

ECE316- Probability and Random Processes Winter 2014  
Problem Set # 1

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**Week 1**

IT IS IN YOUR INTEREST TO DO THE PROBLEMS YOURSELVES

1. Use the definitions of the complement, union, and intersection to prove the following , for any events S and T:

- (a)  $S = (S \cap T) \cup (S \cap T^c)$ .
- (b)  $S \cap T^c \subset T^c$
- (c)  $(S \cap T^c) \cup (S^c \cap T) \cup (S \cap T) = S \cup T$

While using Venn diagrams may be helpful, Venn diagrams do not constitute a proof.

2. Let S and T be two events. Use axioms of probability to prove the following:

- (a)  $P(S^c) = 1 - P(S)$ .
- (b) If  $S \cap T = \emptyset$ , then  $P(S) \leq P(T^c)$ .

3. Let S and T be two events. Use the axioms of probability and your results from Problems 1 and 2 to prove the following:

- (a)  $P(S \cap T) \geq P(S) + P(T) - 1$
- (b)  $P(S) + P(T) = P(S \cap T) + P(S \cup T)$ .
- (c) Show that the probability that one and only one of the events S or T occurs is  $P(S) + P(T) - 2P(S \cap T)$ .

4. Let S and T be two events. Prove the following inequalities:

- (a)  $P(S \cap T) \leq P(S)$ .
- (b)  $P(S) \leq P(S \cup T)$ .
- (c)  $P(S \cup T) \leq P(S) + P(T)$ .

5. Out of the students in the class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.

6. For four tosses of a fair coin, determine the probability of:

- (a) The sequence TTTT.
- (b) A total result of three heads and one tail.
- (c) The event “More heads than tails”

Determine also the conditional probabilities:

- (d) “More heads than tails” given “At least one tail”.
- (e) “More heads than tails” given “Fewer than two tails”.

7. If  $P(S) = 0.4$ ,  $P(T^c) = 0.7$  and  $P(S \cup T) = 0.7$  determine:

- (a)  $P(T)$ .
- (b)  $P(S \cap T)$ .
- (c)  $P(S^c|T^c)$ .

8. Assume that exactly 50% of the population are women and exactly 50% are men.

- (a) Jack has one sibling. What is the probability that the sibling is female?
- (b) Jane has a younger sibling. What is the probability that this sibling is female?

Hint: You need to assume that there are  $2n$  people.