Chapter 1

PROVIDING QOS IN LARGE NETWORKS: STATISTICAL MULTIPLEXING AND ADMISSION CONTROL

Nikolay B. Likhanov
Institute for Problems of Information Transmission, Moscow, Russia
likh1@online.ru

Ravi R. Mazumdar
Purdue University, West Lafayette, USA
Corresponding author
mazum@purdue.edu

Francois Theberge
Communications Research Centre and University of Ottawa , Canada
theberge@ieee.org

Abstract In this paper we consider the problem of providing statistical Quality of Service (QoS) guarantees defined in terms of packet loss when independent heterogeneous traffic streams access a network router of high capacity. By using a scaling technique we show how this problem becomes tractable when the server capacity is large and many traffic streams are present. In particular we show that we can define an effective bandwidth for the sources that allows us to map the model onto a multirate loss model. In particular we show several insights on the multiplexing problem as the capacity becomes large. We also provide numerical and simulation evidence to show how the largeness of networks can be used to advantage in providing very simple admission control schemes. The techniques are based on large deviations, local limit theorems, and the product-form associated with co-ordinate convex policies.

1. Introduction

Quality of Service (QoS) guarantees are going to be distinct features of the services that users will obtain from next generation high-speed networks. In the emerging networks, the QoS issue will be much more complicated since the QoS requirements will differ from user to user. Indeed networks will need to offer heterogeneous QoS. This is an important issue due to the fact that QoS based pricing structures are increasingly being advocated.

One of the challenging problems in networks is to characterize the admissible region of the numbers of connections or flows that can be admitted into the network in order to guarantee a given level of Quality of Service (QoS). QoS is usually specified by loss probability constraints or bounds on the delays incurred by the bits as they traverse the network from source to destination. There are two approaches: providing deterministic or statistical guarantees.

Deterministic guarantees are hard guarantees and the analysis is usually based on a worst-case analysis. When traffic streams are shaped or their sample paths forced to conform to a given envelope, a powerful approach called *network calculus* Chang, 1998; LeBoudec, 1998 has been developed. This approach allows us to consider the end-to-end problem but yields conservative results due to the fact that it is essentially a worst-case approach. Providing statistical QoS is much more efficient in terms of resource utilization (in this context being able to support a larger number of flows) and when networks are large they lead to *economies of scale* Duffield and O'Connell, 1995. This is the phenomenon of *statistical multiplexing*. The maximum number of flows is limited by the stability requirement that the average total rate of the flows using a particular resource must be less than the capacity of that resource.

Figure 1.1 shows the typical scenario when different criteria are selected for admitting users into a network assuming that there are 2 classes of users with a high level of burstiness.

One of the key issues in providing statistical QoS is our ability to estimate and/or measure packet loss at a network element under general traffic assumptions. This is intractable in general except in a few simple cases. However knowing that the statistical performance requirements are fairly stringent, i.e. the probability of packet loss to be in the range $10^{-9} - 10^{-5}$, implies that we are concerned with the tail probability distribution that naturally leads to the study of asymptotics. This makes the problem more tractable. There are basically two types of asymptotics of interest: 1) The large buffer asymptotic when there are

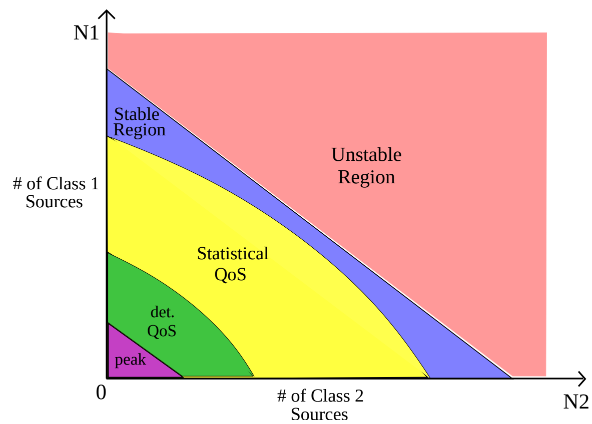

Figure 1.1. Admissible region when using different criteria for the case of 2 classes.

a few traffic streams which share a resource and a given stream can consume a significant amount of the bandwidth. This scenario occurs when there is a substantial amount of non delay sensitive traffic, and 2) The many sources asymptotics when many small sources share the resource. This scenario is of importance in the core of the network or in MPLS (Multipath Label Switching) when virtual routes are established within a network and a flows between the origin-destination (O-D) are aggregated and switched along them. This latter asymptotic is infact an instance of the so-called statistical multiplexing and is the scenario we consider.

Large deviations asymptotics have been studied for a number of scenarios and by now there is quite a lot of know-how from the analytical standpoint. For single buffers based on First In First Out (FIFO) disciplines they can be found in A. Botvich and Duffield, 1995; Choe and Shroff, 1999; Courcoubetis and Weber, 1996; Likhanov and Mazumdar, 1999. For multi-buffered systems large buffer asymptotics for a priority model can be found in O'Connell, 1998 while the many-sources case with HOL priority schemes are reported in Delas et al., 2002. For fair queueing or GPS disciplines large buffer results under restrictive assumptions on the sources can be found in Massoulie, 1999; Bertsimas et al., 1998 while the many sources case can be found in Kotopoulos, 2000 . In the FIFO case the delay distributions are readily obtained from the so-called buffer overflow results. In the HOL and GPS cases it can be shown the delay distributions when a large number of sources are present are also closely related to the loss rate asymptotics Shakkottai and Srikant, 2001; Delas et al., 2002. A recent monograph Ganesh et al., 2004 is quite comprehensive in discussing the various asymptotics mainly in the context of single queues.

In spite of the successes in analyzing the single node case in both the large buffer and many sources contexts, there has been only limited success at identifying the asymptotics for network models. In general, only when the input and output rate functions have "linear geodesics" [*], the end-to-end analysis is feasible by an iterative procedure. This however is not very useful from the point of gaining insights and to perform admisssion control. In Ganesh and O'Connell, 1998; O'Connell, 1997, it is shown that in queues with more than one input, the departure process need not have the linear geodesic property and thus identifying the rate functions of the outputs explicitly in terms of the inputs is not easy. However, there has been some partial success in the many sources case. This is due to the fact that in the many sources context, the resulting measure is a convolution and the buffer occupancy probability goes to zero even for very small buffers. This result has been exploited by Wischik, 1999 who has shown that the moment generating function (m.g.f.) for a single input does not change as it passes through a node when multiplexed with many similar inputs. Since the m.g.f is the necessary information required to determine the overflow asymptotics, one can analyze large in-tree networks where each node receives only a small number of inputs from a large number of other independent nodes. However, such a situation is of limited scope and not valid in networks where many flows utilize many links. Eun and Shroff, 2003 recently extended this result by considering a two-stage queueing system where the first node serves many flows, of which a certain fixed finite set arrive to the second node. In this case, the first node can be ignored for calculating the overflow probability of the second node as the number of flows of the first node (N) increases. But this result does not hold when the number of output flows arriving at the downstream node is of $O(N)$. This situation is however of interest when trying to estimate the end-to-end QoS in the general scenario described in the beginning. Although overflow probability still goes to zero at the downstream node, determining the rate of this decay depends on the particular time scale and on the sample path characteristics of the output flows from the upstream nodes.

When the buffers are small, it has been shown that the instantaneous values of the total input rate determine the asymptotics Likhanov and Mazumdar, 1999; Mandjes and Kim, 2001.

[*] This corresponds to the situation where the most likely path of the buffer occupancy process to an extreme value is a straight line

This small buffer scenario is actually of much interest in today's networks where buffers are small (in comparison to the server capacity). This is the essence of the so-called **rate envelope multiplexing** in networks (see Roberts, 1998) where buffers are small to absorb local fluctuations but essentially the network can be modeled by bufferless nodes. Moreover it is shown in Ozturk et al., 2004 that asymptotically the admissible region can be computed by only knowing the input characteristics and thus from the point of view of admission control it is enough to consider a single buffer scenario.

One of the basic issues associated with the connection acceptance phase is the determination of the bandwidth associated with a given connection. When the sources are CBR (constant bit rate) this is relatively easy to do because the bandwidth is fairly constant modulo jitter introduced in the network. However, when source bandwidth varies randomly over the duration of the connection this is much more difficult given the QoS constraints. Allocating the peak rate for such a connection will negate the gains achievable by statistical multiplexing (see figure 1.1) while the mean rate would be very poor with regard to packet loss. This issue has been addressed in the context of *effective bandwidths* by Hui, 1988 for unbuffered models and more recently by Kelly, 1996 in an excellent survey paper. Packet loss is directly related to the number of connections being carried and thus there is a tight coupling between the connection acceptance and packet level phenomena. One major issue is the great difference in time-scales and due to the extremely high bit rates feedback information is not very useful to control bit loss while feedback is the basic means at the connection level. This coupling leads to a bootstrapping between open loop and closed loop control i.e. packet level phenomena impacts the number of connections allowable for which the a priori statistical information of bit flow must be used and the number of connections in turn alter bit loss. In this paper we address this issue and provide a framework in which this procedure can be done based on apriori information about the arrival rates of connections or sessions.

The basic aim of this paper is to show how the largeness of network capacity and the multiplexing of many independent flows can be used to advantage. It yields simple closed-form results and a very simple admission procedure based on the notion of an *effective bandwidth* of a connection mentioned earlier. This just amounts to approximating the boundary of the acceptance region by a hyperplane constructed at a particular point defined by the equilibrium distribution of the set of connections that lead to a violation of the QoS constraints. Moreover, we show that this is completely determined by knowing their arrival rates into the network leading to a more robust procedure since the in classical effective bandwidth idea the effective bandwidth of sources changes with the connection mix. We then show some asymptotic properties in that as the size of the system increases, the effective bandwidths of sources converge to their mean rates. Finally we show the connection of between the admission control strategy and the problem of estimating connection blocking which is also an important design parameter called the Grade of Service (GoS) used in defining so-called Service Level Agreements (SLAs). In particular, we show that the effective bandwidth as defined in this paper provides the consistent mapping that maps packet level phenomena to the connection level phenomena in that the most likely equilibrium state for the blocking coincides with the most likely equilibrium state for loss.

The organization of the paper is as follows. In Section 1.2 we formulate the problem and then use recent results from local limit large deviations to obtain an estimate of connection acceptance region. In Section 1.3 we develop the notion of the most likely loss configuration and show some of its properties. Section 1.4 stitches together of the previous results developed in the previous sections for the CAC scheme. In Section 1.5 we illustrate the application of the CAC procedure on an example with ON-OFF sources and compare the analytical results with simulations.

2. Problem Formulation and Acceptance Region

Consider a link of capacity C units of bandwidth which is accessed by M types of independent stationary, ergodic connections or flows. It is assumed that there are M heterogeneous classes of connections and a connection of type i; $i \in \{1, 2, \cdots, M\}$ arrives according to a Poisson process with intensity λ_i. A source when connected has a certain bit rate , say, $a_i(t) \in [0, \Pi_i]$ where Π_i denotes the peak rate of the source. Let $r_i = \mathbb{E}[a_i(t)]$ denote the mean bit rate.

We assume that the link has a given configuration of the number of connections of each type being carried at a given time which we assume is held invariant. We will obtain the bit loss† probability for a given fixed configuration.

More precisely, suppose that the given link has $\mathbf{n} = (n_1, n_2, \cdots, n_M)$ number of connections being carried at time t where n_i denotes the number of connections of type i. Let $X_i(t)$ denote the instantaneous load on the link due to source i then by definition $X_i(t) = \sum_{m=1}^{n_i} a_{m,i}(t)$ where

†Although in this paper we refer to the flow in bits, the granularity can be taken to be in packets in which case we have a packet loss measure

$a_{m,i}(t)$ are i.i.d. with common distribution as $a_i(t)$ and $0 \le X_i(t) \le n_i\Pi_i$.

We assume that $\sum_{i=1}^{M} \Pi_i n_i > C$ since otherwise there can be no loss. Furthermore, since we are interested in very small bit loss probabilities we assume that the average load is less than C i.e. $\sum_{i=1}^{M} n_i r_i < C$. The other situations are not of interest although in principle the development carried out below can still be done.

Let $X(t) = \sum_{i=1}^{M} X_i(t)$ denote the instantaneous load on the system. Then the number of bits lost during an interval of length T is just given by: $N(T) = \int_0^T (X(t) - C)^+ dt$

Let $N_1(T) = \int_0^T \sum_{i=1}^{M} n_i a_i(t) dt$ denote the total number of bits which are offered to the system in an interval of length T and $a_i(t)$ is the instantaneous rate of type i calls which is a r.v. with values in $[0, \Pi_i$. Then $\frac{N(T)}{N_1(T)}$ denotes the fraction of bits lost. Since the processes are stationary and ergodic, by the strong law of large numbers, the stationary bit loss probability is given by:

$$\begin{aligned} \mathbb{P}(bit\ loss) &= \lim_{T \to \infty} \frac{N(T)}{N_1(T)} \\ &= \frac{\mathbb{E}[(X - C)^+]}{\sum_{i=1}^{M} n_i r_i} \end{aligned} \tag{2.1}$$

where $x^+ = \max\{x, 0\}$.

Now to compute $\mathbb{E}[(X - C)^+]$ we need to determine the tail distribution $\mathbb{P}(X > x)$ and this is given by a convolution measure since the connections are independent. This is extremely intractable in general.

In the sequel we will show that in fact when the size of the system is large then one can exploit very elegant results from the theory of sums of independent random variables (termed local limit theorems, which can be found in Petrov, 1975 or Korolyuk et al., 1985) to obtain explicit analytical results that are $O(1)$ in complexity.

The notion of a large system can be viewed in many ways. A particularly attractive way is to view a large system as a scaled version of a nominal system. More precisely, we scale both the capacity C and the number of sources $\{n_i\}$ by a factor N i.e. $C(N) = NC$ and $n_i(N) = Nn_i;\ i = 1, 2, \cdots, M$. This scaling keeps the ratio of the number of sources to the capacity constant. This is a purely analytic device which will allow us to determine the accuracy of our results.

Let us now assume the scaled version. Let $P_N(\mathbf{m})$ denote the bit loss probability given by:

$$\begin{aligned} P_N(\mathbf{m}) &= \frac{\mathbb{E}[(X^(N) - NC)^+]}{N\sum_{i=1}^M r_i n_i} \\ &= \frac{\int_{NC}^{\sum N n_i \Pi_i} dF^(N)(x)}{n\sum_{i=1}^M n_i r_i} \end{aligned}$$

where $F^(N)(x)$ is the distribution of $X^(N)(t) = \sum_{i=1}^M X_i^(N)(t)$ and $X_i^(N)(t) = \sum_{j=1}^{Nn_i} a_{i,j}(t)$.

We first begin with the following simple result which is a simple consequence of sums of independent r.v's. The proof is trivial and so we omit it.

Lemma 2.1 *Define $\eta = \sum_{i=1}^M \xi_i$ where ξ_i are independent r.v's with distribution the same as $\sum_{j=1}^{n_i} a_{i,j}(t)$. Let $\{\eta_i\}$ be an independent collection of r.v's with the same distribution as η defined above. Then the random variables $\sum_{i=1}^N \eta_i$ and $\sum_{i=1}^M X_i$, where X_i are independent r.v's with distribution as $\sum_{j=1}^{Nn_i} a_{i,j}$, have the same distribution.*

Remark 2.1 *The importance of this result is to convert the summation with respect to the number of types to a summation in the scale N. Then noting that the probability distribution of sums of N i.i.d. r.v's can be written as the N-fold convolution of the common distribution we can construct useful estimates based on measure changes and local limit theorems.*

As a result of the above Lemma we can write $F^(N)(dx) = \mu^{*N}(dx)$ where $\mu(dx)$ is the measure of η. We can now obtain estimates for the required loss by now invoking the Bahadur-Rao theorem Bahadur and Rao, 1960 from local limit large deviations. We state the theorem below as well as a theorem on local limit large deviations for densities due to Petrov, 1975.

Proposition 2.1 *Let $X^N = \sum_{j=1}^{Nm} X_j$ where $\{X_j\}_{j=1}^{Nm}$ are i.i.d. r.v's with moment generating function $\phi(h)$.*

Then as $N \to \infty$, uniformly for any $u > 0$

(Petrov): *For all $u \in (-\infty, +\infty)$*

$$P^{X^N}(Nu)du = \mathbb{P}\left(X^N \in [Nu, Nu + du)\right) = \frac{e^{-NI(u)}}{\sqrt{2\pi\sigma^2 N}} du \left(1 + O(\frac{1}{N})\right) \tag{2.2}$$

(Bahadur-Rao): *For $u > E[X_i]$:*

$$\mathbb{P}\left\{X^N \geq Nu)\right\} = e^{-NI(u)} \frac{1}{\tau\sqrt{2\pi\sigma^2 N}} \left(1 + O\left(\frac{1}{N}\right)\right) \tag{2.3}$$

where

$$I(u) = u\tau - n_i \log(\phi(\tau)) \tag{2.4}$$

where $\tau(u)$ is the unique solution to:

$$m\frac{\phi'(\tau)}{\phi(\tau)} = u \tag{2.5}$$

and

$$\sigma^2(u) = m\left(\frac{\phi''(\tau)}{\phi(\tau)} - \left(\frac{\phi'(\tau)}{\phi(\tau)}\right)^2\right) \tag{2.6}$$

$I(u)$ is referred to in large deviations theory as the rate function.

We apply the above result by taking $\mu = \mu_\eta$ where μ_η is the distribution corresponding to the r.v. η and $u = C$. By the definition of η the moment generating function is given by

$$\phi_\eta(t) = \prod_{k=1}^{M} (\phi_k(t))^{n_k} \tag{2.7}$$

where $\phi_k(t)$ is the moment generating function of a_k.
and therefore

$$I(C) = C\tau_c - \sum_{i=1}^{M} n_i \ln(\phi_i(\tau_c)) \tag{2.8}$$

where τ_c is the unique (since $\sum_{i=1}^{M} n_i \Pi_i > C$ and $\sum_{i=1}^{M} n_i r_i < C$ by assumption) solution of

$$\sum_{i=1}^{M} \frac{n_i \phi_i'(\tau_c)}{\phi_i(\tau_c)} = C \tag{2.9}$$

Using the Bahadur-Rao theorem Bahadur and Rao, 1960 and the result of Petrov, 1975, we can then show the following result for the bit loss probability necessary to characterize the acceptance region. The proof can be found in Likhanov and Mazumdar, 1999; Likhanov et al., 1996.

PROPOSITION 2.2 *Consider an unbuffered system of capacity NC which carries Nn_i ; $1 \le i \le M$ independent stationary, ergodic sources with a source of Type i having instantaneous rate $a_i(t)$ which is a r.v. which takes values in $[0, \Pi_i]$ with mean rate $r_i = \mathbb{E}[a_i(t)]$ Under the hypotheses that $\sum_{i=1}^M n_i \Pi_i > C$ and $\sum_{i=1}^M n_i r_i < C$ the stationary bit loss probability is given by:*

$$P(bit\ loss) = \frac{e^{-(NI(C)}}{\tau_c^2 C \rho \sqrt{2\pi\sigma^2 N^3}} \left(1 + O(\frac{1}{N})\right) \qquad (2.10)$$

where:

i) $\tau_c > 0$ is the unique solution to

$$\sum_{i=1}^{M} \frac{n_i \phi_i'(\tau_c)}{\phi_i(\tau_c)} = C$$

where $\phi_i(t)$ is the moment generating function of a_i.

ii) $I(C)$ is the rate function given by:

$$I(C) = C\tau_c - \sum_{i=1}^{M} n_i \ln(\phi_i(\tau_c))$$

iii) σ^2 is given by

$$\sigma^2 = \sum_{i=1}^{M} n_i \left(\frac{\phi_i''(\tau_c)}{\phi_i(\tau_c)} - \left(\frac{\phi_i'(\tau_c)}{\phi_i(\tau_c)}\right)^2 \right)$$

and $\rho = \sum_{i=1}^M r_i n_i$.

REMARK 2.2 *If the bit overflow probability is used as the QoS parameter, then the bound is given by the Chernoff bound which is just $e^{-NI(C)}$. This is the starting point of the approach in Hui, 1988; Kelly, 1996*

Examples

We now give explicit relations for some commonly used traffic sources in applications.

ON-OFF Sources: These are the most commonly used source models to represent variable bit rate (VBR) traffic. The importance of these models is that they serve as *worst case* traffic for a given set of traffic models characterized by burst length, peak and mean rates as shown by Doshi, 1995; Guillemin et al., 2002. Given their importance we provide detailed expressions for the required quantities.

By definition an ON-OFF source has an instantaneous rate $a_i \in \{0, \Pi_i\}$ i.e. it is either OFF (and therefore with rate 0) or ON at peak rate Π_i. Let p_i denote the stationary probability that a source is ON, then $r_i = \Pi_i p_i$. (Alternatively p_i is obtained from the mean rate specification by the preceeding relation).

For this particular case the instantaneous load on the system is completely specified by the number of connections which are in their ON state at a given time. Given the independence assumption on the sources equation (1.6) in this case reads:

$$P(\sum_{i=1}^{M} X_i = k) = \sum_{\mathbf{m} \in \mathbf{A}(k)} \prod_{i=1}^{M} \binom{n_i}{m_i} p_i^{m_i} (1-p_i)^{n_i - m_i} \tag{2.11}$$

where

$$\mathbf{A}(k) = \{\mathbf{m} : \sum_{i=1}^{M} m_i \Pi_i = k\} \tag{2.12}$$

In spite of the above explicit form the computational complexity remains for large systems. For this model the quantities necessary to compute the bit loss probability are:

i. $\phi_\eta(t) = \prod_{i=1}^{M} (p_i e^{t\Pi_i} + 1 - p_i)^{n_i}$

ii. τ_c is the unique solution to

$$\sum_{i=1}^{M} \frac{n_i p_i \Pi_i e^{\tau_c \Pi_i}}{p_i e^{\tau_c \Pi_i} + 1 - p_i} = C$$

iii. The rate function $I(C)$ is given by:

$$I(C) = C\tau_c - \sum_{i=1}^{M} n_i \ln(p_i e^{\tau_c \Pi_i} + 1 - p_i)$$

iv. σ^2 (variance under the changed distribution)

$$\sigma^2 = \sum_{i=1}^{M} \frac{n_i p_i \Pi_i^2 e^{\tau_c \Pi_i} (1-p_i)}{(p_i e^{\tau_c \Pi_i} + 1 - p_i)^2}$$

Uniform sources: These correspond to sources when there is complete lack of information regarding the bit flow. In particular the probability that the instantaneous bit flow is equally weighted between all the states i.e. $F_i(x) = \frac{x}{\Pi_i}$ for $x \in [0, \Pi_i]$.

In this case

i) $\phi_i(t) = \frac{1}{\Pi_i}\left(\frac{1-e^{t(\Pi_i)}}{1-e^t}\right)$

ii) $r_i = \frac{\Pi_i}{2}$

iii) τ_c and $I(C)$ can be calculated knowing $\phi_i(t)$

Markov Modulated Sources: This is also a commonly used source model where the instantaneous rate a_i corresponds to the state of the underlying Markov chain. In this case $p_{i,j} = \pi_i(j)$ where $\pi_i(.)$ denotes the stationary distribution of the Markov chain defined on $\{0, 1, \cdots, \Pi_i\}$ for source of type i.

Thus we see that the stationary bit loss probability can easily calculated once the underlying model for the rate process is specified.

In the following subsection we discuss the accuracy of the estimates obtained for the ON-OFF source model by comparing the results with those obtained by the commonly used Gaussian approximations as well as simulations.

2.1 Accuracy of estimates

We now demonstrate the accuracy of the estimate given by Proposition 2.2 by comparing it with simulation results as well as the a Gaussian approximation based on a central limit approximation for the convolution measure.

It is readily seen that the validity of the Gaussian approximation is completely out of the range of bit loss probabilities we are interested in i.e. the Gaussian approximation is only useful for relatively large values of bit loss (in our context when the scale is small). The results reported are for bit loss probabilities of the order 10^{-6} since below this level it is very difficult to obtain any reasonable confidence in simulations. But even at this level the accuracy of the method proposed is obvious.

In the following example, we set the capacity $C = 20$ with two classes of traffic, i.e. $M = 2$. We use the following data

$$n_1 = 20, n_2 = 10, p_1 = .275, p_2 = .8, \Pi_1 = 2, \Pi_2 = 1$$

and we use different values for the multiplier N.

Results in Table 1 are given as base 10 logarithms, so the losses are of the orders $10^{-5} - 10^{-2}$. We note that theorem 1.2 is very precise when bit loss is small, which is not the case for the Gaussian approximation. We have also given the results for based on a simple application of the Chernoff bound.

N	Simulation (99% conf. int.)	Chernoff bound (overflow prob.)	Gaussian	Theorem 1.2
60	(-3.5,-3,3)	-1.4	-4.4	-3.2
80	(-4.0,-3.7)	-1.7	-5.0	-3.6
100	(-4.3,-4.0)	-2.0	-5.6	-4.0
120	(-4.7,-4.4)	-2.3	-6.1	-4.4
140	(-5.0,-4.7)	-2.6	-6.7	-4.7
160	(-5.3,-5.0)	-2.8	-7.2	-5.1
180	(-5.4,-5.3)	-3.1	-7.7	-5.4
200	(-5.7,-5.8)	-3.4	-8.2	-5.7

Table 1.1. Example

Let us now note the importance of the bit loss probability estimates obtained above. For convenience, with regard to Table 1 above, suppose that ε the bound on the loss probability is required to be $\sim 10^{-5}$. The simulations indicate that with a capacity of 3200 the system can handle 3200 connections of type 1 and 1600 connections of type 2. For this configuration the Gaussian estimates suggest bit loss of the order 10^{-7} implying more connections can be admitted while in fact admitting more connections can only drastically reduce performance. Thus for using the Gaussian approximation is too optimistic for bit loss. Keeping the number of connections of Type 1 at 3200 calculations based on the Gaussian estimate keeping capacity at 3200 for loss probabilities of the order 10^{-5} gives the number of type 2 connections to be 1680. For this configuration the simulation results with 99% confidence give bit loss estimates of the order 10^{-3} which is clearly out of the acceptance region. The corresponding results using the results of Theorem 1.2 fall within the margin of error for the simulations. From the values for bit loss using the Chernoff bound it is readily seen that it is too conservative.

2.2 Acceptance Region

In the development so far we assumed that the configuration of the number of connections of each type i.e. $\{n_i\}_{i=1}^{M}$ is known. We then obtained the approximation to the stationary bit loss probability for a given configuration. In reality, the arrivals of connections and whether they are admitted or not implies that the configuration is in fact a random variable (which depends on the admission strategy).

Let $P_L^N(n_1, n_2, \cdots, n_M)$ denote the bit loss probability for the given configuration of $(Nn_1, Nn_2, \cdots, Nn_M)$ viewed as a function of the connection configuration. This will allow us to determine the so-called acceptance region since this mapping defines the bit loss probabilities over all possible connections. The QoS requirements on the bit loss probability are typically of the order $10^{-6} - 10^{-9}$. Let ε denote the QoS bound on the bit loss.

Define:

$$\Omega_\varepsilon = \{\mathbf{n} : P_L^N(\mathbf{n}) \le \varepsilon\} \tag{2.13}$$

where $\mathbf{n} = col(n_1, n_2, \cdots, n_M)$.

Then Ω_ε defines all possible connection configurations which meet the QoS constraints and is referred to as the *acceptance region.*

Let us define the boundary of the acceptance region as

$$\partial\Omega_\varepsilon = \{\mathbf{n} : P_L^N = \varepsilon\} \tag{2.14}$$

Let us see an important property associated with the acceptance region. This is the property of *coordinate convexity* whose definition we recall below:

DEFINITION 2.1 *Let* $\mathbf{S}$ *be the set of all possible configurations. Then* $\mathbf{S}$ *is said to be coordinate convex if for* $\mathbf{n} \in \mathbf{S}$ *implies that the vector* $\mathbf{n} - e_k \in \mathbf{S}$ *for all* $k; 1, 2, .., M$ *such that* $n_k > 0$*, where* e_k *is the unit vector in dimension* M *with a* 1 *in the* $k'th$ *row and O elsewhere.*

Coordinate convexity implies that an arriving connection is accepted if and only if the new configuration obtained after the addition of the new connection remains in the set $\mathbf{S}$ after admittance. The importance of the coordinate convexity is that the equilibrium distribution of the configuration has a so-called *product-form* Ross, 1995 which we will exploit in the following section.

Let us now return to the properties of the acceptance region. First of all it is clear that the acceptance region is coordinate convex under the mapping of the true bit loss probability. This follows directly since:

$$\sum_{j=1}^{n_k} a_{j,k} \ge \sum_{j=1}^{n_k - 1} a_{j,k}$$

where $\{a_{j,k}\}$ are i.i.d. with common distribution as a_k,

Therefore $X = \sum_{i=1}^{M} X_i$ stochastically dominates $Y = \sum_{i \ne k}^{M} X_i + X'_k$ where X'_k corresponds to one less connection of type k. Hence $P(X > a) \ge P(Y > a)$ for all a from which it readily follows that the corresponding bit loss probabilities will dominate since they are specified by the complementary distribution.

We now show that if the result of Proposition 2.2 is used to define the acceptance region then the resulting region is coordinate convex.

Before proceeding with the proof we let us note a few interesting properties associated with the size of the system i.e. scaling. For $\mathbf{m} \in \partial\Omega_\varepsilon$ a little reflection shows that as the scaling increases, keeping the QoS constraint fixed at ε implies that the corresponding n_i used in the unscaled system must decrease which implies that τ_c decreases. In fact with some further analysis it can be shown that the τ_c associated with the measure change goes to 0 at a rate of the order $O(\frac{1}{\sqrt{N}})$. This is important in the sequel in which we retain so-called significant or dominating terms.

PROPOSITION 2.3 *For a given QoS constraint specified by ε, the acceptance region Ω_ε obtained by using the result of Theorem 1.2 for the bit loss probability is coordinate convex for large systems.*

Proof:

To prove this result it is sufficient to show that the following monotonicity result holds:

Let $N\mathbf{n}$ and $N\mathbf{n} - e_k$ be two configurations such that $n_k > 0$, then $P(bit\ loss/N\mathbf{n}) > P(bit\ loss/N\mathbf{n} - e_k)$ for $k \in \{1, 2, \cdots, M\}$ for which $n_k > 0$.

Note for the scaled variables i.e. Nn_k the perturbation e_k is of order $\frac{1}{N}$ implying it is small and therefore the above result will be shown by treating the integer variables $\mathbf{n}$ as continuous and then showing that the partial derivative of the bit loss w.r.t. Nn_k is positive (which implies monotonicity).

Neglecting the $O(\frac{1}{N})$ term in Proposition 2.2 the partial derivative can be shown to be:

$$\begin{aligned} \frac{\partial P(bit\ loss)}{\partial N n_k} &= P(bit\ loss)[-(\frac{\partial I(C)}{\partial n_k} + \frac{\partial \tau_c}{N \partial n_k}) \\ & \quad - \frac{1}{N\ Den} \frac{\partial Den}{\partial n_k}] \end{aligned} \tag{2.15}$$

In the above the term Den refers to the denominator of (2.14) and is given by:

$$Den = \sqrt{2\pi N}\sigma(1 - e^{-\tau_c})^2 \sum_{i=1}^{M} n_i r_i$$

Form the definition of $I(C)$ and τ_c it can be readily seen that:

$$\frac{\partial}{\partial n_k}(I(C) + \tau_c) = -\ln(\phi_k(\tau_c) + \frac{\partial \tau_c}{\partial n_k}$$

Noting that $\tau_c > 0$ this implies that $\ln(\phi_k(\tau_c)) > 0$.

Now from the definition of τ_c it can be shown that $\frac{\partial \tau_c}{\partial n_k}$ is given by the solution to:

$$\sigma^2 \frac{\partial \tau_c}{\partial n_k} = -\frac{\phi_k'(\tau_c)}{\phi_k(\tau_c)}$$

and since $\sigma^2 > 0$ it implies that $\frac{\partial \tau_c}{\partial n_k} < 0$.

From the definition of Den we have:

$$\frac{1}{NDen} \frac{\partial Den}{\partial n_k} = \frac{1}{N\sigma} \frac{\partial \sigma}{\partial n_k} + 2 \frac{1}{N(e^{\tau_c} - 1)} \frac{\partial \tau_c}{\partial n_k} + \frac{r_k}{N \sum_{i=1}^M n_i r_i}$$

In the expression above the first term is $O(\frac{1}{N\sigma^4})$ (from the definition of σ^2 and hence under the condition $\sum_{i=1}^M n_i r_i < C$ is bounded by a constant divided by N. The third term can also be bounded by a constant divided by N while the second term can contribute significantly when τ_c is small since it is of order $O(\frac{1}{\sqrt{N}})$. Hence, for N large we can write:

$$\frac{\partial P(bit\ loss)}{\partial n_k} = P(bit\ loss)[\ln(\phi_k(\tau_c) + \frac{1}{N\sigma^2}(1 + \frac{2}{(e^{\tau_c} - 1)})\frac{\phi_k'(\tau_c)}{\phi_k(\tau_c)} - O(\frac{1}{N})]$$

where the $O(\frac{1}{N})$ term above is positive but smaller in magnitude in comparison to the first two terms for large N. This implies that the positive terms dominate implying that:

$$\frac{\partial P(bit\ loss)}{\partial n_k} > 0$$

for all $n_k > 0;\ k = 1, 2, \cdots, M$ and hence the proof is done. ■

REMARK 2.3 *If the Chernoff bound is used as the approximation then the coordinate convexity is immediate.*

Having established that for large systems the acceptance region is coordinate convex (as a function of the number of connections) when the approximation formula is used for bit loss we now are in a position to further develop the CAC for unbuffered models.

3. Most likely bit loss configuration and its role

Let us recall the model under consideration. A link of capacity NC is accessed by M classes of independent, stationary, ergodic sources. A connection of Type i arrives at Poisson rate $N\lambda_i;\ i = 1, 2, \cdots, M$ and a connection once admitted holds the resources for a random time of unit mean in duration. Once the connection is established, the bit flow is has a random rate $a_i(t)$ as discussed above. The sources are assumed to be mutually independent.

In the previous section given a configuration $(Nn_1, Nn_2, \cdots, Nn_M)$ we gave an $O(1)$ (in complexity) approximation to compute the stationary bit loss probability. Throughout this section and the following sections we assume that the formula given in Theorem 1.2 is used to compute the bit loss probability. We also saw that for large N the acceptance region specified by the QoS denoted by Ω_ε is coordinate convex.

For the model above, it is well known that for coordinate convex state-space the joint distribution of the number of connections under stationarity is given by the following "product-form" distribution which is insensitive to the actual holding time distribution (see Labourdette and Hart, 1992 for example):

$$\Pi(\mathbf{m}) = \frac{1}{G}\prod_{i=1}^{M}\frac{(N\lambda_i)^{m_i}}{m_i!} \tag{3.16}$$

where G is the normalizing constant given by:

$$G = \sum_{\mathbf{m}\in\Omega_\varepsilon}\frac{(N\lambda_i)^{m_i}}{m_i!}$$

and $\mathbf{m}$ is the vector of the number of connections being held of each type.

We now restrict ourselves to the set $\partial\Omega_\varepsilon$ which we denote as the boundary states. By definition, $\partial\Omega_\varepsilon$ is the subset of states in which the bit loss meets the constraints exactly and thus correspond to the allowable states with the maximal bit loss permissible. We now isolate amongst these states the state with the highest probability of occurring which we define to be the *most likely bit loss configuration* i.e.

DEFINITION 3.1 *The state(s)* $\mathbf{m}^* \in \partial\Omega_\varepsilon$ *given by*

$$\mathbf{m}^* = argmax_{\mathbf{m}\in\partial\Omega_\varepsilon}\Pi(\mathbf{m}) \tag{3.17}$$

is (are) said to be the most likely bit loss configuration(s).

Let $P_L(\mathbf{Nm})$ denote the stationary bit loss probability (for the configuration $(Nm_1, \cdots, Nn_M)$. Then the most likely bit loss state can be computed by from the constrained nonlinear optimization problem:

$$\begin{aligned}&Max\ \Pi(\mathbf{Nm})\ \ \textit{subject to}\\&P_L(\mathbf{Nm}) = \varepsilon\end{aligned}$$

The above problem is a constrained nonlinear integer optimization problem. However, due to the size, unit increments are of negligible relative order and hence we can treat it as a constrained nonlinear optimization problem over non-negative reals. Even so the problem as posed is formidable. However, we can exploit the fact the for N large we can approximate the terms by using the Stirling approximation.

Let us multiply the numerator and denominator (i.e. G) in (3.16) by $\prod_{i=1}^{M} e^{-N\lambda_i}$. Now neglecting the normalizing factor G (since it is a constant) we can approximate the numerator using Stirling's approximation for Nm_i by:

$$\prod_{i=1}^{M} \frac{e^{-N\lambda_i}(N\lambda_i)^{Nm_i}}{Nm_i!} = \prod_{i=1}^{M} \frac{e^{-N\lambda_i f(\beta_i)}}{\sqrt{2\pi N m_i}} \tag{3.18}$$

where

$$\beta_i = \frac{m_i}{\lambda_i} \tag{3.19}$$

and

$$f(x) = x\ln(x) - x + 1 \tag{3.20}$$

Hence for large N the optimization problem can be written as :

$$\begin{aligned} &Max \prod_{i=1}^{M} \frac{e^{-N\lambda_i f(\beta_i)}}{\sqrt{2\pi N\lambda_i\beta_i}} \quad subject\ to \\ &P_L(\mathbf{N}\lambda\beta) = \varepsilon \end{aligned}$$

By introducing the Lagrange multiplier $\mathbf{y}$ we can convert this problem to an unconstrained minimization problem as:

$$\min \sum_{i=1}^{M} N\lambda_i f(\beta_i) + \mathbf{y}\left(\ln P_L(\lambda\beta) - \ln(\varepsilon)\right) - \sum_{i=1}^{M} \ln(\sqrt{2\pi N\beta_i\lambda_i}) \tag{3.21}$$

Now noting that

$$\ln(P_L(\lambda\beta)) = -(NI(C)+\tau_c) - \ln(\sqrt{2\pi N^3}) - 2\ln(\sqrt{\sigma}(1-e^{-\tau_c}) - \ln(\sum_{i=1}^{M} N\lambda_i\beta_i r_i)$$

The reason of writing the equality constraint in the variational form in terms of logarithms is now obvious i.e. to make both terms of the same order.

For performing the optimization we only retain terms involving $\mathbf{m}$.

Therefore, with the above observation we define $\mathbf{m}^*$ as the vector which minimizes (neglecting the constant terms i.e. not depending on

m):

$$J(\mathbf{Nm}) = \sum_{i=1}^{M}(N\lambda_i f(\beta_i) - \frac{1}{2}\ln(N\beta_i\lambda_i)) - \mathbf{y}[\ln(P_L(N\lambda\beta) - \ln(\varepsilon)] \quad (3.22)$$

From standard nonlinear optimization theory, see Luenberger, 1984 for example, the necessary first-order conditions that $\mathbf{m}^*$ satisfies are:

$$\frac{\partial(\lambda_i f(\beta_i))}{\partial m_j}\delta_{i,j} - \mathbf{y}\frac{\partial P_L(N\lambda\beta)}{N\partial m_j}|_{\mathbf{m}=\mathbf{m}^*} = 0; \quad i,j = 1,2,\cdots,M \quad (3.23)$$

where the Lagrange multiplier $\mathbf{y}$ is such that the equality constraint is achieved.

Now using the expression for the partial derivative of $P_L(N\lambda\beta)$ we obtain the result that the most likely state $N\mathbf{m}^*$ satisfies:

$$Nm_j^* = N\lambda_j(\phi_j(\tau_c))^{\mathbf{y}}\exp\{\frac{\mathbf{y}}{N\sigma^2}[(1+\frac{2}{e^{\tau_c}-1})\frac{\phi_j'(\tau_c)}{\phi_j(\tau_c)}]\} \quad (3.24)$$

and τ_c is the solution to:

$$\sum_{i=1}^{M}\frac{m_i^*\phi_i'(\tau_c)}{\phi_i(\tau_c)} = C \quad (3.25)$$

which gives $M+1$ equations to compute $\mathbf{m}$ and τ_c as functions of the given parameters and the Lagrange multiplier $\mathbf{y}$.

Finally the Lagrange multiplier $\mathbf{y}$ is chosen to satisfy the constraint:

$$P_L(N\mathbf{m}^*) = \varepsilon \quad (3.26)$$

thus giving us $M+2$ equations for computing the $M+2$ unknowns given the source and arrival characteristics $\phi_i(.)$, $\{\lambda_i\}_{i=1}^M$, C as well as the scaling factor N and the QoS constraint ε.

By computing the Hessian at $N\mathbf{m}^*$ it can be shown that the Hessian is positive definite and thus the solution $N\mathbf{m}^*$ is regular. We omit it for sake of brevity.

REMARK 3.1 *From above it follows that the assumption that the states are of order $O(N)$ as assumed in section 1 is satisfied and the bit loss probability approximation as given in Proposition 2.2 is valid.*

Let us now study the role of the most likely bit loss configuration above.

LEMMA 3.1 *Let $N\mathbf{m}^*$ be the most likely bit loss configuration and $N\mathbf{m}$ be any other configuration in $\partial\Omega_\varepsilon$. Let $\Pi(\mathbf{m})$ denote the stationary distribution for $\mathbf{m}$.*

Then:

$$\frac{\Pi(N\mathbf{m})}{\Pi(N\mathbf{m}^*)} \sim O(e^{-N}) \tag{3.27}$$

Proof:

First note that for N large:

$$\Pi(N\mathbf{m}) = \frac{1}{\prod_{i=1}^M \sqrt{2\pi N m_i}} e^{-N\sum_{i=1}^M \lambda_i f(\beta_i)}$$

where β_i is defined by (2.31) and $f(x)$ by (2.32)

Let $N\mathbf{m} \in \partial\Omega_\varepsilon$. Now from the fact that $N\mathbf{m}^*$ is the unique minimizer of $N\sum_{i=1}^M \lambda_i f(\beta_i)$ for $\mathbf{m} \in \partial\Omega_\varepsilon$ we see that

$$\frac{\Pi(N\mathbf{m})}{\Pi(N\mathbf{m}^*)} = \sqrt{\prod_{i=1}^M \frac{m_i^*}{m_i}} e^{-N\sum_{i=1}^M \lambda_i (f(\beta_i) - f(\beta_i^*))}$$

and

$$\sum_{i=1}^M \lambda_i (f(\beta_i) - f(\beta_i^*)) > 0$$

for $\beta \neq \beta^*$ which proves the result for $\mathbf{m} \in \partial\Omega_\varepsilon$.

■

On the other hand if $\mathbf{m} \in \mathit{interior}(\Omega_\varepsilon)$ the above estimates hold for the bit loss probabilities. Indeed, for a configuration $\mathbf{m} \in \mathit{int}(\Omega_\varepsilon)$ from the definition of the rate function the corresponding rate function denoted by ($\mathcal{I}(C)$) is strictly larger (this follows from the fact that the partial derivative with respect to $\mathbf{m}$ is negative) than $I(C)$. In this case

$$\frac{P_L(\mathbf{m})}{P_L(\mathbf{m}^*)} \sim O(e^{-N(\mathcal{I}(C) - I(C))})$$

which gives the ratio of bit loss probabilities this time of $O(e^{-N})$.

The importance of the above result is that, in so far as we consider the boundary states which correspond to the maximum bit loss permissible, the contribution of the other boundary configurations with respect to the most likely bit loss configuration is exponentially negligible as the size of the system becomes large. This important property will be utilized in defining the connection acceptance control which is addressed in the next section.

4. Effective bandwidths, CAC and Connection Blocking

In this section we develop the connection acceptance control strategy building upon the results in the previous sections.

First note that once we characterize Ω_ε a given connection request is admitted if the new configuration with the connection request added is within Ω_ε. In order to do so one would have to compute the region Ω_ε which in light of the expression for the bit loss probability is a daunting task. Moreover as mentioned in the introduction, the calculation of the connection blocking probability which is needed for bandwidth allocation for a given VP in the MPLS context for a given Grade of Service (GoS) is given by the ratio of the number of arriving connections which cannot be accepted over the total number of arriving requests which involves the computation of $\sum_{\mathbf{m}\in\partial\Omega_\varepsilon}\Pi(\mathbf{m})$ which implies the computation of $\partial\Omega_\varepsilon$ which is also daunting. However, as we have seen in the previous section, the most likely bit loss configuration determines the bit loss. We will exploit this property to define the CAC.

Since the basic problem is related to the computation of the nonlinear hyper surface characterized by $\partial\Omega_\varepsilon$ we now consider the following simpler approximation. The basic idea is that since $N\mathbf{m}^*$ lies on $\partial\Omega_\varepsilon$ we construct the tangent hyperplane to $\partial\Omega_\varepsilon$ at $N\mathbf{m}^*$. The slope of the hyperplane then defines the relative contributions in terms of the necessary incremental bandwidth requirements of the various types of connections. This is what we identify as the *effective bandwidths* of the various types of sources.

Figure 1.2 illustrates the idea of the effective bandwidths.

Define:

$$a_j = \ln(\phi_j(\tau_c)) + \frac{1}{N\sigma^2}(1 + \frac{2}{e^{\tau_c} - 1})\frac{\phi_j'(\tau_c)}{\phi_j(\tau_c)} \tag{4.28}$$

Let $a_{\min} = \min(a_1, a_2, \cdots, a_M)$ and then define:

$$A_j = \frac{a_j}{a_{\min}} \tag{4.29}$$

Note since a_j is just the ratio of the partial derivative of the bit loss to the bit loss probability it represents the sensitivity of the bit loss probability (normalized with respect to the minimum of a_j) and thus represents the change in bit loss as the connection is increased and can be identified with the supplementary bandwidth associated with the connection when we try to admit one more.

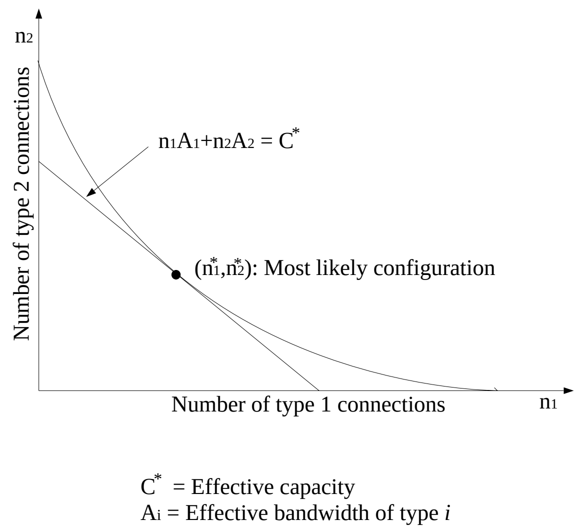

Figure 1.2. Effective bandwidths via hyperplane approximation

Then define C^* as

$$C^* = \sum_{i=1}^{M} m_i^* A_i \tag{4.30}$$

Then the interpretation of $\{A_i\}_{i=1}^{M}$ and C^* is that the A_i denote the *effective bandwidths* of the connection (with the smallest connection assigned a unit bandwidth) and NC^* the *effective capacity* of the Virtual Path.

With the above terms defined the tangent hyperplane to $\partial\Omega_\varepsilon$ can be approximated as:

$$\mathbf{T} = \{\mathbf{n} : \sum_{i=1}^{M} n_i A_i = NC^*\} \tag{4.31}$$

Therefore the Connection Acceptance Control strategy can now be formalized as follows:

1 Compute A_j for a given incoming request of type j.

2 If $A_j + \sum_{ongoing} n_i A_i \leq NC^*$ then admit the connection, else reject the request.

Thus the Connection Acceptance Control strategy involves computing the available bandwidth at the instant of arrival and seeing whether the effective bandwidth of the incoming request is less than the available bandwidth. This linear truncation now allows us to compute the blocking probability for a given connection request. Before doing so let us see some properties of the effective bandwidths and how good the tangent hyperplane approximation is for large systems.

Throughout the following discussion we assume that $N\mathbf{m} \in \partial\Omega_\varepsilon$ i.e. the bit loss probability is held fixed. We will study the behavior of $A_j(N)$, $C^*(N)$, and the tangent hyperplane $T(N)$ (where the explicit dependence of these quantities on the size specified by N is noted) as $N \to \infty$.

Let $N\mathbf{m(N)}$ be a configuration corresponding to bit loss probability ε where $\mathbf{m(N)}^*$ is made to depend on N since we will be changing N.

PROPOSITION 4.1 *Let $P_L(N\mathbf{m(N)})$ be the bit loss probability for configuration $N\mathbf{m(N)}$ which is assumed to be held constant at ε. Then as $N \to \infty$ the following properties hold:*

1. *$\mathbf{m(N)}$ converges to $\mathbf{m}^o$ such that $\sum_{i=1}^M m_i^o r_i = C$ where r_i is the mean rate of a connection of type i.*
2. *$A_j(N)$ converges to $\frac{r_j}{r_{\min}}$*
3. *$C^*(N)$ converges to $\frac{C}{r_{\min}}$.*
4. *The tangent hyperplane $T(N)$ coincides with $\partial\Omega_\varepsilon$*

Proof:
Proof of 1) First note that the bit loss probability $P_L(.)$ can be completely characterized by $(N, \mathbf{m(N)}, \tau_c(N))$. Keeping it fixed and increasing N implies that $\tau_c(N)$ must go to zero since $I(C) \to 0$ and τ_c satisfies:

$$\sum_{i=1}^{M} m_i(N) \frac{\phi_i'(\tau_c(N))}{\phi_i(\tau_c(N))} = C$$

Therefore :

$$\lim_{N\to\infty} \frac{\phi_i'(\tau_c(N))}{\phi_i(\tau_c(N))} = \frac{\phi_i'(0)}{\phi_i(0)} = r_i$$

and hence

$$\lim_{N\to\infty} \mathbf{m(N)} \to \mathbf{m}^o$$

where $\mathbf{m}^o$ satisfies

$$\sum_{i=1}^{M} m_i^o r_i = C$$

Proof of 2) From the definition of A_j we have:

$$A_j(N) = \frac{\ln(\phi_j(\tau_c(N))) + \alpha(\tau_c(N), N)\frac{\phi'_j(\tau_c(N))}{\phi_j(\tau_c(N))}}{\ln(\phi_{\min}(\tau_c(N))) + \alpha(\tau_c(N), N)\frac{\phi'_{\min}(\tau_c(N))}{\phi_{\min}(\tau_c(N))}}$$

where $\alpha(\tau_c(N), N)$ denotes the multiplying factor in the definition of A_j.

Now noting that as $\tau_c(N) \to 0$, $\ln(\phi_j(\tau_c(N)) \to 0$ we have

$$\lim_{N\to\infty} A_j(N) \to \frac{r_j}{r_{\min}}$$

Proof of 3) This follows directly from the definition of C^* and the two results above.

Proof of 4) This follows directly from 1), 2) and 3) since the properties hold for any $N\mathbf{m}(\mathbf{N}) \in \partial\Omega_\varepsilon$. ■

Remark 4.1 *The importance of the above result is that for virtual paths with extremely large capacity in MPLS architectures that can be associated with a very large scaling factor N, the problem becomes completely decoupled with the bandwidth assignment for a given class associated with its mean rate and the effective capacity equal to the given capacity. In this case the only problem is to design capacity allocation to meet GoS requirements using the standard loss model.*

Remark 4.2 *From the result above we see that the boundary of the acceptance region is a hyperplane only in the limit as N goes to infinity. When the system is very large then the linearity of the acceptance boundary implies that the overall performance would be completely insensitive to the choice of the point where the hyperplane is taken and thus in the case of very large systems one would expect very little difference from the results in Elwalid et al., 1995; Kelly, 1996.*

We now obtain the expression for the connection blocking probability which is needed for the GoS determination.

Let us recall the basic assumptions: connection arrivals are Poisson with a connection of type i having intensity $N\lambda_i$. It is assumed without loss of generality that the connections hold the assigned bandwidth for a random amount of time of unit mean. Connections are assumed to be independent.

From the above, a connection of Type i is assigned the effective bandwidth A_i which in the MPLS context corresponds to the number of VC's required. For convenience we assume all quantities are integer valued i.e. we define the quantities:

$$\begin{aligned} A_i &= int(A_i) \\ m_i^* &= int(m_i^*) \end{aligned}$$

where $int(x)$ is the smallest integer $\geq x$.

As a consequence C^* will also be integer valued. From a practical viewpoint the integer valued effective bandwidths will then denote the number of VC's required and NC^* will be the total effective capacity of the VP in terms of number of VC's.

With the above parameters, the consequence of the CAC rule specified by the linear rule renders the model as a classical multi-rate loss model with rates A_i for a connection of Type i. Once again using the fact that the system is large on account of the scaling factor N we can use the approximations for the blocking probabilities for the multi-rate model given in Gazdicki et al., 1993 (noting that since by definition the smallest effective bandwidth is 1 and hence the GCD (greatest common divisor) of $\{A_i\}_{i=1}^M$ is 1). These results can also be found in Mitra and Morrison, 1994 wherein they are obtained using saddle-point techniques. We quote the result below:

PROPOSITION 4.2 *For the multirate loss model specified by arrival rates $N\lambda_i$, effective bandwidths A_i and effective capacity NC^*.*

Then the following expressions for the blocking probability hold:

Light Load: *If $\sum_{i=1}^M \lambda_i A_i < C^*$, then the connection blocking probability for class k; $k = 1, 2, \cdots, M$ is given by:*

$$P_k(N) = \exp(-I(C^*))\frac{1}{\sqrt{2\pi N\sigma}}\left(\frac{1-\exp(t_{c^*}A_k)}{1-\exp(t_{c^*})}\right)(1+O(\frac{1}{N})) \tag{4.32}$$

Crtitcal Load: *If $\sum_{i=1}^M \lambda_i A_i = C^*$ then*

$$P_k(N) = \sqrt{\frac{2}{\pi N}}\frac{A_k}{\sigma}(1+O(\frac{1}{\sqrt{N}}) \tag{4.33}$$

Heavy Load: *If $\sum_{i=1}^M \lambda_i A_i > C^*$ then*

$$P_k(N) = (1-\exp(t_{c^*}A_k))(1+O(\frac{1}{N})) \tag{4.34}$$

where the quantities t_{c^*}*,* $I(C^*)$*, and* σ *are given by:*

i) t_{c^*} *is the unique solution to the equation* $\sum_{i=1}^{M} \lambda_i A_i \exp(t_{c^*} A_i) = C^*$

ii) $I(C^*) = C^* t_{c^*} - \sum_{i=1}^{M} \lambda_i (\exp(t_{c^*} A_i) - 1)$

iii) $\sigma^2 = \sum_{i=1}^{M} \lambda_i A_i^2 \exp(t_{c^*} A_i)$

Thus, from above once the most likely bit loss configuration is identified from the source characteristics and the corresponding values of C^* and $\{A_i\}_{i=1}^{M}$ are determined the connection blocking probability can be determined from above.

It is worth remarking one point with regard to the definition of the effective capacity C^*. In Labourdette and Hart, 1992, they show that the most likely state at capacity for the multi-rate loss model i.e. the most likely configuration $\mathbf{m}$ such that $\sum_{i=1}^{M} m_i A_i = C^*$ is given by:

$$m_i^* = \lambda_i \alpha^{A_i}$$

where α is the unique solution to

$$C^* = \sum_{i=1}^{M} \lambda_i A_i \alpha^{A_i}$$

From the definition of the most likely bit loss configuration above and the definition of A_i the effective bandwidths it is readily seen that in fact the most likely bit loss configuration corresponds to the most likely configuration for the loss model with effective capacity NC^*. To see this: define $\alpha = e^{a_{\min}\mathbf{y}}$. Then by definition $m_i^* = \lambda_i \alpha^{A_i}$ and by definition $C^* = \sum_{i=1}^{M} \lambda_i A_i \alpha^{A_i}$.

This basically shows that in mapping the bit loss phenomena to the connection level multirate loss model we remain consistent i.e. the most likely bit loss configuration remains invariant since it corresponds to the most likely configuration at capacity for the multirate loss model.

Thus the basic problem of CAC to meet a given QoS specified by ε it is equivalent to transforming the model to a multi-rate loss model where the bit level phenomena is mapped into the connection level through the definition of the effective bandwidth and effective capacity.

Let us now recapitulate all the results concerning the CAC and connection blocking:

Assume that the following data is given: connection arrival rates $\{\lambda_i\}_{i=1}^{M}$, bit flow characteristics $\{\phi_i(t)\}_{i=1}^{M}$, link capacity C, QoS parameter ε and scaling factor N.

Step 1: Determine the parameters τ_c, $\{m_i^*\}_{i=1}^M$, and lagrange multiplier $\mathbf{y}$ from the following set of (M+2) equations:

$$\begin{aligned} m_j^* &= \lambda_j(\phi_j(\tau_c))^{\mathbf{y}} \exp\left(\frac{\mathbf{y}}{N\sigma^2}[1+\frac{2}{e^{\tau_c}-1}]\frac{\phi_j'(\tau_c)}{\phi_j(\tau_c)}\right) \\ C &= \sum_{i=1}^{M} m_i^* \frac{\phi_i'(\tau_c)}{\phi_i(\tau_c)} \\ \varepsilon &= P_L(N\mathbf{m}^*) \text{ given in Theorem 1.2} \end{aligned}$$

Step 2: Having obtained τ_c and m_i^* above compute the effective bandwidth A_i and then the effective capacity C^*.

Step 3: Depending upon the loading condition compute the blocking probabilities $P_k(N)$ using appropriate form from Proposition 4.2 with multirate parameters λ_i, A_i and C^*

In the next section we go through an example of the above procedure and compare the results obtained with simulations.

5. Numerical example of QoS and GoS with proposed admission control

In this example, we use the same parameters as in Table 1. We set capacity $C = 20$ with two classes of traffic such that

$$\lambda_1 = \lambda_2 = 14, p_1 = .275, p_2 = .8, \Pi_1 = 2, \Pi_2 = 1$$

and we use multiplier $N = 100$. bit loss tolerance is set to $\epsilon = 10^{-4}$. This value is larger than reality to allow for accurate simulations to be performed. Recall that we suppose that each class of traffic is modeled as an ON-OFF process and therefore,

$$\phi_j(\tau_c) = p_j \exp(\tau_c \Pi_j) + 1 - p_j$$

We numerically solve **Step 1** given at the end of section 3 to find m_1^*, m_2^*, τ_c and $\mathbf{y}$. We obtain the following solution

$$m_1^* = 14.156, m_2^* = 14.217, \tau_c = .06215, \mathbf{y} = .2245$$

With these values, we go to **Step 2** of the procedure and we compute $a_1 = .04948, a_2 = .06852$ which we normalize to get $A_1 = 1.0, A_2 = 1.385$ and $C^* = 33.847$.

In **Step 3** of the procedure, we use the approximation formulae of Theorem 4.2 to evaluate call blocking probabilities. We first notice that $\sum_j A_j \lambda_j < C^*$ so we are in the light load case. The blocking probabilities which are reported in Table 1.5.

Technique	Class 1 blocking	Class 2 blocking
Simul. (95 % conf. int.)	.00427-.00501	.00631-.00724
Theorem 3.1	.00479	.00661

Table 1.2. Call blocking probabilities

This procedure defines an acceptance region of the form

$$\sum_j A_j n_j \leq NC^*$$

To justify the use of such an acceptance region, we used simulation to evaluate bit loss at a number of points on the boundary of the acceptance region. Recall that the objective was to keep bit loss below $10^{-4} = \epsilon$. The results are given in Table 1.2 where we see that our linear acceptance region is conservative and very close to the true (unknown) acceptance region.

Number of Class 1 calls	Number of Class 2 calls	Base 10 logarithm of 95% conf. int. for bit loss
500	2083	(-4.13,-4.03)
1000	1722	(-4.19,-4.11)
1416	1422	(-4.16,-4.09)
1500	1361	(-4.30,-4.23)
2000	1000	(-4.25,-4.17)

Table 1.3. bit loss values

Concluding remarks

In this paper we have proposed a framework for addressing the problem of admission control to offer connections statistical guarantees on their QoS requirements based on the loss probability. This can be readily extended to delay distributions. We have shown how large system size can be used to advantage via the ideas of scaling and moreover we have shown that there is a natural definition for the effective bandwidths that gives a consistent mapping from the fast bit time scale to the slow connection time scale that allows for GoS dimensioning. The approach can be readily extended to more complicated scheduling disciplines provided one can analytically obtain estimates for the loss probabilities or delay distributions. We have also shown how the effects of multiplexing lead to a reduction of effective bandwidths and shown other scaling properties of large systems showing loss concentrates at certain points on the boundary of the acceptance region in equilibrium.

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