Delay and Capacity Trade-off in Wireless Ad Hoc Networks with Random Mobility

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Abstract

In this paper, we study the delay and capacity trade-off in mobile ad hoc networks. We consider an ad hoc network with n nodes distributed uniformly on a sphere. The nodes are mobile, and move in accordance with the random way-point mobility model, used widely in the ad hoc networks literature. We show that the 2-hop relaying algorithm proposed by Grossglauser and Tse (2001), incurs an expected packet delay of $\Theta(nT_p(n))$, where $T_p(n)$ is the packet duration. We show that any protocol that allows only nearest neighbor transmissions, incurs an expected packet delay of $\Omega(T_p(n)\sqrt{n})$. We show that the trade-off: $delay/capacity \geq \Theta(T_p(n)n)$, is both necessary as well as sufficient in mobile ad hoc networks. A protocol which introduces redundancy into the 2-hop relaying algorithm, and offers a throughput of $\Theta(1/k(n))$ and an expected packet delay of $\Theta(nT_p(n)/k(n))$, for any $k(n) = O(\sqrt{n})$, is developed.

Keywords: Ad Hoc Networks, Random Mobility, Delay, Throughput Capacity, Scaling Laws.

1 Introduction

Ad hoc networks are autonomous systems of nodes (routers) which communicate with each other without a fixed infrastructure usually via wireless links. The nodes in an ad hoc network can be either static or mobile. In their seminal work, Gupta and Kumar [1], have shown that for a random wireless network with n static nodes, the per node capacity is $\Theta(\frac{1}{\sqrt{n \log n}})$[1]. Thus each node gets a vanishingly small throughput as the node density grows. This is a negative result as it implies that ad hoc networks are not scalable. Grossglauser and Tse [2], have shown that a constant per node throughput can be achieved in the case of mobile ad hoc networks. However, no bounds on the packet delay are provided in [2].

[1] We use the following asymptotic notation throughout:

$$f(n) = O(g(n)) \leftrightarrow \limsup_{n\to\infty} \frac{f(n)}{g(n)} < \infty, \quad f(n) = o(g(n)) \leftrightarrow \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0,$$
$$f(n) = O(g(n)) \leftrightarrow g(n) = \Omega(f(n)), \quad f(n) = o(g(n)) \leftrightarrow g(n) = \omega(f(n))$$
$$f(n) = \Theta(g(n)) \leftrightarrow f(n) = O(g(n)) \;\; and \;\; f(n) = \Omega(g(n))$$

Clearly, the usefullness of the throughput result is limited since both quantities, the delay and throughput, are important. A little amount of reflection also shows that there is a trade-off between the two quantities in that one can only be improved at the expense of the other. Characterization of this trade-off would not only provide useful insights about the fundamental limits of mobile ad hoc networks, but would also help in the design of novel and superior scheduling and relaying algorithms.

The delay and capacity trade-off in mobile ad hoc networks has attracted a lot of attention recently. Delay limited capacity of ad hoc networks has been addressed in [15]. Their work is motivated by the *diversity coding* approach given in [12]. Geographical routing with maximum delay guarantees is considered in [13]. In [11], the authors derive the delay and capacity trade-off for a cell partitioned network with a so-called instantaneous or *infinite mobility* model, where each node's location in a future time-slot is independent of its present and past locations. This independence assumption greatly simplifies the analysis but at the same time makes the model quite unrealistic. Our work complements their work in the sense that we consider more realistic mobility model.

Toumpis and Goldsmith [5] present a scheme that can achieve a per-node capacity of $\Theta(n^{\frac{d-1}{2}}/\log^{\frac{5}{2}} n)$ under a bound of $O(n^d)$ on the packet delay. Ignoring the logarithmic term, we see that the trade-off, $\lambda^2 \leq O\left(\frac{d}{n}\right)$, is achievable in [5], where λ is the average throughput and d the average delay. Lin and Shroff [10] improved their results and showed that the trade-off, $\lambda^3 \leq O\left(\frac{d}{n}\right)$, is achievable under the same settings. Both these works assume a mobility model similar to the infinite mobility model in [11]. El Gamal et al [6] consider the delay and capacity trade-off in wireless networks using a random walk mobility model on a torus. They consider a cellular network and use a cellular TDMA transmission scheme in which each cell becomes active at regularly scheduled cell time-slots. Their scheme achieves the trade-off given by $\lambda = \Theta(1/\sqrt{na(n)\log n})$ and $d = O(\log^4(1/a(n))/\sqrt{a(n)}v(n))$, where $a(n)$ is the area of each cell and $v(n)$ is the speed of the nodes. It is interesting to note that all these above mentioned trade-offs are between the packet delay and the throughput per node, and different packet size scalings have been assumed in each of these works. It is intuitively clear that the packet delay would be dependent on the size of the packet. How does one compare these trade-offs? Is there any fundamental trade-off for mobile ad hoc networks which holds irrespective of the packet size scaling? These questions along with others are answered in this paper.

The main contributions of this paper are the following. We derive the delay and capacity trade-off with the random way-point mobility model, which is used widely in the ad hoc networks literature, and is much more realistic than the mobility models considered in [11, 6, 5, 10]. Also, the protocols that we consider in this paper are totally distributed, and very simple to implement, unlike the ones in [5, 10, 6], which require co-ordination between large number of nodes.

The remaining paper is organized as follows. In Section 2, we introduce our model and discuss some previous results. In section 3, we analyze our mobility model in detail. The delay performance of the distributed 2-hop relaying protocol is analyzed in section 4. In Section 5, we develop an alternative relaying protocol for delay improvement, and analyze its performance. Delay and capacity trade-offs are considered in Section 6. We end this paper with some concluding remarks in section 7.

2 Model and Prior Results

We consider an ad hoc network formed by n mobile nodes distributed uniformly and independently on the surface of a sphere of radius R at the start. The nodes move in accordance with the random way-point mobility model, which is described in the next subsection. Since we are interested in the asymptotic throughput capacity and delay, we consider n to be large in the sequel.

We consider a typical interference based transmission model as in [3, 2, 15, 1]. Node i is capable of transmitting $W\ bits/sec$ to node j, at time t, if

$$\frac{P_i(t)\gamma_{ij}(t)}{N_o + \frac{1}{L}\sum_{k\neq i} P_k(t)\gamma_{kj}(t)} > \beta,$$

where P_i is the transmit power of node i, $\gamma_{ij}(t)$ is the channel gain from node i to node j at time t, N_o is the background noise power, L is the processing gain of the system, and β is the SINR requirement for successful communication. The channel gain is assumed to be of the form

$$\gamma_{ij}(t) \ = \ \frac{1}{d_{ij}(t)^{\alpha}}$$

where $d_{ij}(t)$ is the distance between the nodes i and j, at time t, and typically, $\alpha \in (2, 4]$.

All nodes generate traffic at the same rate and each node sends packet to a unique destination. This makes sure that no node can become the bottleneck in the network, resulting in a decrease of the per node throughput, like in the multi-access channels.

The two most widely used mobility models in ad hoc networks literature are the random way-point mobility model, and the random walk (Brownian) mobility model (see [14]). In this paper, we focus mainly on the random way-point mobility model. Brownian mobility model has been considered in [3]. We now describe the random way-point mobility model in detail.

2.1 Random Way-point Mobility Model (RWMM)

In this model, at each time-step the mobile node chooses a random destination on the sphere and moves toward it with a random speed. The speed is chosen uniformly from the interval $[v_{min}, v_{max}]$, where $v_{min}(n)$ and $v_{max}(n)$ are strictly positive. The movement is along the great circle that passes through the initial position and the final destination. On reaching the destination, the node pauses for a random amount of time and the process repeats itself. In this paper, we consider the RWMM with no pause times. The pause times can easily be accounted for with only a minor set of changes in the analysis.

We now describe the distributed 2-hop relaying protocol, which is analyzed in detail in the next section.

2.2 Distributed 2-Hop Relaying Protocol

In this protocol, the packets are either transmitted directly from the source to the destination, or via a single relay node. Let $M_i(t)$ be the number of source node i packets that destination $d(i)$ receives, up to time t. Then we say that a long term throughput of $\lambda(n)$ is feasible, if for every $S-D$ pair i, we have,

$$\liminf_{T\to\infty} \frac{M_i(T)}{T} \geq \lambda(n) \tag{2.1}$$

We assume a slotted system. In every time-slot, each node randomly and independently decides to be a sender or a potential receiver. In particular, each node flips a biased coin and decides to be a sender with probability p, and otherwise decides to be a potential receiver. The sender density (p) is a global parameter known to all the nodes in the network. The optimal value of p depends on the size of the neighborhood of a typical node. Larger the size of the neighborhood, the smaller p should be.

After deciding to be a sender, each sender node transmits to its nearest neighbor with a unit transmit power. It is however possible that the nearest neighbor decided to be a sender as well. In that case, it certainly cannot receive the packet. Even when the nearest neighbor decided to be a potential receiver, the packet transmission might fail due to the interference generated by other simultaneous transmissions in the network.

The following strategy is used by each sender node to choose which packet to send to its nearest neighbor:

1. If there is a packet generated locally for the nearest neighbor, then, send it. (S-D transmission)
2. If no such packet is found, then randomly choose one of the following two options with equal probability:
   - If there is a packet destined for some other node in the network, then, send it. Else remain idle. (S-R transmission)
   - If there is packet destined for the nearest neighbor, then, send it. Else remain idle. (R-D transmission)

Let N_t be the number of successful sender-receiver pairs per slot, in [2] it is shown that

$$\lim_{n\to\infty} \frac{E[N_t]}{n} = \phi > 0 \tag{2.2}$$

The main reason why we can have $\Theta(n)$ concurrent nearest neighbor transmissions is that the received power at the nearest neighbor is of the same order as the total interference generated by $\Theta(n)$ number of interferers (see [8], for a similar proof).

For simplicity, we assume that all nodes have the same radius of communication, say, $r(n)$. Thus, two nodes can communicate if and only if they are within distance $r(n)$ of each other. It is clear that $r(n)$ should scale down with n to prevent the interference from becoming excessive as n grows. In [4], we show that this simplified model of communication is equivalent to the interference based model (in terms of the throughput and delay), provided $r(n)$ is $\Theta(1/\sqrt{n})$. In the sequel, we consider this simplified model of communication.

3 Random Way-Point Mobility Model: Contact Time And Inter-Meeting Time

In this section, we define several new concepts, which are required for the analysis of the delay in the following sections. We start by defining the notion of the *contact time*, $T_c(n)$, which characterizes the duration of a typical *wireless link* in the network.

Definition 3.1 (Contact time) *Consider any two arbitrary nodes, say i and j, with positions $X_i(t)$ and $X_j(t)$, respectively, at time t. We define,*

$$T_c(n) = \inf\{t : d_S(X_i(t), X_j(t)) \geq r(n) \mid d_S(X_i(0), X_j(0)) = r(n)/2,\ d_S(X_i(0^-), X_j(0^-)) > r(n)/2\}$$

The use of $r(n)/2$[2] instead of $r(n)$ in the above definition is to ensure that all contacts be at least as long as the packet duration. The following Lemma shows how $T_c(n)$ is related to $r(n)$ and $v(n)$.

Lemma 3.1 *The expected* contact time, *$\mathbb{E}\{T_c(n)\}$, is* $\Theta\left(\frac{r(n)}{v(n)}\right)$.

Proof: The nodes must at least travel a relative distance of $r(n)/2$ in order to move out of contact, and the time for this is at least $\Theta(r(n)/v(n))$. Further, if the nodes travel a relative distance of $3r(n)/2$ then they are guaranteed to be out of contact, and the time for this is at most $\Theta(r(n)/v(n))$. □

Let $T_p(n)$ denote the transmission time or the duration of a packet. Then, in order for a packet transmission to be successful, the communicating nodes must remain in contact for a length of time longer than the packet duration. Hence, $T_p(n)$ must be $O(T_c(n))$. Without much loss of generality, we assume $T_p(n)$ to be $\Theta(T_c(n))$. This is in some sense the best case, as making $T_p(n) = o(T_c(n))$ does not reduces the packet delay, but increases the overhead per packet.

The packet delay under the D2HRP, and other mobility dependent relaying protocols, is related to how frequently any two arbitrary nodes in the network come in contact with each other or their *inter-meeting* time, $I(n)$.

Definition 3.2 (Inter-meeting time) *Consider any two arbitrary nodes, say i and j, with positions*

[2] The factor of half is not really important here, $\epsilon r(n)$, such that $0 < \epsilon < 1$, would also be fine.

$X_i(t)$ and $X_j(t)$, respectively, at time t. We define,

$$I(n) = \inf\{t : d_S(X_i(t), X_j(t)) \leq r(n)/2 \mid d_S(X_i(0), X_j(0)) = r(n),\ d_S(X_i(0^-), X_j(0^-)) < r(n)\}$$

It turns out that in order to estimate the delay under the D2HRP, we need to estimate the first two moments of the inter-meeting time. Our approach is to bound the complementary distribution of the inter-meeting time, and hence obtain a bound on all higher order moments of the inter-meeting time. Without any loss of generality, consider the nodes to be placed on a unit sphere (unit area), and define:

- S^2 = The surface of the unit sphere centred at the origin
- $d_S(x, y)$ = The distance between the two points x and y on S^2.
- $\aleph$ = The north pole.
- $\Re = \{x \in S^2\colon\ d_S(x, \aleph) \leq r(n)/2\}$.

We start with the following Lemma which shows that the probability that a line connecting two random points on S^2 intersects with $\Re$ is $\Theta(r(n))$.

Lemma 3.2 *Let L be a line connecting two uniformly and independently chosen points on S^2. Then, there exists strictly positive constants c_1 and c_2 such that $c_1 r(n) \geq$ P(L intersects $\Re$) $\geq c_2 r(n)$, for large enough n.*

Proof: Let the two points be X and Y. We first establish the upper bound. If X lies at a distance x, $> r(n)/2$, from the north pole then the angle β subtended at X by $\Re$ is no more than $c_3 r(n)/x$. The area of the sector so formed is no more than $c_4 r(n)/x$. If Y does not lie in this sector then L cannot intersect $\Re$. Since, Y is uniformly distributed on S^2, the probability of this is no more than $c_4 r(n)/x$.

Since, X is uniformly distributed on S^2, the probability density that it is at a distance x from the north pole is $\sqrt{\pi}\sin(2\sqrt{\pi}x)$. Integrating over all possible values of x, we get:

$$
\begin{aligned}
P(L\ intersects\ \Re) &\leq c_5 r^2(n) + \int_{\frac{r(n)}{2}}^{\frac{\sqrt{\pi}}{2}} \frac{c_4 r(n)}{x} \sqrt{\pi} \sin{(2\sqrt{\pi}x)} dx \\
&\leq c_5 r^2(n) + c_4 \pi^{\frac{3}{2}} r(n) \quad (Using\ \sin x \leq x,\ for\ x \geq 0) \\
&\leq c_1 r(n) \quad (For\ n\ large\ enough) \qquad (3.1)
\end{aligned}
$$

provided c_1 is chosen to be large enough.

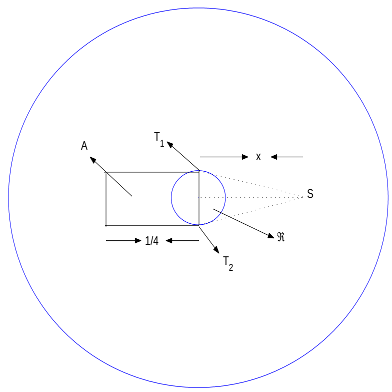

Figure 1: The figure shows the top view of the regions A and $\Re$. For convenience sake, the distances shown are the actual distances and not the ones which one would observe in a top view.

To prove the lower bound, suppose that X is at a distance x, $\leq \sqrt{\pi}/4$, from the north pole. Draw the tangent lines from X to $\Re$. Let T_1 and T_2 be their points of intersection with $\Re$. Consider the rectangular region A with T_1 and T_2 as two of its vertices as shown in the Figure 3. If Y lies in A then L interse cts $\Re$. The area of the region A, and hence the probability that Y lies in A, is at least $c_5 r(n)$. Noting that the probability that x$\leq \sqrt{\pi}/4$ is 1/2, and taking $c_2 = c_5/2$, the result follows. $\square$

The following Lemma establishes the monotonicity property of the conditional intersection probability. This is a key result for establishing the lower bound on the complementary distribution of the inter-meeting time.

Lemma 3.3 *Let X_1 and X_2 be two arbitrary points on S^2, with $0 \leq d_S(X_1, \aleph) \leq d_S(X_2, \aleph) \leq \sqrt{\pi}/2$. Consider the lines L_1 and L_2 joining X_1 to Y_1 and X_2 to Y_2, respectively, with Y_1 and Y_2 distributed uniformly on S^2. Then, $P(L_1\ intersects\ \Re) \geq P(L_2\ intersects\ \Re)$.*

Proof: The proof is constructive. Join X_2 with $\aleph$, along the great circle on which they lie. Denote by T_2, the line segment joining X_2 with $\aleph$. Without any loss of generality, we assume that X_1 lies on this line segment. Let $\mathcal{A}_1$ (respectively $\mathcal{A}_2$) denote the region on S^2 in which Y_1 (respectively Y_2) must belong in order for L_1 (respectively L_2) to intersect with $\Re$. Then, it is easily seen that $\mathcal{A}_1 \supset \mathcal{A}_2$. Since, Y_1 and Y_2 are uniformly distributed on S^2, the result follows. □

Consider a node, say i, distributed uniformly on S^2 at time $t = 0$, and let $X_i(t)$ be its position at time t. Define $F(n) = \inf\{t : X_i(t) \in \Re\}$. Also, define $I_f(n) = \inf\{t : d_S(X_i(t), \aleph) \leq r(n)/2 \mid d_S(X_i(0), \aleph) = r(n)\}$. Then, it is clear that $I(n) = \Theta(I_f(n))$[3]. In what follows, we first bound the complementary distribution of $F(n)$, and then by proving that $I_f(n) = \Theta(F(n))$, we extend those bounds to $I(n)$.

The following Lemma gives an upper bound on the complementary distribution of $F(n)$.

Lemma 3.4 *With F(n) as defined above, we have*

1. $P\left(F(n) > \frac{m\sqrt{\pi}}{v_{min}(n)}\right) \leq (1 - c_2 r(n))^m$
2. $\mathbb{E}\{F(n)\} \leq \frac{\sqrt{\pi}}{c_2 r(n) v_{min}(n)}$
3. $\mathbb{E}\{F^2(n)\} \leq \frac{\pi}{(c_2 r(n) v_{min}(n))^2}$

Proof: Let N denote the random variable indicating the number of trips that node i makes (including any incomplete one) before entering into $\Re$. Define $\Gamma_j = \{j^{th}$ trip does not intersect $\Re\}$. Since the duration of each trip is bounded above by $\sqrt{\pi}/2v_{min}(n)$, we have $P(F(n) > m\sqrt{\pi}/v_{min}(n) | N \leq 2m) = 0$, and hence

$$
\begin{aligned}
P\left(F(n) > \frac{m\sqrt{\pi}}{v_{min}(n)}\right) &= P\left(F(n) > \frac{m\sqrt{\pi}}{v_{min}(n)} | N > 2m\right) P(N > 2m) \\
&\leq P(N > 2m) \\
&\leq P(\Gamma_1 \cap \Gamma_3 \cap \ldots \cap \Gamma_{2m-1}) \\
&\leq (1 - c_2 r(n))^m \qquad (3.2)
\end{aligned}
$$

where the last equation follows by noting that Γ_i for odd i are independent, and from Lemma 3.2, we have $P(\Gamma_i) \geq (1 - c_2 r(n))$. This proves *I*.

[3]This is to be interpreted as $\mathbb{E}\{I^k(n)\} = \Theta(\mathbb{E}\{I_f^k(n)\})$, $\forall k$.

Using the non-negativity of $F(n)$ along with (3.2), we have

$$\begin{aligned}\mathbb{E}\{F(n)\} &= \int_0^\infty P(F(n) > x)dx \\ &\leq \frac{\sqrt{\pi}}{v_{min}(n)} \sum_{m=0}^{\infty} (1 - c_2(r(n))^m \\ &= \frac{\sqrt{\pi}}{c_2 r(n) v_{min}(n)} \end{aligned} \tag{3.3}$$

This proves II. Similarly, we have

$$\begin{aligned}\mathbb{E}\{F^2(n)\} &= \int_0^\infty xP(F(n) > x)dx \\ &\leq \frac{\pi}{v_{min}^2(n)} \sum_{m=0}^{\infty} (m+1)(1 - c_2(r(n))^m \\ &= \frac{\pi}{(c_2 r(n) v_{min}(n))^2} \end{aligned} \tag{3.4}$$

And III is proved. □

To establish a lower bound on the complementary distribution of $F(n)$, we need some more results. The following Lemma shows that the kind of dependency that exists between successive trips (end point of the current trip is the starting point of the next trip) in the RWMM, makes it more likely that the number of trips that a node makes before it enters into $\Re$ would be larger, as compared to the case where the successive trips are independent.

Lemma 3.5 *Let N be defined as before, then $P(N > m) \geq (1 - c_1 r(n))^m$.*

Proof: With Γ_i defined as before, we have

$$\begin{aligned} P(N > m) &= P(\Gamma_1 \cap \Gamma_2 \cap ... \cap \Gamma_m) \\ &= P(\Gamma_1) \prod_{i=2}^{m} P(\Gamma_i | \Gamma_1 \cap \Gamma_2 \cap ... \cap \Gamma_{i-1}) \\ &\geq \prod_{i=1}^{m} P(\Gamma_i) \\ &\geq (1 - c_1 r(n))^m \qquad (From\ Lemma\ 3.2) \end{aligned} \tag{3.5}$$

The only step which requires an explanation is the third step. Consider a point X, distributed uniformly on S^2, and define $U(x) = P(d_S(X, \aleph) > x)$. We say that $F \succ G$ if $F(x) \geq G(x), \ \forall x \in [0, \infty)$. Given that the first i trips do not intersect $\Re$, the distribution of the starting point of the $(i+1)^{th}$ trip, S_{i+1}, is skewed. Let $C(x) = P(d_S(S_{i+1}, \aleph) > x)$, then a simple Bayesian argument, using Lemma 3.3, implies that $C \succ U$. This implies that $P(\Gamma_i | \Gamma_1 \cap \Gamma_2 \cap ... \cap \Gamma_{i-1}) \geq P(\Gamma_i)$. □

Consider an i.i.d sequence of lines, $(L_i)_{i=1}^{\infty}$, connecting two uniformly and independently chosen points on S^2, and define $(l_i)_{i=1}^{\infty}$ to be the corresponding sequence of lengths. The following Lemma is a simple application of the Tchebychev's inequality under the i.i.d case.

Lemma 3.6 $P\left(\sum_{i=1}^m l_i < \frac{m\bar{l}}{2}\right) \leq \frac{\sigma_l^2}{4m\bar{l}^2}$, *where* $\bar{l}$ *and* σ_l^2 *are the common mean and variance, respectively, of* l_i.

Proof: By Tchebychev's inequality, we obtain

$$P\left(\sum_{i=1}^{m}(l_i - \bar{l}) > \frac{m\bar{l}}{2}\right) \leq \frac{\sigma_l^2}{4m\bar{l}^2}$$

Since, $P\left(\sum_{i=1}^m (l_i - \bar{l}) > \frac{m\bar{l}}{2}\right) \geq P\left(\sum_{i=1}^m l_i < \frac{m\bar{l}}{2}\right)$, the result follows. □

Define $\zeta_m = \cap_{i=1}^m \{L_i \text{ does not intersect } \Re\}$. Given that ζ_m has occurred, it is more likely that the first m trips were short in length, rather than long. However, the following Lemma shows that the conditional lengths of the trips are not too different from the unconditional lengths.

Lemma 3.7 $P(\sum_{i=1}^m l_i > x | \zeta_m) \geq P(\sum_{i=1}^m l_i > x)(1 - c_1 r(n))^m$.

Proof: Let us look at the probability that L_i would intersect $\Re$, given that $l_i = l$. It is easy to see that the upper bound on the intersection probability of $c_1 r(n)$, as derived in Lemma 3.2, is valid in this case irrespective of l. This implies that $P(\zeta_i | l_i = l) \geq (1 - c_1 r(n))$ for all possible values of l. Using this, together with the independence of L_i's, we obtain

$$P\left(\sum_{i=1}^{m} l_i > x \mid \zeta_m\right) \geq P\left(\sum_{i=1}^{m} l_i > x \cap \ \zeta_m\right) \tag{3.6}$$

$$\geq (1 - c_1 r(n))^m P(\sum_{i=1}^{m} l_i > x) \tag{3.7}$$

□

We are now ready to give a lower bound on the complementary distribution of $F(n)$.

Lemma 3.8 *1.* $P\left(F(n) > \frac{\bar{l}m}{2v_{max}(n)}\right) \geq \frac{1}{2}(1 - c_1 r(n))^m$.

2. $\mathbb{E}\{F(n)\} \geq \frac{\bar{l}}{16 c_1 r(n) v_{max}(n)}$.

3. $\mathbb{E}\{F^2(n)\} \geq \frac{\bar{l}^2}{64(c_1 r(n) v_{max}(n))^2}$.

Proof: Let $(d_i)_{1=1}^{\infty}$ denote the lengths of the trips that a node makes under the RWMM. We have,

$$
\begin{aligned}
P\left(F(n) > \frac{m\bar{l}}{2v_{max}(n)}\right) &\geq P\left(F(n) > \frac{m\bar{l}}{2v_{max}(n)} \mid N > m\right) P(N > m) \\
&\geq P\left(\sum_{i=1}^{m} d_i > \frac{m\bar{l}}{2} \mid N > m\right) (1 - c_1 r(n))^m \quad (From\ Lemma\ 3.5) \\
&\geq P\left(\sum_{i=1}^{m} l_i > \frac{m\bar{l}}{2} \mid \zeta_m\right) (1 - c_1 r(n))^m \qquad (3.8) \\
&\geq P\left(\sum_{i=1}^{m} l_i > \frac{m\bar{l}}{2}\right) (1 - c_1 r(n))^{2m} \quad (From\ Lemma\ 3.7) \\
&\geq \left(1 - \frac{\sigma_l^2}{4m\bar{l}^2}\right) (1 - c_1 r(n))^{2m} \quad (From\ Lemma\ 3.6) \\
&\geq \frac{1}{2}(1 - c_1 r(n))^m \quad (For\ all\ m \geq m_o, where\ m_o\ is\ chosen\ to\ be\ large\ enough) \qquad (3.9)
\end{aligned}
$$

where (3.8) follows from the fact that the dependence between the trips in the RWMM makes it more likely for a node to spend long times before entering into $\Re$. This proves I.

Non-negativity of $F(n)$ implies:

$$
\begin{aligned}
\mathbb{E}\{F(n)\} &= \int_0^{\infty} P(F(n) > x) dx \\
&\geq \frac{\bar{l}}{4v_{max}(n)} \sum_{m=m_o}^{\infty} (1 - c_1 r(n))^{2m} \\
&\geq \frac{\bar{l}}{4v_{max}(n)} \frac{(1 - c_1 r(n))^{2m_o}}{2c_1 r(n)} \\
&\geq \frac{\bar{l}}{16 c_1 v_{max}(n) r(n)} \qquad (For\ large\ enough\ n) \qquad (3.10)
\end{aligned}
$$

This proves II. To prove III, we use the non-negativity of F(n) once more, to obtain

$$\begin{aligned}
\mathbb{E}\{F^2(n)\} &= \int_0^{\infty} xP(F(n) > x)dx \geq \frac{\bar{l}^2}{8v_{max}^2(n)} \sum_{m=m_o}^{\infty} m(1 - c_1 r(n))^{2m} \\
&\geq \frac{\bar{l}^2}{8v_{max}^2(n)} (1 - c_1 r(n))^{2m_o} \sum_{m=0}^{\infty} m(1 - c_1 r(n))^{2m} \\
&\geq \frac{\bar{l}^2}{64(c_1 r(n) v_{max}(n))^2} \qquad (\textit{For large enough } n) \qquad (3.11)
\end{aligned}$$

□

Now, we are ready to prove that $I_f(n) = \Theta(F(n))$.

Lemma 3.9 $I_f(n) = \Theta(F(n))$.

Proof: Let us consider a node moving under the RWMM. As required in the inter-meeting time definition, assume that the node is at a distance of $r(n)$ from $\aleph$ at time $t = 0$. Let us denote by T_o the time remaining in current trip of the node, and by T_1 the time duration of the next trip, which we denote by $\mathcal{T}_1$. At time $T_o + T_1$ the node will be at a uniformly distributed point on S^2, and it would require $F(n)$ amount of time for the node to come inside $\Re$. Now, since the duration of all trips is upper bounded by $\sqrt{\pi}/2v_{min}(n)$, we obtain

$$P(I_f(n) > x + \sqrt{\pi}/2v_{min}(n)) \leq P(F(n) > x) \qquad (3.12)$$

This establishes the upper bound. To derive the lower bound, we note that given $\mathcal{T}_1$ does not intersect with $\Re$, the distribution of the end point of the trip $\mathcal{T}_1$ is skewed in such a way that it is less likely that the next trip would intersect with $\Re$. Therefore, the time taken by the node to come within $\Re$ would be at least $F(n)$. Since the probability that $\mathcal{T}_1$ would intersect with $\Re$ is less than $c_1 r(n)$, we obtain

$$P(I_f(n) > x) \geq P(F(n) > x)(1 - c_1 r(n)) \qquad (3.13)$$

From (3.12) and (3.13) it follows that all the moments of $I_f(n)$ and $F(n)$ are of the same order, i.e., $I_f(n) = \Theta(F(n))$. □

Using Lemmas 3.4, 3.8, and 3.9, together with the fact that $I(n) = \Theta(I_f(n))$, we arrive at the main result of this section.

Proposition 3.1 *Let $I(n)$ denote the inter-meeting time of an arbitrary pair of nodes under the RWMM. Then there exist positive constants c_1, c_2, c_6, and c_7, such that:*

1. $P\left(I(n) > \frac{c_6 m}{v(n)}\right) \geq \frac{1}{2}(1 - c_1 r(n))^m$.
2. $P\left(I(n) > \frac{c_7 m}{v(n)}\right) \leq (1 - c_2 r(n))^m$.
3. $\mathbb{E}\{I(n)\} = \Theta\left(\frac{1}{r(n)v(n)}\right)$, *and* $\mathbb{E}\{I^2(n)\} = \Theta\left(\frac{1}{r^2(n)v^2(n)}\right)$. *Assuming,* $T_p(n) = \Theta(T_c(n))$, *we note that:* $\mathbb{E}\{I(n)\} = \Theta\left(nT_p(n)\right)$, *and* $\mathbb{E}\{I^2(n)\} = \Theta\left(n^2 T_p^2(n)\right)$.

Here $v(n)$ is the average velocity of the nodes.

So far we have not discussed how the velocity of the nodes should scale with n. In this paper, we consider the following two, perhaps extreme, possibilities:

I. $v_{min}(n) = v_{max}(n) = \Theta(1)$[4]: In this case, Lemma 3.1 implies that the $T_c(n)$ is $\Theta(1/\sqrt{n})$, and hence $T_p(n)$ must be $\Theta(1/\sqrt{n})$ as well. We refer to this as the variable packet size strategy (VPSS).

II. $v_{min}(n) = v_{max}(n) = \Theta(1/\sqrt{n})$: In this case, Lemma 3.1 implies that the $T_c(n)$ is $\Theta(1)$, and hence $T_p(n)$ is $\Theta(1)$ as well. We refer to this as the fixed packet size strategy (FPSS).

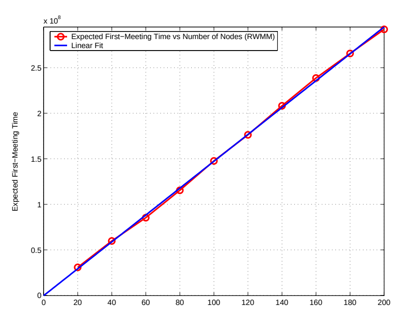

Figure 2: Expected inter-meeting time vs n, for $R = 100$, $T_p(n) = 1ms$, and $v(n) = r(n) = \frac{1}{\sqrt{n}}$.

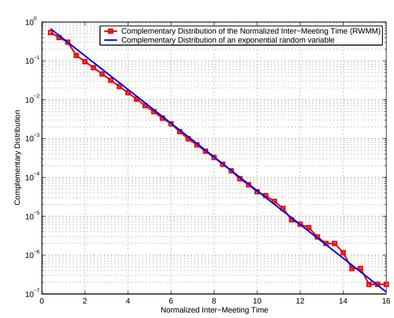

Figure 3: Complementary distribution of the inter-meeting time, for $R = 100$, $n = 50$, $T_p(n) = 1ms$, and $v(n) = r(n) = \frac{1}{\sqrt{n}}$.

Our simulation results (see Figure 2) are in complete agreement with these analytical results. The simulation results seem to suggest that the distribution of the inter-meeting is exponential. The exponential distribution assumption is used in [3] to simplify the analysis, and the results obtained are quite similar to this paper.

[4]This is to be interpreted as: $\exists$ positive constants c_1 and c_2, independent of n, such that, $c_1 < v_m in(n) \leq v_m ax(n) < c_2$.

4 Delay And The D2HRP

Consider a network with n nodes and n S-D pairs, with all nodes following the D2HRP. In most cases, the packet delay has two components, the queuing delay at the source node and the queuing delay at the relay node. Sometimes the packet might be directly transmitted from the source to the destination, eliminating the queuing delay at the relay node. It is easily seen that the S-D transmissions contribute only $\Theta(1/n)$ to the throughput, and hence can be ignored during the analysis. Henceforth, we assume that every packet travels exactly two hops, i.e., from the source to the relay, and then from the relay to the destination.

Consider the source queue at some arbitrary node, say, node i. To account for the queuing delay at the source, we assume that the input to this source queue is a Bernoulli stream of rate λ_i. The following Lemma gives the queuing delay at such a queue.

Lemma 4.1 *If the exogenous packet stream at node i is Bernoulli with rate λ_i, where $\lambda_i < \frac{\phi}{2}$, then a) the expected queuing delay ($\mathbb{E}\{D_i^s\}$) at the source node i, is given by*

$$\mathbb{E}\{D_i^s\} = T_p(n)\frac{1-\lambda_i}{\frac{\phi}{2}-\lambda_i} \tag{4.1}$$

b) and the output process from the queue is Bernoulli stream of rate λ_i.

Proof: We know that in the case of the distributed 2-hop relaying protocol, for large enough n, the total number of successful sender-receiver pairs per slot are roughly $n\phi$. Since the R-D transmissions and the S-R transmissions are given equal priority, the service time at any source is Bernoulli with rate $\frac{\phi}{2}$. Now, (a) follows by noting that the source is a Bernoulli(λ_i)/Bernoulli($\frac{\phi}{2}$) queue. And (b) follows from the reversibility of the Bernoulli/Bernoulli queues. □

Consider any arbitrary S-D pair in the network. The packets from the source can reach the destination via any of the $n-2$ relay nodes. As in [2], we assume that the relay nodes maintain a separate queue for each of the S-D pairs (see Figure 4). Since $T_p(n) = \Theta(T_c(n))$, $\Theta(1)$ packets are exchanged during a contact between any two nodes. Hence, the packet stream from the source is distributed evenly between the relay nodes. Consider one such relay queue, the arrival to the queue occurs when the source sends a packet to the relay node, and the departure from the queue takes place when the relay node delivers a packet to the destination node. It is clear that the inter-arrival time and the

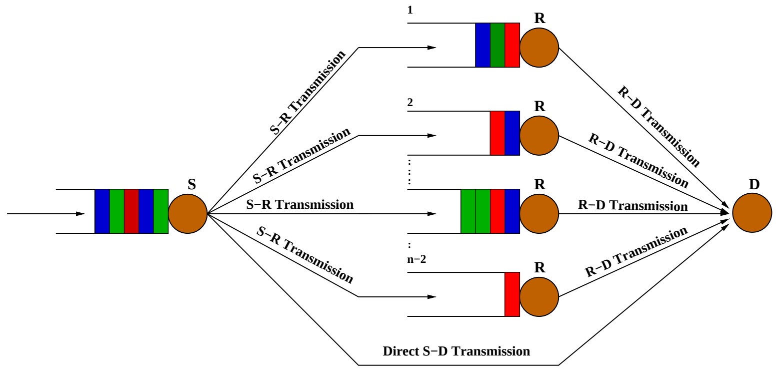

Figure 4: Network model for a particular S-D pair. The source queue is a simple Bernoulli(λ)/Bernoulli($\frac{\phi}{2}$). Each packet either goes directly to the destination or through one of the $n-2$ relay nodes. The relay queues are GI/GI/1-FCFS.

inter-departure time for this queue are of the same order as the inter-meeting time of any two arbitrary nodes in the network. We are now ready to estimate the overall queuing delay.

Proposition 4.1 *If all the nodes move in accordance with the RWMM, and if the exogenous packet stream at node i is Bernoulli with rate λ_i, where $\lambda_i < \frac{\phi}{2}$, then the expected packet delay for the source node i is $\Theta(T_p(n)n)$.*

Proof: Since the inter-meeting time scales as $\Theta(nT_p(n))$, the queuing delay at the relay queue must at least be $\Theta(nT_p(n))$. Hence, the queuing delay at the source node can simply be ignored. Using the first and the second moment of $I(n)$, as given in Proposition 3.1 (part III), along with the Kingman's ([9]) upper bound on delay given for a GI/G/1-FCFS queue (see [7]), the result follows. □

The following corollary is a direct consequence of proposition 4.1.

Corollary 4.1 *If all the nodes move in accordance with the RWMM, then the expected packet delay scales as $\Theta(n)$ under the FPSS, and $\Theta(\sqrt{n})$ under the VPSS.*

Our simulation results are in complete agreement with these theoretical results. Due to space constraints, we do not include them here.

5 Distributed 2-Hop Relaying With Redundancy

The D2HRP provides $\Theta(1)$ throughput per node, but incurs a large packet delay of the order of $\Theta(nT_p(n))$. For most of the applications, such large packet delay would be impossible to tolerate. In this section, we consider a simple distributed protocol that gives a much better delay performance than the D2HRP by introducing redundancy into the system. Before discussing the protocol any further, let us first look at how much the delay can possibly be improved. It turns out that there is a fundamental lower bound on the packet delay which is valid under all protocols which allow only nearest neighbors to communicate with each other.

Proposition 5.1 *For a network with n nodes, moving in accordance with the RWMM, no scheduling or relaying protocol which allows only nearest neighbor to communicate can guarantee an expected delay better than $\Theta(T_p(n)\sqrt{n})$.*

Proof: Note that since the S-D pairs are chosen randomly, each packet travels an expected distance of $\Theta(1)$ before reaching the destination. Further, since $T_c(n) = \Omega(T_p(n))$, the packet travels a distance of $O(1/\sqrt{n})$ per time-slot due to mobility. Since, only nearest neighbor communications are allowed the distance travelled due to relaying is also $O(1/\sqrt{n})$ per time-slot. Hence, in any time-slot the packet travels an aggregate distance of $O(1/\sqrt{n})$ towards the destination, and the result follows. □

We now describe the distributed 2-hop relaying protocol with redundancy (D2HRP-WR), which can achieve the above mentioned lower bound on the delay.

In the D2HRP-WR, each source node sends duplicate copies of the packet to new relay nodes, whenever possible. The packet is delivered to the destination either by the source node or one of the relay nodes. Clearly, once the packet is delivered to the destination, there is no need for any node to hold the packet in its queue. But, notifying all the nodes which currently hold the packet in their queue would result in too much of overhead. To avoid this overhead we follow the same approach as in [11]. All source nodes maintain a send number (SN). The source node increments the SN before sending a new packet, and all subsequent copies of the packet are sent with same SN. Each destination maintains a request number (RN), which is delivered to the transmitter before any R-D or S-D transmission. The source node sends at most $k(n)$ duplicate copies of the packet. This $k(n)$ can be varied depending upon the requirements of the application.

The detailed implementation of the protocol is as follows.

In each time-slot each node independently decides to be a sender or a potential receiver, just like in the case of the D2HRP. Then, one of the following three actions is performed by each successful sender-receiver pair:

I. *S-D Mode:* If the sender node has packets generated locally for the receiver then it asks the receiver for the current RN. All packets with SN < RN are deleted from the source. If a packet with SN=RN is found, then it is transmitted to the receiver, and the sender and the receiver increment their corresponding numbers, i.e., SN=SN+1 and RN=RN+1.

II. If no such packet exists then the sender chooses one of the following two options with equal probability:

- *S-R Mode:* The sender transmits the packet with current SN, and does so upon every transmission opportunity until $k(n)$ replicas have been delivered to distinct nodes. After such a time, SN is incremented to SN+1. If the sender node does not have any such packet to send, then it remains idle for that slot.

- *R-D mode:* As in the case of the S-D Mode, the sender asks the receiver for the current RN, and deletes all the packets with SN < RN. If any packet with SN=RN is found then it is sent to the receiver, and the receiver increments RN to RN+1. If no such packet is found then the sender remains idle for that slot.

Note that the above protocol guarantees an in-order delivery of the packets to the destination. The performance of the protocol can be improved slightly by allowing out of order packet delivery. We do not consider such a possibility in this paper.

The following Proposition states the main result of this section.

Proposition 5.2 *Consider a network with n nodes, moving in accordance with the RWMM, and following the D2HRP-WR. Let $\mathbb{E}\{D(n)\}$ be the expected packet delay, and $\lambda(n)$ be throughput per node. Then, for any $k(n) = O(\sqrt{n})$, we have $\mathbb{E}\{D(n)\} = \Theta\left(\frac{nT_p(n)}{k(n)}\right)$ and $\lambda(n) = \Theta\left(\frac{1}{k(n)}\right)$. In particular, when $k(n) = \Theta(\sqrt{n})$, then, $\mathbb{E}\{D(n)\} = \Theta(T_p(n)\sqrt{n})$ and $\lambda(n) = \Theta\left(\frac{1}{\sqrt{n}}\right)$.*

Proof: Consider a packet generated at an arbitrary source node at time $t = 0$. For the time being, assume that it is the only packet generated at the source, i.e., there is no delay due to queuing at the relay nodes. Once the packet reaches the head of the line at the source queue, the time required to reach the destination is at most $T(n) = T_1(n) + T_2(n)$, where $T_1(n)$ is the time required by the source node to send the packet to $k(n)$ other nodes, and $T_2(n)$ is the time required by one of those

$k(n)$ nodes to deliver the packet to the destination node. It is easy to show that $T_1(n) = \Theta(k(n)T_p(n))$ and $T_1^2(n) = \Theta(k^2(n)T_p^2(n))$. Clearly, the position of these $k(n)$ nodes, having received a packet from the same source node, are correlated, and thus the time taken to reach the destination node are also correlated. However, at time $T_1 + \frac{\sqrt{\pi}}{v_{min}}$[5] these $k(n)$ nodes are guaranteed to be uniformly distributed on S^2. Let $A(n)$ denote the additional amount of time required by one of these nodes to come into contact with the destination node. Clearly, $A(n)$ is the minimum of $k(n)$ i.i.d random variables, each of which is $\Theta(F(n))$ (see the definition of $F(n)$, given in section 3). Using Lemma 3.4 and 3.1, it is easily seen that $\mathbb{E}\{A(n)\} = \Theta(nT_p(n)/k(n))$ and $\mathbb{E}\{A^2(n)\} = \Theta(n^2T_p^2(n)/k^2(n))$. Noting that $\frac{\sqrt{\pi}}{v_{min}} = \Theta(\sqrt{n}T_p(n))$ and $k(n) = O(\sqrt{n})$, we have $E(T(n)) = O(nT_p(n)/k(n))$ and $E(T^2(n)) = O(n^2T_p^2(n)/k^2(n))$.

The SN/RN handshake ensures that newer packets do not interfere with the older packets, but that the replication of the next packet waiting at the source queue begins at or before time $T(n)$ after the current packet. The packets thus view the network as a single queue to which they arrive and are served sequentially. Using the Kingman's [9] upper bound, it follows that the expected packet delay is $O(nT_p(n)/k(n))$.

The lower bound on the expected packet delay follows easily by noting that the expected packet delay is at least as large as $\mathbb{E}\{A(n)\}$, which is $\Theta(nT_p(n)/k(n))$. Hence, the delay is $\Theta(nT_p(n)/k(n))$. Further, since each packet gets relayed to $\Theta(k(n))$ nodes, the throughput per node is $\Theta(1/k(n))$. □

Note that, making $k(n) = \omega(\sqrt{n})$ cannot reduce the delay beyond $\Theta(T_p(n)\sqrt{n})$. This follows from the fact the D2HRP-WR allows only nearest neighbor communications, and hence according to Proposition 5.1, must incur an expected packet delay of $\Omega(T_p(n)\sqrt{n})$.

6 Delay and Capacity Trade-offs

Note that the packet delay and throughput capacity under the D2HRP and the D2HRP-WR satisfy *delay/capacity* $= \Theta(nT_p(n))$. In this section, we show that *delay/capacity* $\geq \Theta(nT_p(n))$ is indeed a necessary trade-off. The fact that the D2HRP and the D2HRP-WR achieve this trade-off with equality establishes the sufficiency of the trade-off.

Proposition 6.1 *Consider a network with n mobile nodes, moving in accordance with the RWMM,*

[5]This is twice the maximum trip duration

and n S-D pairs, each generating traffic at an expected rate of $\lambda(n)$ packets per time-slot. Consider any arbitrary control protocol, which is used to stabilize the network, and guarantee an expected packet delay of $\mathbb{E}\{D(n)\}$. Then,

$$\frac{\mathbb{E}\{D(n)\}}{\lambda(n)} \geq \Theta(T_p(n)n)$$

Proof: The proof requires a lot of algebraic manipulations, and is forwarded to the Appendix. □

It is interesting to note that the above trade-off is not entirely limited to our settings. In fact, it holds under some remarkably different settings as well, like for example the static wireless networks.

7 Concluding Remarks

In this paper, we studied the delay and capacity trade-offs in mobile ad hoc networks with the random way-point mobility model. While there are some results available for random walk models, the RWMM has not been considered in any of the previous works. From a practical standpoint, the random way-point mobility model is perhaps more realistic than the random walk models.

We showed that the distributed 2-hop relaying protocol incurs a delay of $\Theta(nT_p(n))$, and that the delay performance can be significantly improved by introducing some redundancy into the system. This is similar in some sense to the multi-hop in static networks, where the redundancy can be thought of as the number of hops that the packet takes to reach the destination. Although, we considered only the random way-point mobility model in this paper, similar results can be derived under the Brownian mobility model as well (see [3]).

We showed that trade-off, $delay/capacity \geq \Theta(T_p(n)n)$, is both necessary as well sufficient under the RWMM. Using similar arguments, as to those in [1, 6], it is easy to show that the same trade-off holds for static networks as well.

Our results inspire a rich set of questions concerning the fundamental limits of mobile ad hoc networks. It would be nice to characterize the class of mobility models for which this trade-off holds. We believe that the trade-off would hold for all possible mobility models under which the distribution of the nodes remains uniform at all times. Such a result can perhaps be proved using similar techniques as in this paper.

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8 Appendix

Proof of Proposition 6.1

Let $\mathbb{E}\{D_i(n)\}$ be the expected packet delay for the S-D pair i. Then, we have

$$\mathbb{E}\{D(n)\} = \frac{\sum_i \mathbb{E}\{D_i(n)\}}{n}$$

Let $\mathbb{E}\{R_i(n)\}$ be the expected *redundancy* of the packets generated by the S-D pair i. Note that the redundancy of the packet is the number of nodes who receive a copy of the packet, including the destination node. The S-D pair i generates an expected traffic of $\lambda(n)\mathbb{E}\{R_i(n)\}$ per time-slot. Summing over all i, we obtain that the aggregate traffic generated by all the S-D pairs in the network is $\lambda(n)\sum_i \mathbb{E}\{R_i(n)\}$. Since there can be at most $n/2$ successful transmissions per time-slot, we have:

$$\lambda(n)\sum_i \mathbb{E}\{R_i(n)\} \leq \frac{n}{2} \tag{8.1}$$

Now suppose that the expected redundancy, $\mathbb{E}\{R_i(n)\}$, of the packets generated by the S-D pair i is $\Omega(\sqrt{n})$. Since, the Proposition 5.1 implies that $\mathbb{E}\{D_i(n)\}$ must at least be $\Theta(T_p(n)\sqrt{n})$, we obtain

$$\mathbb{E}\{D_i(n)\}\mathbb{E}\{R_i(n)\} \geq \Theta(nT_p(n)) \tag{8.2}$$

Now suppose that $\mathbb{E}\{R_i(n)\} = o(\sqrt{n})$. Consider a packet generated at the S-D pair i. The packet has an expected delay of $\mathbb{E}\{D_i(n)\}$, and an expected redundancy of $\mathbb{E}\{R_i(n)\}$. Let the random variables

$R_i(n)$ and $D_i(n)$ represent the actual redundancy and delay for the packet. Then we have:

$$\begin{aligned}\mathbb{E}\{D_i(n)\} &\geq \mathbb{E}\{D_i(n)|R_i(n) \leq 2^a\mathbb{E}\{R_i(n)\}\}Pr\{R_i(n) \leq 2^a\mathbb{E}\{R_i(n)\}\} \\ &\geq \mathbb{E}\{D_i(n)|R_i(n) \leq 2^a\mathbb{E}\{R_i(n)\}\}\left(1 - \frac{1}{2^a}\right) \end{aligned} \tag{8.3}$$

where (8.3) follows from Markov Inequality, and a is a positive integer to be fixed later.

Consider a virtual system in which there are $2^a\mathbb{E}\{R_i(n)\}$ uniformly distributed nodes holding the packet at the start. Let $T(n)$ be the time required for one of these nodes to deliver the packet to the destination node. Then, $T(n) = \Theta(Z(n))$, where $Z(n)$ is the mininum of $2^a\mathbb{E}\{R_i(n)\}$ i.i.d random variables with the same distribution as $F(n)$ (see the definition of $F(n)$ in section 3). Using Lemma 3.4, it follows that $\mathbb{E}\{T(n)\} = \Theta(nT_p(n)/2^a\mathbb{E}\{R_i(n)\})$. Although, there are more nodes holding the packet in this virtual system, the mean of $T(n)$ does not necessarily bound $\mathbb{E}\{D_i(n)|R_i(n) \leq 2^a\mathbb{E}\{R_i(n)\}\}$. This is because conditioning on the event $\{R_i(n) \leq 2^a\mathbb{E}\{R_i(n)\}\}$ might skew the probabilities associated with the node mobility process. To understand this, consider an extreme case: if the redundancy of a packet is given to be less than or equal to 1, then it would imply that the packet is sent directly from the source to the destination. The expected delay in such a case is much smaller than the expected delay in the case where there is only one node holding the packet at the start.

However, since the event $\{R_i(n) \leq 2^a\mathbb{E}\{R_i(n)\}\}$ occurs with a probability at least $1 - 1/2^a$, we obtain the following bound:

$$\mathbb{E}\{D_i(n)|R_i(n) \leq 2^a\mathbb{E}\{R_i(n)\}\} \geq \inf_{\mathcal{F}:P(\mathcal{F})\geq\left(1-\frac{1}{2^a}\right)} \mathbb{E}\{T(n)|\mathcal{F}\}$$

Since $T(n) = \Theta(Z(n))$, there exists a constant $c_8 > 0$, such that

$$\mathbb{E}\{D_i(n)|R_i(n) \leq 2^a\mathbb{E}\{R_i(n)\}\} \geq c_8 \inf_{\mathcal{F}:P(\mathcal{F})\geq\left(1-\frac{1}{2^a}\right)} \mathbb{E}\{T(n)|\mathcal{F}\}$$

It is clear that the event $\mathcal{F}$ which achieves the infimum must be $\mathcal{F} = \{Z(n) \leq x : P(Z(n) \leq x) = 1 - 1/2^a\}$, and we get

$$\mathbb{E}\{D_i(n)|R_i(n) \leq 2^a\mathbb{E}\{R_i(n)\}\} \geq c_8\mathbb{E}\{Z(n)|Z(n) \leq x\} \tag{8.4}$$

Now since $Z(n)$ is the minimum of $2^a\mathbb{E}\{R_i(n)\}$ i.i.d random variables with the same distribution as $F(n)$, it follows from Lemma 3.4 and 3.8, that

$$P\left(Z(n) > \frac{m\sqrt{\pi}}{v_{min}(n)}\right) \le (1 - c_2 r(n))^{2^a m\mathbb{E}\{R_i(n)\}}$$
$$P\left(Z(n) > \frac{\bar{l}m}{2v_{max}(n)}\right) \ge \frac{1}{2}(1 - c_1 r(n))^{2^a m\mathbb{E}\{R_i(n)\}} \qquad (For\ m \ge m_o\ )$$

In order to simplify the exposition, assume that $v_{max}(n) = v_{min}(n) = 1$ (this corresponds to the VPSS). A little more careful analysis along the lines of Lemma 3.2 shows that c_1 and c_2 can be taken to be 4 and 1/4, respectively. Since $\sqrt{\pi} < 2$ and $\bar{l} > 1/4$, we obtain

$$\begin{aligned} P\left(Z(n) > 2m\right) &\le (1 - 4r(n))^{2^a m\mathbb{E}\{R_i(n)\}} \\ &\le e^{-2^{a+2} m r(n)\mathbb{E}\{R_i(n)\}} \quad (Using\ e^{-x} \ge (1-x),\ for\ 0 \le x \le 1) \end{aligned} \tag{8.5}$$

$$\begin{aligned} P\left(Z(n) > \frac{m}{8}\right) &\ge \frac{1}{2}(1 - \frac{1}{4}r(n))^{2^a m\mathbb{E}\{R_i(n)\}} \qquad (For\ m \ge m_o) \\ &\ge \frac{1}{2}e^{-2^{a-1} m r(n)\mathbb{E}\{R_i(n)\}} \qquad (For\ m \ge m_o\ ,\ using\ e^{-2x} \le (1-x),\ for\ 0 \le x \le 1/2) \end{aligned} \tag{8.6}$$

Now since $P(Z(n) \le x) = 1 - 1/2^a$, from (8.6), we obtain

$$x \ge \frac{(a-1)\log 2}{2^{a+2} r(n)\mathbb{E}\{R_i(n)\}} = x_1 \tag{8.7}$$

Since $r(n) = \Theta(1/\sqrt{n})$ and $\mathbb{E}\{R_i(n)\} = o(\sqrt{n})$, x_1 goes to ∞ with n. This implies that for any positive integer b (to be fixed later) there exists an integer, say x_2, such that $x_1/2 < x_2 < x_1$ and x_2 is an integer multiple of $2b$. Since x_2 is an integer multiple of $2b$, x_2/b is an even integer, and equation (8.6) implies that

$$\begin{aligned} P\left(Z(n) > \frac{x_2}{b}\right) &\ge \frac{1}{2}e^{-\frac{2^{a+2} x_2 r(n)\mathbb{E}\{R_i(n)\}}{b}} \\ &\ge \frac{1}{2}e^{-\frac{(a-1)\log 2}{b}} \qquad (Using\ x_2 < x_1) \\ &= \left(\frac{1}{2}\right)^{\frac{a+b-1}{b}} \end{aligned} \tag{8.8}$$

Using (8.5), we obtain

$$\begin{aligned}
P(Z(n) > x_2) &\leq e^{-2^{a+1}x_2 r(n)\mathbb{E}\{R_i(n)\}} \\
&\leq e^{-\frac{(a-1)\log 2}{4}} \qquad (Using\ x_2 > x_1/2) \\
&= \left(\frac{1}{2}\right)^{\frac{a-1}{4}}
\end{aligned} \tag{8.9}$$

Setting $a = 13$ and $b = 12$, we obtain $P(Z(n) > x_2) \leq 1/8$ and $P(Z(n) > x_2/b) \geq 1/4$. Now, we have

$$\begin{aligned}
\mathbb{E}\{Z(n)|Z(n) \leq x\} &\geq \mathbb{E}\{Z(n)|Z(n) \leq x_2\} \quad (Using\ x > x_2) \\
&\geq \frac{x_2}{b}\frac{P(\frac{x_2}{b} \leq Z(n) \leq x_2)}{P(Z(n) \leq x_2)} \\
&\geq \frac{x_1}{2b}\left(P\left(Z(n) > \frac{x_2}{b}\right) - P(Z(n) > x_2)\right) \\
&= \frac{(a-1)\log 2}{2^{a+6}r(n)\mathbb{E}\{R_i(n)\}b} \\
&= \frac{\log 2}{2^{19}r(n)\mathbb{E}\{R_i(n)\}}
\end{aligned} \tag{8.10}$$

Since $v_{max}(n) = v_{min}(n) = 1$, we have $T_p(n) = \Theta(r(n)) = \Theta(1/\sqrt{n})$. Using (8.3), (8.4), and (8.10), we obtain

$$\mathbb{E}\{D_i(n)\}\mathbb{E}\{R_i(n)\} \geq \Theta(nT_p(n))$$

Hence, $\mathbb{E}\{D_i(n)\}\mathbb{E}\{R_i(n)\} \geq \Theta(nT_p(n))$ for all possible values of $\mathbb{E}\{R_i(n)\}$. Taking the sum over all i, and multiplying with $\lambda(n)$, we obtain

$$\begin{aligned}
\lambda(n)\Theta(nT_p(n))\sum_{i=1}^{n}\frac{1}{\mathbb{E}\{D_i(n)\}} &\leq \lambda(n)\sum_{i=1}^{n}\mathbb{E}\{R_i(n)\} \\
&\leq \frac{n}{2} \qquad (Using\ (8.1))
\end{aligned} \tag{8.11}$$

Now using the convexity of $1/x$, for $x \geq 0$, it follows that

$$\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\mathbb{E}\{D_i(n)\}} \geq \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\mathbb{E}\{D_i(n)\}} = \frac{1}{\mathbb{E}\{D(n)\}} \tag{8.12}$$

Using (8.11) and (8.12), the result follows.