ECE316- Probability and Random Processes Winter 2011
Problem Set # 6

R. Mazumdar **Last week**

This is the last problem set. It covers ideas on the Weak Law of Large Numbers (WLLN) and the Central Limit Theorem (CLT) as well as some problems involving sums of random variables. A copy of the standard normal tables is attached. IT IS IN YOUR INTEREST TO DO THE PROBLEMS YOURSELVES

1. Let $\{X_i\}_{i=1}^{\infty}$ be i.i.d. r.v's with mean $E[X_i] = \mu$ and $var(X_i) = \sigma^2$. Use the weak law of large numbers to show that:

$$\lim_{n\to\infty} \frac{1}{\binom{n}{2}} \sum_{1\leq i\leq j\leq n} X_i X_j = \mu^2$$

   Hint:

$$2 \sum_{1\leq i\leq j\leq n} X_i X_j = (\sum_{i=1}^{n} X_i)^2 - \sum_{i=1}^{n} X_i^2$$

2. Let $\{S_i\}$ be i.i.d with $E[S_i] = \frac{1}{\lambda}$. Define :

$$T_n = \sum_{i=1}^{n} S_i$$

   and

$$N_t = \max\{n : T_n \leq t\}$$

   By definition of N_t it follows that $N_{T_n} = n$.

   Use the WLLN to show that

$$\lim_{t\to\infty} \frac{N_t}{t} = \lambda$$

3. The number of students who enroll in a psychology course is a Poisson random variable with mean 100. The professor in charge of the course has decided that if the number enrolling is greater than 120 or more she will teach two separate sections whereas if the number of students is less than 120 only one section will be needed. What is the probability that the professor will have to teach 2 sections?

4. In an orientation week contest, 100 freshmen independtly estimate the height in meters of a picket fence. Assume the standard deviation for the individual guesses is less than 0.1meters. Give a lower bound on the probability tat the average of their guesses is off by less than 1cm.

5. To prove a coin is unfair, a judge tosses it 2000 times. How many heads would he need to get to prove with 95% confidence that the coin is unfair?

6. One hundred reservations have been confirmed for the 98-set flight. If generally 3% of the confirmed passengers do not show up, what is the probability that somone will be bumped in the flight?

7. Show that :

$$\lim_{n\to\infty} \sum_{k=0}^{n} e^{-n} \frac{n^k}{k!} = \frac{1}{2}$$

Hint: Use the fact that if X_i are i.i.d Poisson r.v's with mean 1 and variance 1 then via the CLT

$$\lim_{n\to\infty} \Pr\left(\frac{\sum_{i=1}^{n} X_i - n}{\sqrt{n}} > 0\right) = \frac{1}{2}$$

8. Suppose that a fair die is rolled 100 times. Let X_i be the value obtained on the $i-th$ roll. Compute an approximation for

$$P\{\prod_{i=1}^{100} X_i \leq a^{100}\}$$

where $1 < a < 6$

9. An insurance company has 10,000 policy holders. The average yearly claim per policy holder is \$250 and the standard deviation is \$750.

i) Find the probability that the total claims in a year will exceed \$2.5 million.

ii) What is the probability that the number of claims of at least \$1000 will not exceed 1000.?

$\phi(x)$ vs. x , The Normalized Gaussian Distribution Function

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.50	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.60	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.70	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.80	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000