ECE316- Probability and Random Processes Winter 2011
Problem Set # 1

R. Mazumdar **Week 1**

IT IS IN YOUR INTEREST TO DO THE PROBLEMS YOURSELVES

1. Use the definitions of the complement, union, and intersection to prove the following , for any events S and T:

   (a) $S = (S \cap T) \cup (S \cap T^c)$.

   (b) $S \cap T^c \subset T^c$

   (c) $(S \cap T^c) \cup (S^c \cap T) \cup (S \cap T) = S \cup T$

   While using Venn diagrams may be helpful, Venn diagrams do not constitute a proof.

2. Let S and T be two events. Use axioms of probability to prove the following:

   (a) $P(S^c) = 1 - P(S)$.

   (b) If $S \cap T = \emptyset$, then $P(S) \leq P(T^c)$.

3. Let S and T be two events. Use the axioms of probability and your results from Problems 1 and 2 to prove the following:

   (a) $P(S \cap T) \geq P(S) + P(T) - 1$

   (b) $P(S) + P(T) = P(S \cap T) + P(S \cup T)$.

   (c) Show that the probability that one and only one of the events S or T occurs is $P(S) + P(T) - 2P(S \cap T)$.

4. Let S and T be two events. Prove the following inequalities:

   (a) $P(S \cap T) \leq P(S)$.

   (b) $P(S) \leq P(S \cup T)$.

   (c) $P(S \cup T) \leq P(S) + P(T)$.

5. Out of the students in the class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.

6. For four tosses of a fair coin, determine the probability of:

   (a) The sequence TTTT.

   (b) A total result of three heads and one tail.

   (c) The event "More heads than tails"

   Determine also the conditional probabilities:

   (d) "More heads than tails" given "At least one tail".

   (e) "More heads than tails" given "Fewr than two tails".

7. If $P(S) = 0.4, P(T^c) = 0.7$ and $P(S \cup T) = 0.7$ determine:

   (a) $P(T)$.

   (b) $P(S \cap T)$.

   (c) $P(S^c|T^c)$.

8. Assume that exactly 50% of the population are women and exactly 50% are men.

   (a) Jack has one sibling. What is the probability that the sibling is female?

   (b) Jane has a younger sibling. What is the probability that this sibling is female?

   Hint: You need to assume that there are 2n people.

9. Problem 14 page 54 –Chapter 2 (of text by A. Haddad).

10. Problem 15 Chapter 2 page 54 of text.