ECE603- Statistical Signal Processing Problem Set 3
Winter 2014

Note: Reminder midterm next Friday, Feb 28 .

These problems are related to filtering and representations of w.s.s. processes.

1. Let $\sigma_n^2 = E|\xi_n - \hat{\xi_n}|^2$ where $\hat{\xi_n} = E[\xi_n|H_0]$. Show that the sequence is singular or deterministic if $\sigma_n^2 = 0$ for some $n \geq 1$, and if $\sigma_n^2 \to R(0) < \infty$ as $n \to \infty$ then the sequence is regular.

2. Show that the sequence $\xi_n = e^{in\varphi}$ where φ is uniform on $[0, 2\pi]$ is regular. Find the etimator $\hat{\xi_n} = E[\xi_n|H_0]$ and σ_n^2 and show that the nonlinear estimator :

   $$\tilde{\xi_n} = \left(\frac{\xi_0}{\xi_{-1}}\right)^n$$

   provides a correct estimate of ξ_n given the past $(\cdot, \xi_{-1}, \xi_0)$ i.e. $E|\tilde{\xi_n} - \xi_n|^2 = 0$.

   Here we see the best estimator in the mean squared sense is a non-linear estimator.

3. Let an observation sequence be given by:

   $$\xi_k = \theta_k + n_k$$

   where θ_k is the signal and n_k is the noise sequence independent of the signal sequence.

   Both θ_k and n_k are correlated sequences with spectral densities for $z = e^{-2\pi\lambda}$:

   $$p_\theta(z) = \frac{1}{|1 + bz|^2}$$

   and

   $$p_n(z) = \frac{1}{|1 + cz|^2}$$

   .

   Find $\bar{[}\theta_{n+m}|\xi_k, k \leq n]$.

   Repeat the problem if :

   $$p_\theta(z) = |2 + z|^2$$

   and

   $$p_n = 1$$

   .

4. Let $\{W_t; t \geq 0\}$ be a standard Brownian motion process and A be a r.v. with $E[A] = 0$ and $E[A^2] = 1$ independent of $\{W_t\}$. Suppose we observe the process:

$$X_t = At + W_t \; ; \quad t \geq 0$$

Let $\hat{A}_t$ denote the linear MMSE estimator of A given $X_s, 0 \leq s \leq t$. Find the explicit form of the estimator in the form:

$$\hat{A}_t = \int_0^t h(t,s) dX_s$$

i.e. find $h(t,s)$.