ECE316- Probability and Random Processes Winter 2014
Problem Set # 2

R. Mazumdar **Week 2**

The following are problems on independence and conditional probabilities. The corresponding material can be found in Chapters 2 and 3 of Ross.

1. In each of the following cases, determine whether the events A and B are independent:

   (a) $P(A) = 0.1,\ P(B) = 09,\ P(A \cup B) = 0.91$

   (b) $P(A) = 0.4,\ P(B) = 0.6.\ P(A \cup B) = 0.76$

   (c) $P(A) = 0.5,\ P(B) = 0.7,\ P(A \cup B) = 0.73$

2. Show the following:

   (a) For any events A and B , $P(A|A \cup B) \geq P(A|B)$. Will your answer be true if we consider the situation: $P(A|B \cup C) \geq P(A|B)$ for arbitrary events A, B and C?

   (b) Define $P_C(A) = P(A|C)$. Show that $P_C(.)$ defines a probability on $\Big(\Omega, \mathcal{F}\Big)$

   (c) Show that $P_C(A|D) = P(A|C \cap D)$. The rhs is often written as $P(A|C, D)$.

3. Let a peripatetic professor flies from Toronto to Tokyo changing planes at Chicago and LA. At each airport his luggage is misplaced with probability p. On arriving in Tokyo the professor finds his luggage missing. What are the chances that his luggage was misplaced in Toronto, Chicago, and LA respectively?

4. Two factories A and B manufactures watches. Factory A has one defective watch manufactured in a 100 while Factory B manufactures one defective watch in 200. A retailer receives a consignment of watches but does not know from which factory. He checks the first watch and finds it OK. What is the probability that the second watch he checks is also fine? Are the two watches that the retailer examines independent?

5. There are 3 cards identical in all respects but colour. The first one is red on both sides, the second is white on both sides and the third has one side of each colour A dealer selects one card at random and puts it on the table without looking. You look at the exposed face of the card and see it is red. What is the probability that the hidden face is also red? (it is not 0.5 !)

6. Here's a problem involving Bayes' rule.

   With probability 0.8, Al is guilty of the crime for which he is about to be tried. Rob and Tom, each of whom knows whether or not Al is guilty, have been called to testify. Rob is a friend of Al's and will tell the truth if Al is innocent but will lie with probability 0.2 if Al is guilty. Tom hates everybody but the judge and will tell the truth if Al is guilty but will lie with probability 0.3 if Al is innocent.

   (a) Determine the probability that the witnesses will give conflicting testimony.

   (b) Which witness is more likely to commit perjury (i.e. give false testimony)?

   (c) What is the conditional probability that Al is innocent given that Rob and Tom give conflicting testimony?

   (d) Are the events "Rob commits perjury" and "Tom commits perjury" independent?

   Hint: Define the events:

   A= Al is innocent; B= Rob testifies that Al is innocent; C= Tom testifies Al is innocent.

   Using these three events draw a tree diagram for the events and their complements.

   Now let X= Witnesses give conflicting testimony, Y= Rob commits perjury, Z= Tom commits perjury. From the first part collect events that correspond to X and find $P(X)$. Find $P(Y)$ and $P(Z)$ using a similar procedure to solve Part (b). Find $P(A \cap X)$ to solve Part (c). Finally find $P(Z \cap Y)$ to solve for part d).

7. To detect a particular disease doctors can apply a particular test. If the patient suffers from such a disease the test gives a positive result 99% of the time but also gives a positive result in 2% of the cases of healthy subjects. Statistical data shows that 1 in a 1000 of the population has the disease. What is the probability that a person who tests negative has the disease? Let M denote the event that the patient is ill and $+$ and $-$ denote the events that the test is positive or negative. You need to compute $P(- \mid M)$.

8. Two numbers are selected independently at random from the interval $[0, 1]$. The smaller one is known to be less than $\frac{1}{3}$. What is the probability that the larger one is greater than $\frac{3}{4}$.