Assignment 1 Solution

Problem 6

(a) There are $2^4 = 16$ sequences in total. The probability of getting each sequence is 1/16. Thus, $P(TTTT) = 1/16$

(b) There are 4 sequences with three heads and one tail. The probability of getting this pattern is 4/16=1/4

(c) There are two patterns satisfying "more heads than tails": no tails or one tail. $P(more\ heads\ than\ tail) = P(no\ tail) + P(3\ heads\ and\ one\ tail) = 1/16 + 1/4 = 5/16$

(d)$P(more\ heads\ than\ tails|at\ least\ one\ tail) = \frac{P(more\ heads\ than\ tails \bigcap at\ least\ one\ tail)}{P(at\ least\ one\ tail)}$
$= \frac{P(3\ heads\ and\ one\ tail)}{P(at\ least\ one\ tail)} = \frac{1/4}{1-1/16} = 4/15$

(e)$P(more\ heads\ than\ tails|fewer\ than\ two\ tails) = 1$

Problem 7

(a) $P(T) = 1 - P(T^c) = 0.3$

(b) $P(S \bigcap T) = P(S) + P(T) - P(S \bigcup T) = 0.4 + 0.3 - 0.7 = 0$

(c) $P(S^c|T^c) = \frac{P(S^c \bigcap T^c)}{P(T^c)} = \frac{1-P(S \bigcup T)}{P(T^c)} = \frac{0.3}{0.7} = 3/7$

Problem 8

Assume the total population is $2n$

(a) $P(Jack's\ sibling\ is\ female|Jack\ is\ male) = \frac{n}{2n-1}$

(b) $P(Jane's\ sibling\ is\ female|Jane\ is\ female) = \frac{n-1}{2n-1}$