Downlink Power Allocation for Multi-class CDMA Wireless Networks

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***Abstract*—In this paper we consider the downlink power allocation problem for multi-class CDMA wireless networks. We use a utility based power allocation framework to treat multi-class services in a unified way. The goal of this paper is to obtain a power allocation which maximizes the total system utility. In the wireless context, natural utility functions for each mobile are non-concave. Hence, we cannot use existing techniques on convex optimization problems to derive a social optimal solution. We propose a simple distributed algorithm to obtain an approximation to the social optimal power allocation. The proposed distributed algorithm is based on dynamic pricing and allows partial cooperation between mobiles and the base station. The algorithm consists of two stages. At the *mobile selection* stage, the base station selects mobiles to which power is allocated, considering the partial-cooperative nature of mobiles. This is called partial-cooperative optimal selection, since in a partial-cooperative setting and pricing scheme considered in this paper, this selection is optimal and satisfies system feasibility. At the *power allocation* stage, the base station allocates power to the selected mobiles. This power allocation is a social optimal power allocation among mobiles in the partial-cooperative optimal selection, thus, we call it a partial-cooperative optimal power allocation. We compare the partial-cooperative optimal power allocation with the social optimal power allocation for the single class case. From these results, we infer that the system utility obtained by the partial-cooperative optimal power allocation is quite close to the system utility obtained by the social optimal allocation.**

I. Introduction

Radio resources are scarce and the demand for wireless services keeps increasing, hence the efficient management of the radio resources in wireless networks is important in achieving a high level of utilization. Power control is an important component in the resource management problem.

In recent years, power control has been given extensive attention in both academic and industrial research, because of its critical role in code division multiple access (CDMA) networks. Most research efforts have been devoted to voice systems, since voice service has been the main service provided by wireless networks. In a voice system, all users have the same quality of service (QoS) requirements and it is important that the signal to interference ratio (SIR) exceeds some minimum threshold. Hence, the main purpose of power control in such systems is to eliminate the near-far effect by equalizing the SIR of each user setting it at the minimum SIR threshold [1], [2].

This research has been supported in part by NSF grants ANI-0073359 and ANI-9805441.

In the next generation of wireless networks, it is expected that services will have significantly differing characteristics from the current voice-dominated wireless networks. Already, the demand for various services with different QoS requirements such as video and data is increasing. The required bandwidth for these services is much higher than that for voice services, further compounding the scarcity of resources in wireless systems. Therefore, to accommodate services with different characteristics more efficiently, we need a different approach to power control in the next generation wireless networks. Moreover, such services are highly asymmetric, requiring more bandwidth in the downlink than the uplink. This implies that, in next generation wireless networks, efficient resource allocation of the downlink becomes a very important issue.

Recently, utility (and pricing) based network control algorithms have extensively been studied in the literature. These are not new concepts and have been studied in economics. The utility represents the degree of a user's satisfaction when it acquires certain amount of the resource and the price is the cost per unit resource which the user must pay for this resource. The basic idea of these algorithms is to control a user's behavior through the price of resources to obtain the desired results, e.g., high utilization for the overall system and fairness among users.

In wired networks, utility and pricing based algorithms are well studied for distributed flow control of best effort services. Kelly *et al.* [3] obtain the social optimal solution which maximizes the summation of all the users' utilities by allocating the resources according to the notion of *proportional fairness per unit charge*. Yäiche *et al.* [4] obtain a Nash bargaining solution which is Pareto-optimal and yields the *proportionally fair solution*. In these works, the utility function is assumed to be a concave function of the allocated rate, which makes the problem a convex programming problem. Hence, the Karush-Kuhn-Tucker (KKT) conditions are used to obtain the optimal solution.

The utility (and pricing) based control algorithms can also be applied to the power control problem in wireless networks. But, the main difficulty in solving the problem is that, in general, the problem cannot be formulated as a convex programming problem. Thus, the KKT condition cannot be used for the sufficient condition of the optimal solution. In most of works on utility and pricing for power control, only Nash equilibria, which are inefficient [5], have been obtained.

Utility based algorithms without pricing are considered in [6], [7]. Oh and Wasserman [6] consider an uplink power and spreading gain control problem for the non-real time services. They use an instantaneous throughput for each mobile as a utility function and obtain a global optimal solution which maximizes the aggregate throughput by jointly optimizing power control and spreading gain. But, their algorithm can be applied only for the system with one class of mobiles. Moreover, they do not consider any constraint on the spreading gain. Ji and Huang [7] formulate an uplink power control problem as a non-cooperative N-person game in which each user transmits a power level maximizing its utility without considering the behavior of other users. Under certain assumptions on the utility function, they show that there exists a Nash equilibrium.

Utility based algorithms with pricing are considered in [8], [9], [10], [11]. Sarayda *et al.* [8] formulate an uplink power control problem for a single-class wireless data system as a non-cooperative N-person game. They use the number of bits which can be transmitted using a Joule of energy as a utility function. They show that there exists a Nash equilibrium but it is inefficient in the sense that there exists another power allocation which Pareto dominates the Nash equilibrium allocation. To improve efficiency, they introduce pricing. The base station informs each user of a fixed price for unit power. Each user transmits a power level which maximizes its net utility (utility minus cost for power allocation). They show that the game with pricing converges to Nash equilibria under some conditions on the strategy set and present an algorithm which converges to a Pareto-dominant equilibrium, even though the social optimum cannot be obtained. In addition, they show that the choice of price impacts on the system utilization significantly. However, they do not provide a systematic algorithm to find an optimal price. Xiao *et al.* [9] formulate a downlink power control problem for multi-class wireless networks as a non-cooperative N-person game with pricing. In this setting, they do not allow constraints on the power. They use a sigmoid function as a utility function. By adjusting parameters of the sigmoid function, the utility functions for heterogeneous services are treated in a unified way. As in [8], the base station informs each user of a fixed price for unit power and each user requests a power level which maximizes its net utility value. They show that their algorithm is *standard* [2] under mild conditions and that the algorithm does not diverge even when the system is infeasible. In the numerical results, they show that the system utilization depends on the price, but they too do not provide an algorithm on how to obtain the optimal price. Liu *et al.* [10] consider a downlink resource allocation problem for the voice service. They use a step function as a utility function and as a pricing scheme, they use price per unit power and price per code. They obtain the optimal prices to maximize either total system utility or total revenue. This work is extended by Zhang *et al.* in [11].

In this paper we study downlink power allocation problem in multi-class CDMA based wireless networks. We use a utility based framework mentioned above. However, the situation considered here differs from the previous works in many aspects. Primarily, we consider a multi-class system while a single class data system is considered in [6] and a voice system in [10], [11]. This heterogeneous case requires much more and different analysis. We study the problem of maximizing total system utility for heterogeneous users which is a social optimum and differs from the Nash equilibrium considered in [7], [8], [9]. In general, the operating points are different. Furthermore, we consider the downlink case which imposes a global power constraint rather than the uplink case treated in [6], [7], [8] for which there are only individual power constraints on each user. This completely changes the structure of the optimization problem for which the previous (and simpler) algorithms are not applicable. It can be shown that in the absence of a total power constraint, the algorithms developed in [9] can be used with some modification. However, in practice, any transmitter has a maximum power level that it can transmit at and so it is necessary to develop algorithms for the power constrained case as is done in this paper.

As mentioned before, the goal of this paper is to obtain a power allocation which maximizes the total system utility. However, due to the non-convexity of the problem, it is difficult to obtain a social optimal power allocation and even if we could obtain it, it could require a very complex algorithm. Therefore, in this paper, we propose a simple algorithm to obtain a power allocation which is Pareto optimal as well as a good approximation of the social optimal power allocation. This algorithm can be implemented in a distributed way and, in this case, our problem can be expressed as a *partial-cooperative M-person power allocation game with dynamic pricing*. From the utility and pricing point of view, dynamic pricing is one of the distinguishing features of this work compared with other utility and pricing based power control algorithms [8], [9].

The rest of the paper is organized as follows. In Section II, we describe the system model considered in this paper and formulate the basic problem. In Section III, we present the proposed power allocation algorithm which consists of the mobile selection stage and the power allocation stage. We generalize the algorithm to the case when each mobile has the minimum SIR requirement in Section IV. In Section V, we study a special case when all mobiles are homogeneous and compare our power allocation with the social optimal power allocation for this case in Section VI. Finally, we conclude in Section VII.

II. SYSTEM MODEL AND PROBLEM DESCRIPTION

We consider downlink power allocation in a multi-class CDMA wireless network and focus on a single cell, consisting of a single base station and M mobiles. Each mobile communicates with the base station. For downlink communication, the base station has a maximum power limit, P_T. It allocates power to each mobile within the power limit (i.e., the summation of the power allocated to each mobile cannot exceed the power limit). Each mobile i, $i = 1, 2, \cdots M$, has its own utility function, U_i, which represents the degree of mobile i's satisfaction of the received QoS. We assume that U_i has the following properties.

Assumptions:

(a) U_i is an increasing function of γ_i, the SIR of mobile i.
(b) U_i is twice continuously differentiable.
(c) $U_i(0) = 0$.
(d) U_i is bounded above.

(e) $\frac{\partial^2 U_i(\gamma_i)}{\partial \gamma_i^2}(N_i+\gamma_i)+2\frac{\partial U_i(\gamma_i)}{\partial \gamma_i}=0$ has at most one solution for $\gamma_i > 0$, where N_i is processing gain, which is defined by the ratio of the chip rate to the data rate.

(f) If $\frac{\partial^2 U_i(\gamma_i)}{\partial \gamma_i^2}(N_i+\gamma_i)+2\frac{\partial U_i(\gamma_i)}{\partial \gamma_i}=0$ has one solution at $\gamma_i^o > 0$, $\frac{\partial^2 U_i(\gamma_i)}{\partial \gamma_i^2}(N_i+\gamma_i)+2\frac{\partial U_i(\gamma_i)}{\partial \gamma_i} > 0$ for $\gamma_i < \gamma_i^o$ and $\frac{\partial^2 U_i(\gamma_i)}{\partial \gamma_i^2}(N_i+\gamma_i)+2\frac{\partial U_i(\gamma_i)}{\partial \gamma_i} < 0$ for $\gamma_i > \gamma_i^o$.

By assumptions (e) and (f), the utility function can be one of three types[1]: a sigmoidal-like function of its own power allocation [2], a concave function of its own power allocation, or a convex function of its own power allocation. In general, most utility functions used in wired or wireless networks can be represented by these three functions [9], [12].

Even though we define the utility function as a function of the SIR, the SIR is a function of the power allocation of all mobiles given the path gain from the base station to the mobile, interference, and noise. We can represent γ_i, the SIR for mobile i, as follows:

$$\begin{aligned} \gamma_i(\bar{P}) &= \frac{N_i G_i P_i}{G_i \sum_{m=1}^{M} P_m - G_i P_i + I_i} \\ &= \frac{N_i P_i}{\sum_{m=1}^{M} P_m - P_i + \frac{I_i}{G_i}} \\ &= \frac{N_i P_i}{\sum_{m=1}^{M} P_m - P_i + A_i}, \end{aligned} \quad (1)$$

where

- P_i : Allocated power for mobile i.
- $\bar{P}$: Power allocation vector, $(P_1, P_2, \cdots, P_M)$ for mobiles, $1, 2, \cdots, M$, respectively.
- N_i : Processing gain for mobile i.
- G_i : Path gain from the base station to mobile i.
- I_i : Background noise and intercell interference to mobile i.
- M : Number of mobiles in the cell.

Note that the utility value of mobile i depends on not only its own power allocation but also on the power allocations of all the other mobiles.

The goal of this paper is to obtain the power allocation for each mobile which maximizes the total system utility (i.e., the summation of utilities of all mobiles). The basic formulation of this problem is given by the following optimization problem:

$$(\mathbf{A}) \qquad \max_{\bar{P}} \sum_{i=1}^{M} U_i(\gamma_i(\bar{P})) \quad (2)$$

$$\text{subject to} \sum_{i=1}^{M} P_i \le P_T, \quad (3)$$

$$P_i \ge 0, \quad i = 1, 2, \cdots, M. \quad (4)$$

We call the solution of problem $(\mathbf{A})$ the *social optimal power allocation* and the selection of mobiles which is allocated positive power at the social optimal power allocation the *social optimal selection*. Note that, in general, the objective function of problem $(\mathbf{A})$ in (2) is not a concave function.

[1] we will show this in Lemma 2.

[2] A sigmoidal-like function means a function, $f(x)$ which has one inflection point, x^o and $\frac{d^2 f(x)}{dx^2} > 0$ for $x < x^o$ and $\frac{d^2 f(x)}{dx^2} < 0$ for $x > x^o$.

III. PARTIAL-COOPERATIVE OPTIMAL POWER ALLOCATION

Our power allocation algorithm consists of two stages. At the first stage, mobiles to which power is allocated are selected, and at the second stage, power is allocated optimally to the selected mobiles. Before we describe the details of our power allocation algorithm, we first decompose problem $(\mathbf{A})$ as mobile problems and a base station problem.

The next proposition tells us that to maximize the total system utility, the base station must transmit at its maximum power limit, P_T.

Proposition 1: If $\bar{P} = (P_1, P_2, \cdots, P_M)$ is a power allocation and $\sum_{i=1}^{M} P_i < P_T$, then we can find another power allocation, $\bar{P}^* = (P_1^*, P_2^*, \cdots, P_M^*)$ such that $\sum_{m=1}^{M} P_m^* = P_T$ and $\sum_{i=1}^{M} U_i(\gamma_i(\bar{P}^*)) > \sum_{i=1}^{M} U_i(\gamma_i(\bar{P}))$.

Proof: If $\sum_{i=1}^{M} P_i < P_T$, there exists an $\alpha > 1$ such that

$$\sum_{i=1}^{M} P_i < \alpha \sum_{i=1}^{M} P_i = P_T.$$

We define $P_i^* = \alpha P_i$ for $i = 1, 2, \cdots, M$, then

$$\begin{aligned} \gamma_i(\bar{P}^*) &= \frac{N_i P_i^*}{\sum_{j=1}^{M} P_j^* - P_i^* + A_i} \\ &= \frac{\alpha N_i P_i}{\sum_{j=1}^{M} \alpha P_j - \alpha P_i + A_i} \\ &> \frac{\alpha N_i P_i}{\sum_{j=1}^{M} \alpha P_j - \alpha P_i + \alpha A_i} \\ &= \gamma_i(\bar{P}), \qquad i = 1, 2, \cdots, M. \end{aligned}$$

Therefore, $U_i(\gamma_i(\bar{P}^*)) > U_i(\gamma_i(\bar{P}))$ for all i, since U_i is an increasing function of γ_i. ■

By this property, problem $(\mathbf{A})$ is equivalent to the following problem.

$$(\mathbf{B}) \qquad \max_{\bar{P}} \sum_{i=1}^{M} U_i(\gamma_i(P_i))$$

$$\text{subject to} \sum_{i=1}^{M} P_i \le P_T,$$

$$P_i \ge 0, \quad i = 1, 2, \cdots, M,$$

where $\gamma_i(P_i) = \frac{N_i P_i}{P_T - P_i + A_i}$. Note that the utility function for each mobile does not depend on the power allocation for other mobiles in problem $(\mathbf{B})$, while the utility function for each mobile depends on the power allocation for all mobiles in problem $(\mathbf{A})$.

To solve problem $(\mathbf{B})$, we state the following result from [13] (page 213) that gives us optimality condition for general optimization problem.

Lemma 1: Let $f : \mathbf{R}^N \rightarrow \mathbf{R}$ and $g_i : \mathbf{R}^N \rightarrow \mathbf{R}$, $i = 1, 2, \cdots, K$ be arbitrary functions. We define

$$L(\bar{x}, \bar{\lambda}) = f(\bar{x}) + \bar{\lambda}^T g(\bar{x}),$$

$$w(\bar{\lambda}) = \max_{\bar{x}} \{L(\bar{x}, \bar{\lambda})\},$$

and

$$Y(\bar{\lambda}) = \{\bar{x} | L(\bar{x}, \bar{\lambda}) = w(\bar{\lambda})\},$$

where $\bar{x} = (x_1, x_2, \cdots, x_N)^T$, $\bar{\lambda} = (\lambda_1, \lambda_2, \cdots, \lambda_K)^T$, and $g(\bar{x}) = (g_1(\bar{x}), g_2(\bar{x}), \cdots, g_K(\bar{x}))^T$. If $\bar{x}(\bar{\lambda}) \in Y(\bar{\lambda})$ for any $\bar{\lambda} \geq 0$, then $\bar{x}(\bar{\lambda})$ is a global optimal solution of the following optimization problem.

$$\max_{\bar{x}} f(\bar{x})$$
$$\text{subject to } g(\bar{x}) \geq g(\bar{x}(\bar{\lambda})).$$

To use this lemma in our problem, we define

$$U_i'(\gamma_i(P_i)) = \begin{cases} U_i(\gamma_i(P_i)), & \text{if } 0 \leq P_i \leq P_T, \\ -\infty, & \text{otherwise.} \end{cases}$$

Then problem $(\mathbf{B})$ is equivalent to the following problem:

$$\max_{\bar{P}} \sum_{i=1}^{M} U_i'(\gamma_i(P_i))$$
$$\text{subject to } \sum_{i=1}^{M} P_i \leq P_T.$$

Note that the constraints $P_i \geq 0$, $i = 1, 2, \cdots, M$ do not appear in the above problem. Now, we define

$$L(\bar{P}, \lambda) = \sum_{i=1}^{M} U_i'(\gamma_i(P_i)) + \lambda(P_T - \sum_{i=1}^{M} P_i).$$

Then, for any $\lambda \geq 0$, $\bar{P}(\lambda) \in Y(\lambda)$ is a global optimal solution of the following optimization problem.

$$\max_{\bar{P}} \sum_{i=1}^{M} U_i'(\gamma_i(P_i))$$
$$\text{subject to } \sum_{i=1}^{M} P_i \leq \sum_{i=1}^{M} P_i(\lambda). \quad (5)$$

where $\bar{P}(\lambda) = (P_1(\lambda), P_2(\lambda), \cdots, P_M(\lambda))$. If we find a λ^* above such that $\sum_{i=1}^{M} P_i(\lambda^*) = P_T$ (when P_T is the threshold in (3)), the social optimal solution of problem $(\mathbf{A})$ can be obtained. Further, if $|P_T - \sum_{i=1}^{M} P_i(\lambda^*)|$ is small, a good approximation to the solution of problem $(\mathbf{A})$ can be obtained. Therefore, to obtain a good approximation to the solution of problem $(\mathbf{A})$, we will solve the following optimization problem.

$$(\mathbf{C}) \qquad \min_{\lambda} F(\lambda) = \min_{\lambda} \{P_T - \sum_{i=1}^{M} P_i(\lambda)\} \quad (6)$$
$$\text{subject to } \bar{P}(\lambda) = \arg\max\{L(\bar{P}, \lambda)\}, \quad (7)$$
$$\sum_{i=1}^{M} P_i(\lambda) \leq P_T. \quad (8)$$

By Lemma 1, if $\min_\lambda F(\lambda) = 0$ at λ^*, then $\bar{P}(\lambda^*)$ is the social optimal solution of problem $(\mathbf{A})$ and if $\min_\lambda F(\lambda) \approx 0$ at λ^*, then $\bar{P}(\lambda^*)$ could be a good approximation to the optimal solution of problem $(\mathbf{A})$ satisfying the feasibility condition. To solve problem $(\mathbf{C})$, we first consider (7). Since $L(\bar{P}, \lambda)$ is separable in $\bar{P}$, $\bar{P}(\lambda)$ solves (7) if and only if it solves the following problem.

$$\begin{aligned} (\mathbf{D_i}) \qquad P_i(\lambda) &= \arg\max\{U_i'(\gamma_i(P)) - \lambda P\}, \\ &= \arg \max_{0 \leq P \leq P_T} \{U_i'(\gamma_i(P)) - \lambda P\}, \\ &= \arg \max_{0 \leq P \leq P_T} \{U_i(\gamma_i(P)) - \lambda P\}, \\ & \quad i = 1, 2, \cdots, M. \end{aligned}$$

Note that the parameters in problem $(\mathbf{D_i})$ are correspond only to mobile i. By this property, we can decompose problem $(\mathbf{C})$ as the mobile problem $(\mathbf{D_i})$ for each mobile i and the following base station problem.

$$(\mathbf{E}) \qquad \min_{\lambda} F(\lambda)$$
$$\text{subject to } \sum_{i=1}^{M} P_i(\lambda) \leq P_T.$$

Each mobile i solves problem $(\mathbf{D_i})$ independently one another and the base station solves problem $(\mathbf{E})$.

We can interpret the decomposed problems as follows. Based on λ, the price per unit power, from the base station, each mobile i tries to maximize its net utility, (i.e., the utility minus the cost) by solving problem $(\mathbf{D_i})$. This implies that, given the price, λ, mobile problems are equivalent to a non-cooperative M-person game with a fixed price [8], [9]. However, in our formulation, by solving problem $(\mathbf{E})$ based on the power request of each mobile, the base station adjusts the price, λ dynamically to obtain a good approximation to the social optimal power allocation by minimizing $F(\lambda)$. Therefore, this problem can be interpreted as a *non-cooperative M-person game with dynamic pricing* and the pricing scheme which we use is linear pricing with the same unit price. The linear pricing with the same unit price means that the unit price for each user is same and the total cost for power is obtained by *unit price* $\times$ *the amount of allocated power*.

Using this interpretation, we can implement the power allocation algorithm to obtain a good approximation to a social optimal power allocation in a distributed way. However, by the discontinuity of $P_i(\lambda)$, there may be no equilibrium allocation for this problem. Moreover, by Proposition 1, $\sum_{i=1}^{M} P_i^*$ must be P_T, where P_i^* is allocated power for mobile i. To take care of these two facts, we divide the algorithm in two stages. First is the mobile selection stage. In this part, mobiles which are allocated positive power are selected. If problem $(\mathbf{E})$ can be solved, the selected mobiles are mobiles which are allocated positive power at the solution of $(\mathbf{E})$. Otherwise, the selected mobiles are mobiles which are allocated positive power at the approximation of the solution of $(\mathbf{E})$. Second is the power allocation stage. At this stage, only selected mobiles participate in the power allocation game and power is allocated to the mobiles optimally. To make unselected mobiles not participate in the power allocation game and, thus, to guarantee the convergence of the power allocation algorithm, the base station needs cooperation of mobiles. Therefore, in our algorithm, each mobile is assumed to have a partial-cooperative property, since it has both the non-cooperative property and the cooperative property and we call our problem a *partial-cooperative M-person game with dynamic pricing*.

A. Mobile selection problem

In this subsection, we consider the mobile selection problem. First, we study properties of $P_i(\lambda)$. We define P_i^o as

$$P_i^o = \begin{cases} P^*, & \text{if } \frac{\partial^2 U_i(\gamma_i(P))}{\partial P^2}|_{P=P^*} = 0,\ 0 \le P^* \le P_T, \\ 0, & \text{if } \frac{\partial^2 U_i(\gamma_i(P))}{\partial P^2} < 0 \text{ for } 0 \le P \le P_T, \\ P_T, & \text{if } \frac{\partial^2 U_i(\gamma_i(P))}{\partial P^2} > 0 \text{ for } 0 \le P \le P_T, \end{cases}$$

and γ_i^o as

$$\gamma_i^o = \gamma_i(P_i^o).$$

In the next lemma, we show that each mobile i has a unique P_i^o.

Lemma 2: Using the above definition of P_i^o, each mobile i has a unique P_i^o.

Proof:

$$\begin{aligned} \frac{\partial^2 U_i(\gamma_i(P_i))}{\partial P_i^2} &= \frac{\partial^2 U_i(\gamma_i)}{\partial \gamma_i^2}\left(\frac{\partial \gamma_i(P_i)}{\partial P_i}\right)^2 \\ &\quad + \frac{\partial U_i(\gamma_i)}{\partial \gamma_i}\frac{\partial^2 \gamma_i(P_i)}{\partial P_i^2} \\ &= \frac{N_i(P_T + A_i)}{(P_T - P_i + A_i)^3} \\ &\quad \times \left\{\frac{\partial^2 U_i(\gamma_i)}{\partial \gamma_i^2}(N_i + \gamma_i) + 2\frac{\partial U_i(\gamma_i)}{\partial \gamma_i}\right\}. \end{aligned}$$

By assumptions (b) and (e), $\frac{\partial^2 U_i(\gamma_i)}{\partial P_i^2} = 0$ is continuous and it has at most one solution for $0 \le P \le P_T$. This implies that each mobile has a unique P_i^o. ■

By Lemma 2 and the definition of P_i^o,

$$U_i \quad \text{is} \quad \begin{cases} \text{a sigmoidal-like function}, & \text{if } 0 < P_i^o < P_T, \\ \text{a concave function}, & \text{if } P_i^o = 0, \\ \text{a convex function}, & \text{if } P_i^o = P_T, \end{cases}$$

of its own power allocation P_i. Hence, P_i^o can be interpreted as an inflection point of a sigmoidal-like function.

The next lemma shows that if mobile i requests positive power, $P_i(\lambda)$ at price λ, then $P_i(\lambda) = P_T$ or $U_i(\gamma_i(P_i(\lambda)))$ is in the concave region.

Lemma 3: If $P_i(\lambda) = \arg\max_{0 \le P \le P_T}\{U_i(\gamma_i(P)) - \lambda P\}$, $P_i(\lambda) = 0$, $P_i(\lambda) = P_T$ or $U_i(\gamma_i(P_i(\lambda)))$ is in the concave region.

Proof: If $0 < P(\lambda) < P_T$, it must satisfy the first and the second order conditions, i.e., $\frac{\partial U_i(\gamma_i(P))}{\partial P}|_{P=P(\lambda)} = \lambda$, $\frac{\partial^2 U_i(\gamma_i(P))}{\partial P^2}|_{P=P(\lambda)} < 0$, since $P(\lambda)$ is an interior point. This implies that $P_i(\lambda) = 0$, $P_i(\lambda) = P_T$ or $U_i(\gamma_i(P_i(\lambda)))$ is in the concave region. ■

Lemma 3 tells us that if the utility function U_i, of mobile i, is a convex function for $0 \le P \le P_T$, mobile i always requests a power level of 0 or P_T.

In the next proposition, we show that each mobile i has the maximum willingness to pay per unit power, λ_i^{max}.

Proposition 2: There exists a unique λ_i^{max} for mobile i such that

$$\lambda_i^{max} = \arg\min_{0 \le \lambda \le \infty}\{\max_{0 \le P \le P_T}\{U_i(\gamma_i(P)) - \lambda P\} = 0\}$$

and for $\lambda > \lambda_i^{max}$, $P_i(\lambda) = 0$.

Proof: First, consider the case when $0 < P_i^o < P_T$. By Lemmas 2 and 3,

$$\begin{aligned} w_i(\lambda) &= \max_{0 \le P \le P_T}\{U_i(\gamma_i(P)) - \lambda P\} \\ &= \max\{0, \max_{P_i^o \le P \le P_T}\{U_i(\gamma_i(P)) - \lambda P\}\} \end{aligned}$$

Let $w_i'(\lambda) = \max_{P_i^o \le P \le P_T}\{U_i(\gamma_i(P)) - \lambda P\}$, then $w_i'(0) > 0$, $w_i'(\infty) < 0$ and $w_i'(\lambda)$ is a decreasing function of λ. This implies that $w_i'(\lambda_i^{max}) = 0$ and $w_i'(\lambda) < 0$ for $\lambda > \lambda_i^{max}$ Therefore, for $\lambda > \lambda_i^{max}$, $P_i(\lambda) = 0$.

Now, consider the case when $P_i^o = 0$. In this case, $U_i(\gamma_i(P))$ is a concave function. Hence, $\frac{\partial U_i(\gamma_i(P))}{\partial P}$ is a decreasing function for $0 \le P \le P_T$. It then follows that

$$P_i(\lambda) = \begin{cases} P_T, & \text{if } \lambda < \frac{\partial U_i(\gamma_i(P))}{\partial P}|_{P=P_T}, \\ P^*, & \text{if } \lambda = \frac{\partial U_i(\gamma_i(P))}{\partial P}|_{P=P^*},\ 0 \le P^* \le P_T, \\ 0, & \text{if } \lambda > \frac{\partial U_i(\gamma_i(P))}{\partial P}|_{P=0}. \end{cases}$$

Therefore, $\lambda_i^{max} = \frac{\partial U_i(\gamma_i(P))}{\partial P}|_{P=0}$ and this is unique.

Finally, consider the case when $P_i^o = P_T$. Then, $U_i(\gamma_i(P))$ is a convex function and

$$w_i(\lambda) = \max\{0, U_i(\gamma_i(P_T)) - \lambda P_T\}.$$

λ_i^{max} can be determined by

$$\lambda_i^{max} = \frac{U_i(\gamma_i(P_T))}{P_T}.$$

Therefore, there exists a unique λ_i^{max}. ■

Each mobile i can calculate λ_i^{max} as follows.

$$\lambda_i^{max} = \begin{cases} \frac{\partial U_i(\gamma_i(P))}{\partial P}|_{P=0}, & \text{if } P_i^o = 0, \\ \frac{\partial U_i(\gamma_i(P))}{\partial P}|_{P=P^*}, & \text{if } 0 < P_i^o < P_T \\ & \text{and } P^* \text{ exists}, \\ \frac{U_i(\gamma_i(P_T))}{P_T}, & \text{otherwise}, \end{cases}$$

where P^* is a solution of the following equation.

$$U_i(\gamma_i(P)) - P\frac{\partial U_i(\gamma_i(P))}{\partial P} = 0, \qquad P_i^o \le P \le P_T.$$

When the price is λ_i^{max}, $P_i(\lambda)$ can have two values. One is zero and the other is positive. But, we take only a positive value of $P_i(\lambda)$ in the sequel.

In the next proposition, we show the relation between price and the requested power of mobile i.

Proposition 3: $P_i(\lambda)$ is a non-increasing function of λ for $\lambda \ge 0$. Moreover, $P_i(\lambda)$ is a decreasing function of λ for $\lambda_i^{min} \le \lambda \le \lambda_i^{max}$, where $\lambda_i^{min} = \max\{\lambda \ge 0 | P_i(\lambda) = P_T\}$.

Proof: By the definition of λ_i^{max}, $P_i(\lambda) = 0$ for $\lambda > \lambda_i^{max}$. Now, suppose $\lambda_1 < \lambda_2 \le \lambda_i^{max}$. If $U_i(\gamma_i(P))$ is a convex function for $0 \le P \le P_T$, then, by Lemma 3, $P_i(\lambda_1) = P_i(\lambda_2) = P_T$. If $U_i(\gamma_i(P))$ is a concave function or a sigmoidal-like function for $0 \le P \le P_T$, then, by Lemma 3, $U_i(\gamma_i(P_i(\lambda_1)))$ and $U_i(\gamma_i(P_i(\lambda_2)))$ must be in concave region, i.e., $P_i(\lambda_1) \ge P_i^o$ and $P_i(\lambda_2) \ge P_i^o$. Let $f_i(P,\lambda) = U_i(\gamma_i(P)) - \lambda P$. Then,

$$\begin{aligned} 0 &\le \frac{\partial f_i(P,\lambda_2)}{\partial P}|_{P=P_i(\lambda_2)} \\ &= \frac{\partial U_i(\gamma_i(P))}{\partial P}|_{P=P_i(\lambda_2)} - \lambda_2 \\ &< \frac{\partial U_i(\gamma_i(P))}{\partial P}|_{P=P_i(\lambda_2)} - \lambda_1 \\ &= \frac{\partial f_i(P,\lambda_1)}{\partial P}|_{P=P_i(\lambda_2)}. \end{aligned}$$

This implies that

$$\frac{\partial f_i(P,\lambda_1)}{\partial P} > 0 \text{ for } P_i^o \le P \le P_i(\lambda_2),$$

since $U_i(\gamma_i(P))$ is a concave function for $P_i^o \le P \le P_i(\lambda_2)$. Therefore, if $P_i(\lambda_2) < P_T$, then $P_i(\lambda_1) > P_i(\lambda_2)$ and if $P_i(\lambda_2) = P_T$, then $P_i(\lambda_1) = P_T$. ■

By the previous results, we summarize the properties of $P_i(\lambda)$ as

$$P_i(\lambda) \text{ is } \begin{cases} \text{discontinuous at } \lambda = \lambda_i^{max}, \text{ if } U_i \text{ is a convex function or a sigmoidal-like function.} \\ \text{continuous, if } U_i \text{ is a concave function.} \\ \text{positive and a decreasing function of } \lambda \text{ for } \lambda_i^{min} \le \lambda \le \lambda_i^{max}. \\ \text{zero for } \lambda > \lambda_i^{max}. \\ P_T \text{ for } \lambda \le \lambda_i^{min}. \end{cases}$$

Using these properties of $P_i(\lambda)$, we can select the mobiles to which positive power is allocated as follows.

Mobile selection algorithm

Suppose that there are M mobiles and $\lambda_1^{max} > \lambda_2^{max} > \cdots > \lambda_M^{max}$ [3].

[3]If some mobiles have the same λ_i^{max}, they are ordered randomly.

(i) The base station broadcasts its maximum power limit, P_T to all mobiles.
(ii) Each mobile i reports its λ_i^{max} to the base station.
(iii) Let $k = 1$.
(iv) The base station broadcasts price, λ_k^{max}.
(v) Each mobile i reports its power request $P_i(\lambda_k^{max})$ to the base station.
(vi) If $k = 1$ and $P_1(\lambda_k^{max}) = P_T$, select mobile 1 and stop,
else if $k = 1$ and $P_1(\lambda_k^{max}) < P_T$, go to (ix),
else go to (vii).
(vii) If $\sum_{j=1}^{k-1} P_j(\lambda_k^{max}) > P_T$, select from mobile 1 to mobile $k-1$ and stop,
else go to (viii).
(viii) If $\sum_{j=1}^{k-1} P_j(\lambda_k^{max}) \le P_T$ and $\sum_{j=1}^{k} P_j(\lambda_k^{max}) > P_T$, select from mobile 1 to mobile $k-1$ and stop,
else go to (ix).
(ix) Let $k = k+1$. If $k \le M$, go to (iv),
else select from mobile 1 to mobile $k-1$ and stop.

Therefore, the mobiles are selected in a descending order of λ_i^{max}.

We now study the characteristics of the mobile selection of our mobile selection algorithm. If the stop condition in (vi) is satisfied, the total power is allocated to mobile 1 with the price λ_1^{max}. In this case, $\sum_{j=1}^{M} P_j(\lambda_1^{max}) = P_T$, since $P_1(\lambda_1^{max}) = P_T$ and $P_j(\lambda_1^{max}) = 0$ for $j > 2$. Thus, this is a social optimal allocation by Lemma 1. By the stop condition in (vii), $\sum_{j=1}^{k-1} P_j(\lambda_{k-1}^{max}) \le P_T$ and $\sum_{j=1}^{k-1} P_j(\lambda_k^{max}) > P_T$. Furthermore, $\sum_{j=1}^{k-1} P_j(\lambda)$ is continuous for $\lambda_k^{max} \le \lambda \le \lambda_{k-1}^{max}$. Therefore, we can find λ^* such that $\sum_{j=1}^{k-1} P_j(\lambda^*) = P_T$, $\lambda_k^{max} < \lambda^* \le \lambda_{k-1}^{max}$. This implies that $\sum_{j=1}^{M} P_j(\lambda^*) = P_T$, since $P_j(\lambda^*) = 0$ for $j = k, \cdots, M$ and, thus, the mobile selection is a social optimal selection by Lemma 1. If the stop condition in (viii) is satisfied, $\sum_{j=1}^{M} P_j(\lambda_k^{max}) = \sum_{j=1}^{k} P_j(\lambda_k^{max}) > P_T$ and $\sum_{j=1}^{M} P_j(\lambda_k^{max} + \epsilon) = \sum_{j=1}^{k-1} P_j(\lambda_k^{max} + \epsilon) < P_T$ for all $\epsilon > 0$. Thus, in this case, we cannot find λ' such that $\sum_{j=1}^{M} P_j(\lambda') = P_T$ and the mobile selection may not be a social optimal selection. Moreover, we cannot find an equilibrium solution of problem $(\mathbf{E})$. However, to obtain a good approximation of the solution of $(\mathbf{E})$ satisfying the constraint, we must select mobiles from 1 to $k-1$. For the selected mobiles, we can find λ^* such that $\sum_{j=1}^{k-1} P_j(\lambda^*) = P_T$ and $\lambda^* \le \lambda_k^{max}$, since $\sum_{j=1}^{k-1} P_j(\lambda_k^{max}) \le P_T$ and $\sum_{j=1}^{k-1} P_j(\lambda)$ is continuous for $\lambda \le \lambda_k^{max}$. If the stop condition in (ix) is satisfied, $\sum_{j=1}^{M} P_j(\lambda_M^{max}) \le P_T$ and $P_j(\lambda)$ for all j is continuous for $\lambda \le \lambda_M^{max}$. Thus, we can find λ^* such that $\sum_{j=1}^{M} P_j(\lambda^*) = P_T$, $\lambda^* \le \lambda_M^{max}$ and, thus, the mobile selection is a social optimal selection by Lemma 1.

Therefore, the mobile selection may not be a social optimal selection. But, we will show that with our pricing scheme and the partial-cooperative property of mobiles, the mobile selection of our algorithm is optimal selection and, thus, we call the selection the *partial-cooperative optimal selection*.

Theorem 1: The mobile selection of the mobile selection algorithm is an optimal selection satisfying the power constraint under the partial-cooperative property of mobiles and linear pricing with the same unit price.

Proof: We prove the result only for the case when the stop condition in (viii) is satisfied, since if any other stop condition is satisfied, the selection is a social optimal selection. Suppose that mobiles 1 through l are selected. If $l > k-1$, by the non-cooperative property, $\lambda \leq \lambda_l^{max}$. In this case, $\sum_{i=1}^{l} P_i(\lambda) > P_T$. Thus, the power constraint cannot be satisfied. Now consider the case when $l < k-1$. In this case,

$$\max_{\sum_{i=1}^{l} P_i \leq P_T} \{\sum_{i=1}^{l} U_i(\gamma_i(P_i))\}$$

is equivalent to

$$\max_{\sum_{i=1}^{k-1} P_i \leq P_T} \{\sum_{i=1}^{k-1} U_i(\gamma_i(P_i))\}$$

with additional constraints, $P_i = 0$ for $i = l+1, \cdots, k-1$. But, by the way in which mobiles are selected, all mobiles from 1 to $k-1$ are allocated positive power at a global optimal power allocation for

$$\max_{\sum_{i=1}^{k-1} P_i \leq P_T} \{\sum_{i=1}^{k-1} U_i(\gamma_i(P_i))\}.$$

Thus,

$$\begin{aligned} &\max_{\sum_{i=1}^{l} P_i \leq P_T} \{\sum_{i=1}^{l} U_i(\gamma_i(P_i))\} \\ < &\max_{\sum_{i=1}^{k-1} P_i \leq P_T} \{\sum_{i=1}^{k-1} U_i(\gamma_i(P_i))\} \end{aligned}$$

and the case when $l < k-1$ cannot be optimal. Therefore, the selection of the mobile selection algorithm is an optimal selection satisfying the power constraint under the partial-cooperative property of mobiles and linear pricing with the same unit price. ■

B. Power allocation for the partial-cooperative optimal mobile selection

After the base station selects mobiles using the mobile selection algorithm in the previous subsection, it allocates its power to the selected mobiles. If the stop condition in (vi) in the mobile selection algorithm is satisfied, the power allocation algorithm is not needed, since the total power, P_T must be allocated to mobile 1. Therefore, in this subsection, we assume that the stop condition in (vii), (viii), or (ix) in the mobile selection algorithm is satisfied. Suppose that mobiles i, $i = 1, 2, \cdots, k-1$ are selected and $\lambda_1^{max} > \lambda_2^{max} > \cdots > \lambda_{k-1}^{max}$. Then, the base station problem $(\mathbf{E})$ can be rewritten as

$$(\mathbf{F}) \qquad \min_{\lambda} P_T - \sum_{i=1}^{k-1} P_i(\lambda)$$
$$\text{subject to } \sum_{i=1}^{k-1} P_i(\lambda) \leq P_T,$$
$$\lambda_{min} \leq \lambda \leq \lambda_{max},$$

where

$$\lambda_{min} = \begin{cases} \lambda_k^{max}, & \text{if the stop condition (vii) in the mobile selection algorithm is satisfied,} \\ 0, & \text{if the stop condition (viii) or (ix) in the mobile selection algorithm is satisfied,} \end{cases}$$

$$\lambda_{max} = \begin{cases} \lambda_{k-1}^{max}, & \text{if the stop condition (vii) or (ix) in the mobile selection algorithm is satisfied,} \\ \lambda_k^{max}, & \text{if the stop condition (viii) in the mobile selection algorithm is satisfied,} \end{cases}$$

and each mobile i, $i = 1, 2, \cdots, k-1$ solves its problem $(\mathbf{D_i})$. The next theorem tells us that the solution of problem $(\mathbf{F})$ and problem $(\mathbf{D_i})$ is a social optimal solution given that a partial-cooperative optimal selection and, thus, we call it the *partial-cooperative optimal power allocation*.

Theorem 2: If a power allocation, $\bar{P}^{k-1}(\lambda^*) = (P_1(\lambda^*), P_2(\lambda^*), \cdots, P_{k-1}(\lambda^*))$ is a solution of problem $(\mathbf{F})$ and problem $(\mathbf{D_i})$, it is a global optimal solution of the following optimization problem.

$$(\mathbf{G}) \qquad \max_{\bar{P}} \sum_{i=1}^{k-1} U_i(\gamma_i(P_i))$$
$$\text{subject to } \sum_{i=1}^{k-1} P_i \leq P_T,$$
$$P_i \geq 0, \qquad i = 1, 2, \cdots, k-1.$$

Proof: From the way the base station selects mobiles, λ^* which satisfies $\sum_{i=1}^{k-1} P(\lambda^*) = P_T$ always exists. Therefore, by Lemma 1, $\bar{P}^{k-1}(\lambda^*) = (P_1(\lambda^*), P_2(\lambda^*), \cdots, P_{k-1}(\lambda^*))$ is a global optimal solution for problem $(\mathbf{G})$. ■

Therefore, if the partial-cooperative optimal selection is the same as the social optimal selection, i.e., the stop condition in (vi), (vii) or (ix) of the mobile selection algorithm is satisfied, the partial-cooperative optimal power allocation is the same as the social optimal power allocation. Otherwise, i.e., the stop condition in (viii) is satisfied, the partial-cooperative optimal power allocation could be a good approximation to the social optimal power allocation. Moreover, as the next theorem shows, the partial-cooperative optimal power allocation is Pareto optimal.

Definition 1: A power allocation vector, $\bar{P}^* = (P_1^*, P_2^*, \cdots, P_M^*)$ is called a Pareto optimal power allocation vector, if there is no other power allocation vector, $\bar{P} = (P_1, P_2, \cdots, P_M)$ such that $U_i(\gamma_i(\bar{P})) \geq U_i(\gamma_i(\bar{P}^*))$, for all $i = 1, 2, \cdots, M$ and $U_j(\gamma_j(\bar{P})) > U_j(\gamma_j(\bar{P}^*))$ for some j.

Theorem 3: The partial-cooperative optimal power allocation, $\bar{P}^* = (P_1^*, P_2^*, \cdots, P_{k-1}^*, 0, \cdots, 0)$, is Pareto optimal.

Proof: Suppose that there exists a power allocation, $\bar{P}'$ such that $\sum_{i=1}^{M} P_i' \leq P_T$, $U_i(\gamma_i(P_i')) \geq U_i(\gamma_i(P_i^*))$, $i = 1, 2, \cdots, M$ and $U_j(\gamma_j(P_j')) > U_j(\gamma_j(P_j^*))$ for some j. First, assume that $1 \leq j \leq k-1$. Then, $\sum_{i=1}^{k-1} U_i(\gamma_i(P_i')) > \sum_{i=1}^{k-1} U_i(\gamma_i(P_i^*))$, which is contradiction, since $(P_1^*, P_2^*, \cdots, P_{k-1}^*)$ is a social optimal solution for problem $(\mathbf{G})$. Now, assume that $k \leq j \leq M$. By the previous result, $U_i(\gamma_i(P_i')) = U_i(\gamma_i(P_i^*))$, $i = 1, 2, \cdots, k-1$ and $P_j' > 0$. Then, by redistributing P_j' to mobile i, $i = 1, 2, \cdots, k-1$, we can find a power allocation $\bar{P}''$ such that $\sum_{i=1}^{k-1} U_i(\gamma_i(P_i'')) > \sum_{i=1}^{k-1} U_i(\gamma_i(P_i^*))$, which is contradiction, since $(P_1^*, P_2^*, \cdots, P_{k-1}^*)$ is a social optimal solution for problem $(\mathbf{G})$. Therefore, there exists no power allocation such as $\bar{P}'$ and, thus, $\bar{P}^*$ is a Pareto optimal power allocation. ■

The power allocation algorithm can be implemented in several ways. If we consider problem $(\mathbf{F})$, we can use a simple line search algorithm such as a bisect algorithm and a golden section algorithm [13]. If we consider problem $(\mathbf{G})$, we can use a gradient based algorithm [4] or a penalty based algorithm [3], since problem $(\mathbf{G})$ is equivalent to the following convex programming problem.

$$
\begin{aligned}
(\mathbf{H}) \qquad & \max_{\bar{P}} \sum_{i=1}^{k-1} U_i(\gamma_i(P_i)) \\
& \text{subject to} \sum_{i=1}^{k-1} P_i \leq P_T, \\
& \qquad P_i \geq P_i(\lambda_i^{max}), \\
& \qquad i = 1, 2, \cdots, k-1,
\end{aligned}
$$

where $P_i(\lambda_i^{max}) \geq P_i^o$. Thus, $U_i(\gamma_i(P_i))$ is a concave function for $P_i(\lambda_i^{max}) \leq P_i \leq P_T$, which makes problem $(\mathbf{H})$ a convex programming problem.

In this subsection, we implement the power allocation algorithm using a bisect algorithm.

Power allocation algorithm

Suppose that mobiles from 1 to $k-1$ are selected by the mobile selection algorithm and let ϵ be a small positive constant.

(i) Set $a^{(1)} = \lambda_{min}$, $b^{(1)} = \lambda_{max}$ and $n = 1$.
(ii) The base station broadcasts the price $\lambda^{(n)} = \frac{a^{(n)}+b^{(n)}}{2}$. to all selected mobiles
(iii) Each mobile i reports its power requests $P_i(\lambda^{(n)})$ to the base station.
(iv) If $|\sum_{i=1}^{k-1} P_i(\lambda^{(n)}) - P_T| < \epsilon$, stop, else go to (v).
(v) If $\sum_{i=1}^{k-1} P_i(\lambda^{(n)}) - P_T > 0$, set $a^{(n+1)} = \lambda^{(n)}$, $b^{(n+1)} = b^{(n)}$, else set $a^{(n+1)} = a^{(n)}$, $b^{(n+1)} = \lambda^{(n)}$.
(vi) $n = n + 1$ and go to (ii).

IV. Partial-cooperative optimal power allocation with the minimum SIR requirement

From the previous section, we know that the partial-cooperative optimal power allocation is Pareto optimal and could be a good approximation to the social optimal power allocation. However, the allocation could be unfair to some mobiles, since the algorithm gives higher priority to mobiles with higher λ_i^{max}. Therefore, with only partial-cooperative optimal power allocation, QoS requirements for every mobile might not be satisfied.

To alleviate this situation, we can introduce a minimum SIR requirement, γ_i^{min} for each mobile i. Therefore, problem $(\mathbf{B})$ can be modified as

$$
\begin{aligned}
(\mathbf{I}) \qquad & \max_{\bar{P}} \sum_{i=1}^{M} U_i(\gamma_i(P_i)) \\
& \text{subject to} \sum_{i=1}^{M} P_i \leq P_T, \\
& \qquad P_i \geq P_i^{min}, \qquad i = 1, 2, \cdots, M,
\end{aligned}
$$

where $\gamma_i(P_i^{min}) = \gamma_i^{min}$. In this problem, we assume that the system is feasible, i.e., $\sum_{i=1}^{M} P_i^{min} \leq P_T$. The system feasibility can be maintained by the call admission control and the scheduling. The optimization problem $(\mathbf{I})$ is equivalent to the following problem.

$$
\begin{aligned}
(\mathbf{J}) \qquad & \max_{\bar{P}'} \sum_{i=1}^{M} U_i(\gamma_i(P_i^{min} + P_i')) - U_i(\gamma_i(P_i^{min})) \\
= \; & \max_{\bar{P}'} \sum_{i=1}^{M} U_i'(\gamma_i'(P_i')) \\
& \text{subject to} \sum_{i=1}^{M} P_i' \leq P_T - \sum_{i=1}^{M} P_i^{min} = P_T', \\
& \qquad P_i' \geq 0, \qquad i = 1, 2, \cdots, M,
\end{aligned}
$$

where $\gamma_i'(P_i') = \frac{N_i(P_i^{min}+P_i')}{P_T - P_i^{min} - P_i' + A_i}$ and $U_i'(\gamma_i'(P_i')) = U_i(\gamma_i'(P_i^{min}+P_i')) - U_i(\gamma_i(P_i^{min}))$. Problem $(\mathbf{J})$ has the same structure as problem $(\mathbf{B})$ and we can apply the mobile selection algorithm and the power allocation algorithm in section III with the utility function for mobile i, U_i' and the power limit,P_T'. In this case, the mobiles selected by the mobile selection algorithm are allocated additional power to the minimum power requirement.

V. Special case: Single class of mobiles

In this section, we study, for illustration, a special case of our method, i.e., when all mobiles are homogeneous (each mobile has the same utility function, U, and the same processing gain N). We present this case because it provides some insight. We compare the partial-cooperative optimal selection and the social optimal selection in this case. Proofs are omitted for the sake of brevity.

In the following proposition, we show the relation between A_i and λ_i^{max}. Recall that A_i defined by I_i/G_i in (1) indicates the "goodness" of the environment of mobile i.

Proposition 4: Suppose that all mobiles have the same utility function U, the same processing gain, N, and $A_i < A_j$, then $\lambda_i^{max} > \lambda_j^{max}$.

Proposition 4 tells us that, in the homogeneous mobile case, mobiles are selected with ascending order of A_i by the partial-cooperative mobile selection algorithm since mobiles are selected with descending order of λ_i^{max} by the partial-cooperative mobile selection algorithm. This implies that the mobile in better environment has more chance to be selected by the partial-cooperative optimal selection algorithm.

Now, we study the social optimal selection. By the next proposition, in the homogeneous mobile case, we can order each mobile i according to A_i.

Proposition 5: Suppose that all mobiles have the same utility function U, the same processing gain, N, and $A_1 < A_2 < \cdots < A_M$. If $\bar{P}^* = (P_1^*, \cdots, P_M^*)$ is a social optimal power allocation and $A_i < A_j$, then $\gamma_i(P_i^*) \geq \gamma_j(P_j^*)$.

Corollary 1: Suppose that all mobiles have the same utility function U, the same processing gain, N, and $A_1 < A_2 < \cdots < A_M$. If $\bar{P}^* = (P_1^*, \cdots, P_M^*)$ is a social optimal power allocation and $P_k^* = 0$, $P_j^* = 0$ for all j such that $A_j > A_k$.
Corollary 1 implies that, in the social optimal selection, mobiles are selected in ascending order of A_i.

Proposition 4 and Corollary 1 tell us that the order of mobile selection of the partial-cooperative optimal selection is the same as that of the social optimal selection. Therefore, the set of mobiles selected by the partial-cooperative optimal selection is a subset of the set of mobiles selected by the social optimal selection and the relation between mobiles in each set is as follows.

$$A_j \quad < \quad A_i, \text{ for } i, j \in V, j \in Z \text{ and } i \notin Z,$$

where V is the set of mobiles selected by the social optimal selection and Z is the set of mobiles selected by the partial-cooperative optimal selection. This implies that the partial-cooperative optimal selection excludes the mobiles which obtain relatively low utility by the social optimal selection and, thus, the difference between the system utilities of them is small. In the next section, we provide numerical examples to show this difference with simulation.

VI. The Comparison of the Partial-Cooperative Optimal Power Allocation and the Social Optimal Power Allocation for the Single Class Case

In this section, we compare the partial-cooperative optimal power allocation and the social optimal power allocation for the single class case by the computer simulation. Each mobile is assumed be homogeneous, which has the same utility function and the same processing gain. We model the cellular network with 9 square cells, as shown in Fig. 1. We assume that the base station is located at the center of each cell and each base station has the same maximum power limit, P_T. We focus on the cell at the center of the system.

We model the path gain from a base station i to a mobile j, $G_{i,j}$ as follows.

$$G_{i,j} \quad = \quad \frac{K_{i,j}}{d_{i,j}^{\alpha}},$$

where $d_{i,j}$ is the distance from the base station i to mobile j, α is a distance loss exponent and $K_{i,j}$ is the log-normally distributed random variable with mean 0 and variance σ^2 (dB),

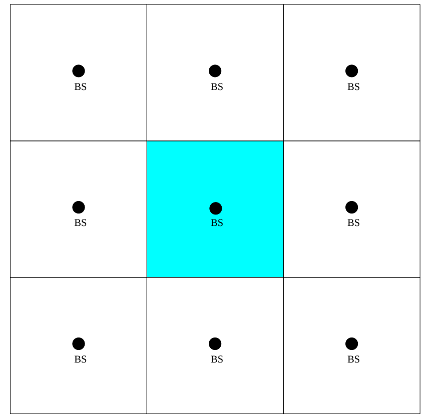

Fig. 1. Cellular network model.

TABLE I
Parameters for the system.

Maximum power (P_T)	10
Processing gain (N)	128
Distance loss exponent (α)	4
Variance of log-normal distribution (σ^2)	8
Length of the side of the cell	1000

which represents shadowing [14]. The parameters for the system are summarized in Table I. For the simulation, we use a sigmoid utility function. The sigmoid utility function is expressed as

$$U(\gamma) \quad = \quad c\{\frac{1}{1+e^{-a(\gamma-b)}} - d\}. \tag{9}$$

We normalize the sigmoid utility function as $U(0) = 0$ and $U(\infty) = 1$ by setting $c = \frac{1+e^{ab}}{e^{ab}}$ and $d = \frac{1}{1+e^{ab}}$. The property of the sigmoid utility function is well studied in [9].

In Tables II – IV, we show the total system utilities for each power allocation and the ratio, varying the values of a, b, and N. For each case, we assume that the mobiles are located independently according to uniform distributions within the cell and run the simulations 10^4 times. The results tells us that the system utility achieved by the partial-cooperative optimal power allocation is quite close to that achieved by the social optimal power allocation.

VII. Conclusions

In this paper, we focus on downlink communication in wireless systems. The downlink is expected to support higher bandwidth applications than the uplink and multi-class services. Considering these service requirements, we have proposed a downlink power allocation algorithm for multi-class CDMA wireless networks. We adopted a utility based framework and tried to maximize the total system utility. The proposed algorithm can be implemented in a distributed way using a partial-cooperative M-person power allocation game with dynamic pricing. It provides a partial-cooperative optimal power allocation which is Pareto optimal and a good approximation of the social optimal power allocation.

TABLE II

COMPARISON OF UTILITY FOR THE HOMOGENEOUS CASE ($b = 7(dB)$, $N = 64$, $M = 10$).

a	0.5	1	2	4	8
Partial	5.967	7.00	7.756	8.256	8.539
Social	6.012	7.093	7.885	8.392	8.661
Partial/Social	0.992	0.987	0.984	0.984	0.986

TABLE III

COMPARISON OF UTILITY FOR THE HOMOGENEOUS CASE($a = 3$, $N = 64$, $M = 10$).

$b(dB)$	3	5	7	9	11
Partial	9.887	9.391	8.072	6.302	4.697
Social	9.907	9.475	8.213	6.459	4.803
Partial/Social	0.998	0.991	0.983	0.976	0.978

TABLE IV

COMPARISON OF UTILITY FOR THE HOMOGENEOUS CASE ($a = 3$, $b = 7(dB)$, $M = 10$).

N	8	16	32	64	128
Partial	1.987	2.982	5.040	8.065	9.884
Social	1.995	2.991	5.253	8.210	9.913
Partial/Social	0.996	0.997	0.959	0.982	0.997

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