor equipped nodes collecting independent information that needs to be communicated to a designated collector or sink node; called the data downloading problem in [26]. Such sensor networks can accordingly be modelled as multisource single-sink networks.

In the example depicted in Fig. 6, three source nodes, s_1, s_2 and s_3, desire to send information to the same destination node d, using, as relays, the nodes r_1, r_2 and r_3. By Theorem 3.1, the rate vector $(R^{(1)}, R^{(2)}, R^{(3)})$ is achievable if there exist some product distribution

$$p\,(u_{s_1}^{(1)}, u_{r_1}^{(1)}, u_d^{(1)})p\,(u_{s_2}^{(2)}, u_{r_2}^{(2)}, u_{r_3}^{(2)}, u_d^{(2)})p\,(u_{s_3}^{(3)}, u_{r_3}^{(3)}, u_d^{(3)})$$

and some functions

$$\begin{aligned} x_{s_1} &= f_{s_1}(u_{s_1}^{(1)}), \quad x_{s_2} = f_{s_2}(u_{s_2}^{(2)}), \quad x_{s_3} = f_{s_3}(u_{s_3}^{(3)}) \\ x_{r_1} &= f_{r_1}(u_{r_1}^{(1)}), \quad x_{r_2} = f_{r_2}(u_{r_2}^{(2)}) \\ x_{r_3} &= f_{r_3}(u_{r_3}^{(2)}, u_{r_3}^{(3)}) \\ x_d &= f_d(u_d^{(1)}, u_d^{(2)}, u_d^{(3)}) \end{aligned}$$

such that for node r_1,

$$R^{(1)} < I(U_{s_1}^{(1)}; Y_{r_1} | U_{r_1}^{(1)}, U_d^{(1)})$$

for node r_2

$$R^{(2)} < I(U_{s_2}^{(2)}; Y_{r_2} | U_{r_2}^{(2)}, U_{r_3}^{(2)}, U_d^{(2)})$$

for node r_3

$$\begin{cases} R^{(2)} < I(U_{s_2}^{(2)}, U_{r_2}^{(2)}; Y_{r_3} | U_{r_3}^{(2)}, U_d^{(2)}, U_{s_3}^{(3)}, U_{r_3}^{(3)}, U_d^{(3)}) \\ R^{(3)} < I(U_{s_3}^{(3)}; Y_{r_3} | U_{r_3}^{(3)}, U_d^{(3)}, U_{s_2}^{(2)}, U_{r_2}^{(2)}, U_{r_3}^{(2)}, U_d^{(2)}) \\ R^{(2)} + R^{(3)} < \\ \quad I(U_{s_2}^{(2)}, U_{r_2}^{(2)}, U_{s_3}^{(3)}; Y_{r_3} | U_{r_3}^{(2)}, U_d^{(2)}, U_{r_3}^{(3)}, U_d^{(3)}) \end{cases}$$

while, for node d, the inequalities (21) at the bottom of the page hold.

The set of inequalities characterizing the achievable rate region may seem very complicated, especially if there are many multirelay routes crisscrossing each other in the network. However, the rules to follow when writing down the inequalities (20) for each node are actually quite simple. For any node, only the routes passing it are of any concern, and all the other routes with the corresponding sources and inputs appear invisible. If a node i is on only one route, say, $\mathcal{N}^{(k_i)}$ of the source k_i, then only one inequality applies

$$R^{(k_i)} < I(\mathcal{U}_{i-}^{(k_i)}, Y_i | U_i^{(k_i)}, \mathcal{U}_{i+}^{(k_i)})$$

where $\mathcal{U}_{i-}^{(k_i)}$ and $\mathcal{U}_{i+}^{(k_i)}$ are the inputs (corresponding to the source k_i) of the upstream nodes and the downstream nodes respectively. Actually, $U_i^{(k_i)}$ can be equivalently replaced by X_i, since node i has no other auxiliary inputs. On the other hand,

$$\begin{cases} R^{(1)} < I(U_{s_1}^{(1)}, U_{r_1}^{(1)}; Y_d | U_d^{(1)}, U_{s_2}^{(2)}, U_{r_2}^{(2)}, U_{r_3}^{(2)}, U_d^{(2)}, U_{s_3}^{(3)}, U_{r_3}^{(3)}, U_d^{(3)}) \\ R^{(2)} < I(U_{s_2}^{(2)}, U_{r_2}^{(2)}, U_{r_3}^{(2)}; Y_d | U_d^{(2)}, U_{s_1}^{(1)}, U_{r_1}^{(1)}, U_d^{(1)}, U_{s_3}^{(3)}, U_{r_3}^{(3)}, U_d^{(3)}) \\ R^{(3)} < I(U_{s_3}^{(3)}, U_{r_3}^{(3)}; Y_d | U_d^{(3)}, U_{s_1}^{(1)}, U_{r_1}^{(1)}, U_d^{(1)}, U_{s_2}^{(2)}, U_{r_2}^{(2)}, U_{r_3}^{(2)}, U_d^{(2)}) \\ R^{(1)} + R^{(2)} < I(U_{s_1}^{(1)}, U_{r_1}^{(1)}, U_{s_2}^{(2)}, U_{r_2}^{(2)}, U_{r_3}^{(2)}; Y_d | U_d^{(1)}, U_d^{(2)}, U_{s_3}^{(3)}, U_{r_3}^{(3)}, U_d^{(3)}) \\ R^{(1)} + R^{(3)} < I(U_{s_1}^{(1)}, U_{r_1}^{(1)}, U_{s_3}^{(3)}, U_{r_3}^{(3)}; Y_d | U_d^{(1)}, U_d^{(3)}, U_{s_2}^{(2)}, U_{r_2}^{(2)}, U_{r_3}^{(2)}, U_d^{(2)}) \\ R^{(2)} + R^{(3)} < I(U_{s_2}^{(2)}, U_{r_2}^{(2)}, U_{r_3}^{(2)}, U_{s_3}^{(3)}, U_{r_3}^{(3)}; Y_d | U_d^{(2)}, U_d^{(3)}, U_{s_1}^{(1)}, U_{r_1}^{(1)}, U_d^{(1)}) \\ R^{(1)} + R^{(2)} + R^{(3)} < I(U_{s_1}^{(1)}, U_{r_1}^{(1)}, U_{s_2}^{(2)}, U_{r_2}^{(2)}, U_{r_3}^{(2)}, U_{s_3}^{(3)}, U_{r_3}^{(3)}; Y_d | U_d^{(1)}, U_d^{(2)}, U_d^{(3)}). \end{cases} \tag{21}$$

for a node at the intersection of $\ell > 1$ routes, its environment is similar to a multiple-access channel with ℓ sources: with each source providing a set of inputs of the corresponding upstream nodes, while the inputs of the downstream nodes are known.

Remark 4.1: The advantage of applying the multisource multirelay scheme to such multisource single-sink networks, as the one shown in Fig. 3, is obvious. Since multiple sources converge to a single sink, traffic gets concentrated and increases as one gets closer to the sink. If a traditional multihop scheme that does not exploit information theory is used, then the bottleneck of the whole network would be the area around the sink where the links carry the heaviest traffic. However, by utilizing a multirelay scheme, each node makes use of the inputs of all the upstream nodes. As one gets closer to the sink, there are more upstream nodes to help, which means higher received signal power and thus higher achievable rates. For the sink node especially, all the inputs of all the other nodes can be used. Importantly, and fortunately, we note that for networks with such a tree structure, the condition A1) does indeed hold, which means that joint backward decoding can be used.

V. Some Capacity Results

In this section, we consider a special case of the above mentioned data downloading problem with AWGN channel models, where the exact capacity can be determined. Our motivation comes from [11], where it was shown that under the assumption of phase fading, with the phase not known to the transmitter, so that transmitters cannot achieve coherent beamforming, the capacity is indeed achieved by the decode-and-forward relay scheme, whenever the relays are located close enough to the source. This was the first capacity result for wireless relay channels under realistic assumptions.

Here, we try to extend the same idea to the case of multiple sources. Consider a two-source one-sink wireless network as depicted in Fig. 4. Denote the distance between any two nodes i and j by d_{ij}. Let P_i be the power constraint of node i. Let N_i be the variance of the additive white Gaussian noise at node i. Then, from (20), we have the following achievable rate region (a simplified version has been given in (9)–(13)).

For node 3

$$R^{(1)} < \log\left(1+\frac{P_1/d_{13}^{\alpha}}{N_3+P_2/d_{23}^{\alpha}+P_4/d_{43}^{\alpha}}\right) \tag{22}$$

for node 4

$$R^{(2)} < \log\left(1+\frac{P_2/d_{24}^{\alpha}}{N_4+P_1/d_{14}^{\alpha}+P_3/d_{34}^{\alpha}}\right) \tag{23}$$

and for node 5

$$\begin{cases} R^{(1)} < \log\left(1+\dfrac{P_1/d_{15}^{\alpha}+P_3/d_{35}^{\alpha}}{N_5}\right) & (24)\\ R^{(2)} < \log\left(1+\dfrac{P_2/d_{25}^{\alpha}+P_4/d_{45}^{\alpha}}{N_5}\right) & (25)\\ R^{(1)}+R^{(2)} < \\ \quad \log\left(1+\dfrac{P_1/d_{15}^{\alpha}+P_2/d_{25}^{\alpha}+P_3/d_{35}^{\alpha}+P_4/d_{45}^{\alpha}}{N_5}\right) & (26)\end{cases}$$

where α denotes the path-loss exponent. We assume phase fading, with the phase unknown to the transmitter, so that no coherent beamforming can be or is exploited, as can be seen from the above. On the other hand, the phase fading assumption also leads to the conclusion that the inequality (26) is actually the cut-set bound for node 5, since the RHS of (26) indeed features the maximum signal power that can be received by node 5. Therefore, the throughput capacity, maximizing $R^{(1)}+R^{(2)}$, is achieved once (26) is tight among these five inequalities, i.e.,

$$\begin{aligned} &\min\{\text{The RHS of } (22),\ \text{The RHS of } (24)\}\\ &\quad+\min\{\text{The RHS of } (23),\ \text{The RHS of } (25)\}\\ &\geq \text{The RHS of } (26). \end{aligned} \tag{27}$$

Therefore, the joint backward decoding actually achieves the throughput capacity (26), provided the condition (27) holds. It can be easily seen that the condition (27) depends on the distances between the nodes, and also the signal-to-noise-ratios (SNRs) P_i/N_j. For fixed SNRs, obviously, a special geometry for (27) to hold is that node 5 is far away from the other nodes, such that the distances $\{d_{i5}\}$ are sufficiently larger than the other distances $\{d_{ij}, j \neq 5\}$.

In a similar fashion, one can determine when the joint backward decoding achieves the throughput capacity for more complicated networks.

Finally, an observation worth pointing out is that the bounds (22) for node 3, and (23) for node 4, are determined by signal-to-interference-plus-noise-ratio (SINR) instead of just SNR as in the bounds (24)–(26) for node 5. The reason is that node 3 is not supposed to decode source 2, and node 4 is not supposed to decode source 1. Therefore, the signals transmitted by the second route (namely, node 2 and node 4) cause interference to node 3, and vice versa for node 4.

VI. Improvements by Time-Sharing

The achievable rate regions stated in Theorem 2.1 and Theorem 3.1 can be further improved by introducing a time-sharing random variable Q, as is a standard technique in multi-user information theory [22, Sec. 14.3.3]. Hence the rate region (9)–(13) can be expanded to

$$\begin{aligned} R_1 &< I(X_1;Y_3|X_3,Q)\\ R_2 &< I(X_2;Y_4|X_4,Q) \end{aligned}$$

and

$$\begin{cases} R_1 < I(X_1,X_3;Y_5|X_2,X_4,Q)\\ R_2 < I(X_2,X_4;Y_5|X_1,X_3,Q)\\ R_1+R_2 < I(X_1,X_3,X_2,X_4;Y_5,Q) \end{cases}$$

for some joint distribution $p(q)p(x_1,x_3|q)p(x_2,x_4|q)$. Similarly, the rate region (20) can be expanded to

$$\sum_{k\in\mathcal{S}} R^{(k)} < I(\mathcal{U}_{i-}^{(\mathcal{S})};Y_i|\mathcal{U}_i^{(\mathcal{S})},\mathcal{U}_{i+}^{(\mathcal{S})},\mathcal{U}^{(\mathcal{M}_i\setminus\mathcal{S})},Q)$$

for some joint distribution $p(q)\prod_{k\in\mathcal{M}} p(u_j^{(k)},\ j\in\mathcal{N}^{(k)}|q)$, and some functions $x_i = f_i(u_i^{(k)}, k\in\mathcal{M}_i, q)$, for $i\in\mathcal{N}$.

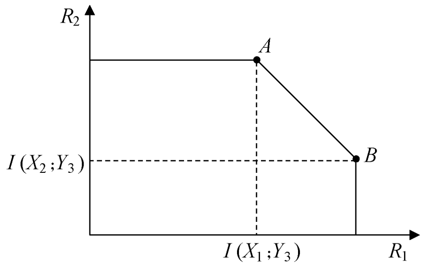

Fig. 7. The capacity region of a multiple-access channel.

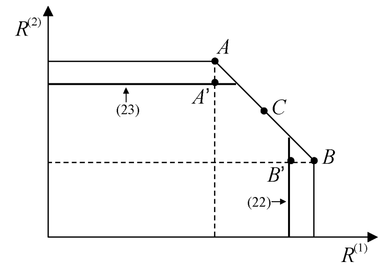

Fig. 8. An achievable rate region of a two-source relay network.

As we have discussed, for the general multisource multirelay networks considered in this paper, among the several known decode-and-forward coding schemes, the joint backward decoding provides the largest achievable rate region with a nice and compact formula. However, a major drawback with backward decoding is the excessive delay, especially when the number of relays is large. Therefore, it would be of great interest to develop simpler coding schemes that can achieve the same rate region with less delay.

An idea to study in this regard is inspired by the simple multiple-access channel, for which it is well known that the capacity region can also be achieved by time-sharing between the corner points. Consider, for example, the capacity region (6)–(8) of a multiple-access channel with two sources, depicted in Fig. 7 as a pentagon. Any rate pair (R_1, R_2) corresponding to a point inside this pentagon can be achieved directly by joint decoding of both sources. However, actually, instead of joint decoding, the corner point A can also be achieved by first decoding source 1 with source 2 treated as purely interference at the rate $R_1 < I(X_1; Y_3)$, and then decoding source 2 with source 1 completely known, at the rate $R_2 < I(X_2; Y_3|X_1)$. Similarly, the reverse decoding order leads to the corner point B. Then, with all the corner points so achieved, the rest of the points inside the pentagon can be achieved by time sharing.

Such a successive decoding strategy can also be applied to the multisource, multirelay network considered in this paper. With each source treated individually in a sequential manner, sliding-window decoding can be used for each relay route, and the resulted coding scheme can be much simpler than the joint backward decoding. But unfortunately, except for some simple scenarios like the multiple-access channel in Fig. 3, the achievable rate region is generally smaller if there are relays.

For example, consider the two-source relay network with the achievable rate region given by (22)–(26). The constraints (24)–(26) define a pentagon like the one shown in Fig. 7. With the additional constraints (22)–(23) of the relays, the achievable rate region can be further confined as depicted in Fig. 8. While the point C is still achievable, it is not achievable by time sharing between A' and B', which are the corner points resulted from successive decoding now.

For the special multisource one-relay network studied in [23], [24], an offset encoding scheme has been proposed in [27], which, combined with sliding-window decoding, can achieve the corner points with much less delay compared to backward decoding. Although time sharing was used in [27] to achieve the entire backward decoding region, a recent discovery in [28] shows that at least for the two-source case, time sharing is not needed if both offset encoding and nonoffset encoding are used. For the example discussed above, it is easy to check that combining offset and nonoffset regions recovers the entire backward decoding region (22)–(26). It remains an interesting open question whether for the general networks considered in this paper, the backward decoding can be replaced.

VII. Concluding Remarks

We have considered here problems involving wireless networks with multiple sources, each with perhaps multiple destinations as in multicast and multiple-relay nodes. These problems feature aspects of both relay channels as well as multiple-access channels. We have shown that while in relay channels, several variants of decode-and-forward can all achieve the same feasible region, in contrast, when there are multiple sources and routes that intersect, backward decoding can actually achieve superior rates. For certain special scenarios of the data downloading problem in sensor networks, such backward decoding can indeed achieve the exact capacity region for some geometries when there is phase fading that is unknown at the transmitter.

The main theme of this study, exploiting "interference" rather than succumbing to it, has been the motivation to develop more sophisticated coding schemes for wireless networks. Clearly, besides the relay and multiple-access schemes studied in this paper, other basic schemes in multiuser information theory also have the potential to be incorporated into the framework for further improvement. For example, much research has been done on relay broadcast channels (see [29] and the references therein), and it is certainly of interest to develop general schemes which feature relay, multiple-access and broadcast altogether.

Another potential improvement may come from network coding. For example, the two-way multirelay network shown in Fig. 5 is not a structure where joint backward decoding can be applied, but it is well suited for network coding where information flows of different directions can be combined at the intermediate nodes (see e.g., [30]).

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