Assignment 1 Solution

PROBLEM 8

(a) $R_u = \{$a transmitted bit is received as "undecied" $\}$

$$\begin{aligned} P(R_u) &= P(R_u|T_0)P(T_0) + P(R_u|T_1)P(T_1) \\ &= 0.5 \times 0.09 + 0.5 \times 0.09 \\ &= 0.09 \end{aligned}$$

(b) Let $E = \{$a bit is received in error $\}$

$$\begin{aligned} P(E) &= P(R_1|T_0)P(T_0) + P(R_0|T_1)P(T_1) \\ &= 0.5 \times (1 - P(R_0|T_0) - P(R_u|T_0)) + 0.5 \times (1 - P(R_1|T_1) - P(R_u|T_1)) \\ &= 0.01 \end{aligned}$$

(c)

$$\begin{aligned} P(T_0|R_0) &= \frac{P(T_0)P(R_0|T_0)}{P(R_0)} \\ &= \frac{P(T_0)P(R_0|T_0)}{P(T_0)P(R_0|T_0) + P(R_0|T_1)P(T_1)} \\ &= \frac{0.5 \times 0.9}{0.5 \times 0.9 + 0.5 \times 0.01} \\ &= \frac{90}{91} \end{aligned}$$

and

$$P(T_1|R_0) = 1 - P(T_0|R_0) = \frac{1}{91}$$

(d) If $P(T_0) = 0.6$, then recalculate part (a) and (b). We conclude that (a) $P(R_u) = 0.09$ and (b) $P(E) = 0.01$.

PROBLEM 9

(a) Let $E = \{$a transmitted bit is received in error $\}$

$$\begin{aligned} P(E) &= P(R_{11}|T_{00})P(T_{00}) + P(R_{00}|T_{11})P(T_{11}) \\ &= p^2 \\ &= 0.01 \end{aligned}$$

(b) $R_u = \{$a transmitted bit is received as "undecied" $\}$

$$\begin{aligned} P(R_u) &= P(R_u|T_{00})P(T_{00}) + P(R_u|T_{11})P(T_{11}) \\ &= 2p(1-p) \\ &= 0.18 \end{aligned}$$

(b) $C = \{$a transmitted bit is received correctly$\}$

$$\begin{aligned} P(C) &= P\{\text{both two bits are decided correctly}\} \\ &= (1-p)^2 \\ &= 0.81 \end{aligned}$$