End-to-end loss estimates in networks with GPS servers handling many traffic streams

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Abstract

We consider a network of servers with small buffers (in comparison to the speed) accessed by a large number of stationary independent flows. The server is shared according to a Generalized Processor Sharing (GPS) discipline. It is assumed that the route of each flow is acyclic but flows need not be independent inside the network. We prove that a large deviations (LD) principle holds and find the large deviations rate functions for the buffer overflow at each node in terms of the external input LD characteristics. We then use these results to obtain (asymptotic in the number of flows) end-to-end packet loss for each flow. When each type of flow is supposed to have a Quality of Service (QoS) defined in terms of packet loss we obtain the admissible region for sources which access the network based on these QoS requirements. The efficacy of the analytical results are validated via simulations showing that the estimates obtained are extremely accurate even when the number of flows is of the order of 100, a situation that is not abnormal for flows in MPLS architectures or in the core of a network.

Index Terms

Generalized Processor Sharing (GPS), Large Deviation Principle, many sources asymptotics, small-buffer.

I. INTRODUCTION

Providing tight service differentiation to flows within a high speed network is quite difficult without providing detailed state information about the flows. Going back to the INTSERV architecture, one way to provide service differentiation is to provide minimum bandwidth guarantees that is easily implementable via a Generalized Processor Sharing (GPS) scheme to share server bandwidth. This was first proposed by Parekh and Gallager [21].

Generalized Processor Sharing is thus advocated for providing minimum bandwidth guarantees and service differentiation to the traffic flows at a node. Its approximate packetized implementation Weighted Fair Queueing (WFQ) is already implemented in most routers. A related discipline, the discriminatory processor sharing (DPS), is also natural as a model of TCP bandwidth sharing. GPS is a work conserving discipline in which each flow is assigned a weight that determines the amount of service capacity it can get. At any time, capacity is distributed to the active flows in proportion to their weights. This way, the choice of weight factors determines bandwidth allocation levels and provides service differentiation and priority for different classes.

Most network applications have constraints on the packet loss and delay incurred by the bits as they traverse the network from source to destination. These guarantees can be provided in deterministic or statistical settings. Deterministic guarantees are hard guarantees and the analysis is usually based on a worst-case scenario. Using network calculus techniques in [21] deterministic bounds on the delay at nodes was provided. Since then network calculus techniques have been extended to more general convex traffic envelopes and there has been a number of papers that have analyzed networks of GPS servers with infinite buffers. The monographs of Le Boudec and Thiran [3] and Chang [4] provide a comprehensive treatment of both deterministic and stochastic network calculus approaches. Deterministic bounds allow us to treat the end-to-end problem but the results are quite conservative due to the fact that input sources are usually

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assumed to satisfy a deterministic envelope or a stochastic type of envelope referred to a exponentially bounded burstiness (EBB) [24]. An analysis using the EBB approach can be found in [10].

Providing statistical QoS is much more efficient in terms of resource utilization (in this context being able to support a larger number of flows) but the analysis is much more complicated. This is due to the fact that traffic flows undergo changes in their statistics when "filtered" through queues and precise characterization of the statistical behavior of flows within a network is possible only in a few simple cases.

An exact stochastic analysis of GPS systems has been given only for the case of two buffers [12]. To the best of our knowledge, there are no exact results in the literature for the case of more than two buffers or general stationary inputs. However in most cases we are interested in designing networks where losses are small that is often well captured by studying the overflow region of the queues that form. Mathematically it leads us naturally to characterize the asymptotics of the buffer overflow or packet loss distributions.

There are basically two types of asymptotics of interest: 1) The large buffer asymptotic when there are a few traffic flows which share a resource and a given flow can consume a significant amount of the bandwidth, and 2) The many sources asymptotics when each source uses a small amount of the resources. See [11] and [19] for a discussion of the two regimes.The latter many sources asymptotic is of interest in applications involving the so-called statistical multiplexing and this is the scenario we will consider in this paper.

Large buffer asymptotics for the GPS system have been considered in [25], [18]. Arrival traffic with long range dependence or heavy tails were examined in [14], [2].

The many sources asymptotic is better suited when there are many flows arriving into a node and each flow has a small bandwidth requirement when compared to the capacity. A large deviations framework has been studied in a number of papers for FIFO queues in [6], [15], for HOL priority systems in [23], [8]. In this framework the capacity and buffers are scaled in the same way as the number of sources, i.e. by a scaling factor of $O(N)$. The many independent flows scenario can also be approached via a Gaussian framework by invoking a Central Limit Theorem (CLT) argument and this has been studied by a number of researchers such as [5], [1] in the FIFO case and in [17] for the GPS case.

In this paper, we analyze the GPS discipline in a network setting. We consider the many sources regime where a large number of flows are involved between origin-destination pairs in a network with fixed routing. Such a scenario occurs in the MPLS architecture where virtual pipes (Label Switched Paths (LSP)) are established for connections which are identified by their "routes". Superposition of a large number of flows is also a natural assumption for modeling traffic in the core of a large network. We also assume that the buffer sizes in the network are small in comparison to the server capacities. This small buffer scenario is actually of much interest in today's networks where the link bandwidths and traffic loads are increasing much faster than the buffer memory sizes. This is also the essence of the so-called *rate envelope multiplexing* in networks (see [22]) where small buffers are used just to absorb the local fluctuations but essentially the network can be modeled by bufferless nodes. A large deviations analysis in the many sources setting for networks with FIFO and small buffers servers can be found in [20]. This paper is an application of these ideas to the case of GPS servers where the situation is more complex because of bandwidth sharing.

We will show that when the buffers are small, only the instantaneous values of the total input rate determine the overflow asymptotics under the many sources assumption. This has already been shown in the FIFO context in [15], [16] for a single node. Then a large deviation principle will be derived for a network with a fairly realistic routing policy in that we only require that the route for a given flow be acyclic but otherwise flows can interact in an arbitrary manner within the network. We also only assume that the flows are initially independent when they enter the network. We first find the overflow probability asymptotics in terms of the large deviation rate functions of the inputs. We then define the packet loss rates and find their LD rate functions along with the corresponding acceptance region. For regulated traffic, a lower bound for the their LD rate functions (and hence an upper bound for the overflow probabilities) can be easily calculated.

The outline of this paper is as follows: In Section II, the network model and the GPS discipline are introduced. In Section III, we obtain the LD asymptotics for the buffer occupancy. In Section IV, we determine the asymptotics of the loss process and then determine the admissible region. An example of a two node network is considered in Section V with numerical results. In Section VI, we discuss the results and provide possible extensions and some approximations.

II. Model and preliminaries

Consider a network composed of K nodes which is accessed by M types (or classes) of traffic flows or sources (see Figure 1 below). The server at node k can serve at the rate of NC_k and the arriving work of type $m \in \mathcal{M} = \{1, \dots, M\}$ which cannot be served is queued in a buffer of size $B_k^m(N)$. We assume that $B_k(N) \doteq \sum_m B_k^m(N) = o(N)$, i.e., $B_k(N)/N \to 0$ as $N \to \infty$. This corresponds to a network with (asymptotically) negligible buffers in relation to the capacity. The work which cannot be accommodated in the buffer is lost.

The external arrivals into the network from different classes are assumed to be mutually independent. We consider a discrete time fluid model where traffic arrivals and services take place in slots indexed by $t \in \mathbb{Z}$. Traffic is served under the Generalized Processor Service (GPS) discipline. Under GPS, each flow m is assigned a weight $0 \le \phi_m \le 1$ such that $\sum_{m=1}^M \phi_m = 1$. The server is work conserving, i.e., does not idle when there is traffic to be sent. A flow is called *backlogged* when it has data in its buffer. For a GPS server which operates at a fixed rate, let $C_m(t_1, t_2)$ be the amount of flow m traffic served during an interval $(t_1, t_2]$. Then for any flow m continuously backlogged during $(t_1, t_2]$ and any other flow n, it holds that

$$\frac{C_m(t_1, t_2)}{C_n(t_1, t_2)} \ge \frac{\phi_m}{\phi_n}.$$

We remark that GPS is an idealized model and in practice a packetized approximation such as Weighted Fair Queueing must be implemented.

Let $X_t^{m,N},\ m = 1, \dots, M$, denote the aggregate amount of work due to the sources of type m which are transmitting at time t. Here N is a scaling parameter and we will be interested in the situation when $N \to \infty$. The stochastic process $X_t^{m,N}$ is assumed to be stationary and ergodic for each m and N. Let $\rho_m^N = \mathbb{E}[X_t^{m,N}]/N$ and $X^{m,N}(t_1, t_2) = \sum_{t=t_1}^{t_2-1} X_t^{m,N}$. We assume that $\rho_m^N \to \rho_m$ as $N \to \infty$ and $X^{m,N}(0, t)/N$ satisfies the following Large Deviation Principle (LDP) with good rate function $I_t^{X^m}(x)$:

$$\begin{aligned} -\inf_{x \in B^\circ} I_t^{X^m}(x) &\le \underline{\lim}_{N \to \infty} \frac{1}{N} \log \mathbb{P}\left\{X^{m,N}(0, t) \in NB\right\} \\ &\le \overline{\lim}_{N \to \infty} \frac{1}{N} \log \mathbb{P}\left\{X^{m,N}(0, t) \in NB\right\} \le -\inf_{x \in \bar{B}} I_t^{X^m}(x) \end{aligned} \tag{II.1}$$

where $B \subset \mathbb{R}$ is a Borel set with interior B° and closure $\bar{B}$ and $I_t^{X^m} : \mathbb{R} \to [0, \infty]$ is a continuous mapping with compact level sets [9]. Only the ordinary topology will be used throughout this paper. In many cases of practical interest, $X_t^{m,N}$ results from the superposition of N independent, identically distributed (i.i.d.) sources. In this case, assumption (II.1) follows from Cramer's theorem [9]. However, it will be sufficient for us to consider $X_t^{m,N}$ just as a sequence of processes and use only the LD assumption (II.1) while deriving the LD rate functions of overflow and total packet loss probabilities. Nevertheless, our motivation comes from the case when $X_t^{m,N}$ is the sum of N i.i.d. processes and we will impose this assumption when we look at the finer asymptotics at the end of Proposition 3.1 and discuss the acceptance region in Corollary 4.1.

It is assumed that the network has fixed routing. Type m flow has a fixed route without any loops and its path is represented by the vector $\pi^m = \left(\pi_1^m, \dots, \pi_{l_m}^m\right)$ where $\pi_i^m \in \{1, \dots, K\}$ and $\pi_i^m \neq \pi_j^m$ for $i \neq j$. Hence type m traffic traverses the nodes $\{\pi_i^m\}$ by entering the network at node π_1^m and leaving after node $\pi^m_{l_m}$. Let the set $\mathcal{M}_k$ denote the types of traffic which pass through the node k, i.e., $\mathcal{M}_k = \{m : \pi^m_i = k,\ 1 \leq i \leq l_m\}$. It is assumed that:

$$\sum_{m \in \mathcal{M}_k} \rho_m < C_k \tag{II.2}$$

This assumption is not needed to prove the results but otherwise there will be at least one flow whose packet loss probability does not go to 0 as $N \to \infty$.

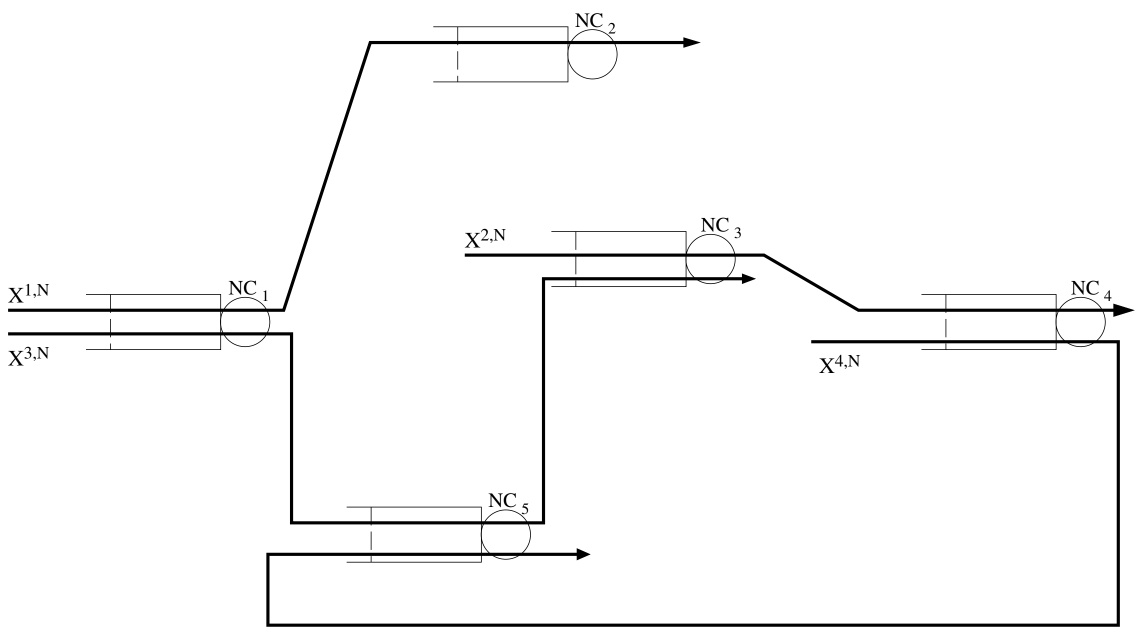

Fig. 1. A typical network considered in the paper

We now present the dynamics of the GPS scheduling for a discrete time fluid system considered in this paper.

Lemma 2.1: Consider a node with service capacity C and assume that its queue is empty at $t = -1$. Let x_m be the amount of type $m = 1, \ldots, M$ traffic arriving to this node at $t = 0$. Choose a permutation s on $\{1, \ldots, M\}$ such that $x_{s(1)}/\phi_{s(1)} \leq x_{s(2)}/\phi_{s(2)} \leq \ldots \leq x_{s(M)}/\phi_{s(M)}$ and define

$$n_0 = \arg\max_n \{(C - \sum_{i<n} x_{s(i)}) \frac{\phi_{s(n)}}{\sum_{i \geq n} \phi_{s(i)}} \geq x_{s(n)}\} \tag{II.3}$$

where $\sum_{\emptyset} = 0$ and $0 = \arg\max_n\{\}$ by assumption. Let $c(m)$ be the service capacity used by type m traffic at $t = 0$ and define $\mathcal{A}(x, C) = \{s(1), \ldots, s(n_0)\}$. Then

$$c(m) = \begin{cases} x_m & m \in \mathcal{A}(x, C) \\ (C - \sum_{i<n_0} x_{s(i)}) \frac{\phi_m}{\sum_{i>n_0} \phi_{s(i)}} & otherwise \end{cases} \tag{II.4}$$

Proof: If $n_0 = 0$, then $x_{s(1)} > C\phi_{s(1)}$ and since $x_{s(1)}/\phi_{s(1)} \leq x_m/\phi_m$, we have $x_m > C\phi_m$ for all type m. Thus all the flows are backlogged, i.e., each type m flow has more traffic than $C\phi_m$ and therefore it receives a capacity of $C\phi_m$. When $n_0 > 0$, we will use induction. For $n = 1$, since $x_{s(1)} \leq C\phi_{s(1)}$, all of type $s(1)$ traffic is served. Assume that this holds for all types $s(n)$ with $n < n_0$. A flow of type $\{s(n+1), \ldots, s(M)\}$ will share the remaining capacity $C - \sum_{i \leq n} x_{s(i)}$ in proportion to its weight. But since $(C - \sum_{i \leq n} x_{s(i)})\phi_m / \sum_{i>n} \phi_{s(i)} \geq x_{s(n+1)}$, it follows that type $n+1$ is also served to the completion. Now consider the types $n > n_0$. Let $C' = C - \sum_{i \leq n_0} x_{s(i)}$. By definition $C'\phi_{s(n_0+1)} / \sum_{i>n_0} \phi_{s(i)} < x_{s(n_0+1)}$ and thus for all $n > n_0$ we have $C'\phi_{s(n)} / \sum_{i>n_0} \phi_{s(i)} < x_{s(n)}$. Again, since the remaining capacity C' is distributed to the flows $n > n_0$ according to their weights, they will all be backlogged and the capacity each will get is given as in equation (II.4).

□

III. BUFFER OVERFLOW ASYMPTOTICS

Let $X_{k,t}^{m,N}$ ($Y_{k,t}^{m,N}$) be the amount of input (output) traffic of type m at node k and time t. If node k is not on the path of input m, set $X_{k,t}^{m,N} = Y_{k,t}^{m,N} = 0$. Also define $\mathbf{Z}_{k,t}^{N} = (X_{k,t}^{1,N}, \dots, X_{k,t}^{M,N})$ and $\mathbf{Z}_{t}^{N} = (X_{t}^{1,N}, \dots, X_{t}^{M,N})$.

We will show that the buffer asymptotics are governed by the instantaneous rates of the inputs to the node due to the assumption $B_k(N) = o(N)$. Now we give the following principal result which relates the LDP rate functions associated with instantaneous internal inputs and the overflow asymptotics at each node to the LDP rate functions associated with the instantaneous external inputs into the network.

Proposition 3.1: There exists a continuous function $g_k^m : \mathbb{R}^M \to \mathbb{R}$, relating the instantaneous input rate at node k for type m flow to all of the instantaneous external input traffic rates such that

$$X_{k,t}^{m,N}/N = g_k^m(X_t^{1,N}/N, \dots, X_t^{M,N}/N) + o(1) \tag{III.5}$$

Let $\mathcal{F}_k^m = \{\text{overflow for type } m \text{ at node } k\}$. Then

$$\lim_{N\to\infty} \frac{1}{N} \log \mathbb{P}\{\mathcal{F}_k^m\} \doteq -\mathbf{I}_k^m = \\ -\inf\{I_1^X(x) : m \notin \mathcal{A}(\mathbf{g}_k(x), C_k)\} \tag{III.6}$$

where $\mathcal{A}(.,.)$ as defined in Lemma 2.1 ,$I_1^X(x) \doteq \sum_{m=1}^{M} I_1^{X^m}(x_m)$ for $x = (x_m) \in \mathbb{R}^M$ and $\mathbf{g}_k(x) = (g_k^1(x), \dots, g_k^M(x))$.

Proof:

We will first show the existence of functions g_k^m. To this end, let us first obtain the relation between the instantaneous rates of inputs and outputs at a node. For $n = 1, \dots, M$, we define the function $f_n : \mathbb{R}^{M+1} \to \mathbb{R}$ as follows: For a given $x \in \mathbb{R}^M$, consider the model in Lemma 2.1 with capacity y and arriving traffic x. Then $f_n(x, y)$ is taken as the amount of capacity used by type n flow (denoted by $c(n)$ in the Lemma). If all the buffers of node j was empty at time $t-1$, then the amount of capacity used by type n flow at time t would be equal to $f_n(X_{j,t}^{1,N}, \dots, X_{j,t}^{M,N}, NC_j)$. If not, the buffered traffic before time t will share the service capacity with the arriving traffic at t. By comparing these two cases (empty and non-emtpy buffers at time $t-1$), we can find the following upper and lower bounds for $Y_{j,t}^{n,N}$, which was defined as the amount of type n traffic served at time t:

$$B_j^n(N) - B_j(N) \le Y_{j,t}^{n,N} - f_n(\mathbf{Z}_{j,t}^N, NC_j) \le B_j^n(N). \tag{III.7}$$

Note that the upper bound is found under the scenario when all of the buffered type n traffic is served. In the lower bound case, the capacity that the type n would receive if the backlogged traffic belonging to other types at time $t-1$ were served. Since all the buffers are of $o(N)$ and f_n is linear in N due to the GPS dynamics given in Lemma 2.1, we can write

$$Y_{j,t}^{n,N}/N = f_n(X_{j,t}^{1,N}/N, \dots, X_{j,t}^{M,N}/N, C_j) + o(1). \tag{III.8}$$

We will now obtain the mapping between the instantaneous rates of traffic inside the network ($X_{k,t}^{m,N}$) and the external input traffic ($X_t^{n,N}$). In a feedforward network this can be obtained via an induction argument by starting at the peripheral nodes and working towards the inner nodes. But in the non-feedforward case which we consider, the instantaneous rates cannot be directly obtained. Indeed, they depend on themselves through a functional relationship. Therefore we need to define such a function and show that it has a unique fixed point.

Let us now describe the function whose fixed point will give the instantaneous traffic rates in the network. Recall that the path of type m flow is represented by the vector $\pi^m = \left(\pi_1^m, \ldots, \pi_{l_m}^m\right)$. Let $q = \sum_{m=1}^M l_m$, $s_1 = 1$, $s_m = \sum_{n=1}^{m-1} l_n$, $1 < m \leq M$ and define

$$\Omega = \{v \in \mathbb{R}^q : v_{s_m+j} \in [0, C_{\pi_j^m}], \quad m = 1, \ldots, M, \quad j = 1, \ldots, l_m - 1\}$$

Let $T : \Omega \to \Omega$ be a mapping such that

$$T(v)_{s_m} = v_{s_m}, \; T(v)_{s_m+j+1} = f_m(\bar{v}, C_{\pi_j^m}) \tag{III.9}$$

where $\bar{v}_n = v_{s_n+n_{j,m}}$ for $n \in \mathcal{M}_{\pi_j^m}$ with $\pi^n_{n_{j,m}} = \pi_j^m$ and $\bar{v}_n = 0$ for $n \notin \mathcal{M}_{\pi_j^m}$.

Here vector v corresponds to the instantaneous traffic rates and T expresses the input rates to a node in terms of the output rates of the upstream nodes (ignoring the o(1) term). Since each input is either an external flow or the output of another node, we must have the relation $T(v) = v$. Now we need to show that this is indeed the case and for a fixed vector w with $w_m = v_{s_m}$, i.e., for fixed values of the external input rates, this solution is unique and a continuous function of w. It is easy to check that $T(\Omega) \subset \Omega$ since $f_n(., y) \leq y$. Let T_w be equal to T for a fixed w, i.e., $T_w = T|_{\Omega_w}$ with $\Omega_w = \{v \in \Omega : v_{s_m} = w_m\}$. Now we will show that T_w has a unique fixed point denoted by $v^0(w)$. From an extension of Banach fixed point theorem [7, p. 187], it is enough to show that T_w is a condensing map which means that for a given metric d and for $u, v \in \Omega_w$, $d(T_w(u), T_w(v)) < d(u, v)$. To prove that T_w is condensing, it is sufficient to show that the transformation at each node between input and output instantaneous rates is a condensing mapping. Indeed T_w can be written as a disjoint sum of such transformations by choosing the appropriate permutation of $\{v_i\}$ since the path of each flow has no loops.

Hence we will consider a generic mapping of the form $F : D \to D$, $F(x)_m = f_m(x, C)$ where $D \subset \mathbb{R}^M$ is a compact, convex set. For $x = (x_i) \in \mathbb{R}^M$, define the norm $\|x\| \doteq \max(\sum_{x_i \geq 0} x_i, -\sum_{x_i < 0} x_i)$ and let d be the corresponding metric. Since D is convex, it is enough to show that F is condensing in a neighborhood of $x \in D$. From Lemma 2.1, there exists a set $\mathcal{A}_x \subseteq \mathcal{M}$ such that $F(x)_m = x_m$ $[F(x)_m < x_m]$ for $m \in \mathcal{A}_x$ $[m \notin \mathcal{A}_x]$. Furthermore there exists a sufficiently small neighborhood $B(x)$ of x such that $\mathcal{A}_y = \mathcal{A}_x$ for $y \in B(x)$. Then for $y \in B(x)$ we have $(F(x) - F(y))_m = (x - y)_m$ if $m \in \mathcal{A}_x$ and $(F(x) - F(y))_m = (C - \sum_{n \in \mathcal{A}_x} (x - y)_n)\phi_m / \sum_{n \notin \mathcal{A}_x} \phi_n$ if $m \notin \mathcal{A}_x$. Thus F is condensing in the region $\{x \in D : \mathcal{A}_x \neq \mathcal{M}\}$.

As mentioned above, T_w can be written as the disjoint sum of F type transformations. Therefore T_w is also a condensing mapping if $\sum_{n \in \mathcal{M}_k} v^0(w)_{s_n+n_k} > C_k$ holds for at least one k. But if this does not hold, it follows that $v^0(w)_j = w_m$ for $s_m \leq j < s_{m+1}$. Thus we have shown that for every w, there exists a unique $v^0(w)$ satisfying $T_w(v^0(w)) = v^0(w)$ Furthermore $v^0(w)$ is also a continuous function of w. Indeed assume that this is not true and there exists $n \to \infty$, $w_n \to w$ but $v^0(w_n) \nrightarrow v^0(w)$. But since $v^0(w_n)$ lies in a compact region, there exists $\bar{v}$ s.t. $v^0(w_{n_k}) \to \bar{v} \in \Omega_w$ and because T is continuous, $T(\bar{v}) = \bar{v}$. But this is in contradiction to the uniqueness of the fixed point in Ω_w and thus proves the continuity of $v^0(.)$.

From the analysis above, we conclude that the unique fixed point corresponding to the instantaneous rates of flows in the network is a continuous function of the instantaneous rates of the external inputs to the network if we ignore the $o(1)$ term in (III.8) due to the buffering. For every sample path, the above analysis of finding the unique fixed point still applies since shifting T by a constant (which is the $o(1)$ buffer amount here) will not effect any of the arguments. But because of the continuity of T, each of these fixed points differs from the one of the bufferless case by an amount of $o(1)$. Thus we conclude that there exists a continuous mapping $g_k^m : \mathbb{R}^M \to \mathbb{R}$ such that

$$X_{k,t}^{m,N}/N = g_k^m(X_t^{1,N}/N, \ldots, X_t^{M,N}/N) + o(1). \tag{III.10}$$

From the above result and the contraction principle, $X_{k,t}^{m,N}/N$ satisfies an LDP of type (II.1) with the good rate function $I_1^{X_k^m}$ given by

$$I_1^{X_k^m}(y) = \inf\{I_1^X(x) : x \in \mathbb{R}^M, g_k^m(x_1, \ldots, x_M) = y\} \tag{III.11}$$

The rate function $I_1^{X_k^m}$ is also continuous. For this, it is enough to show that it is upper semicontinuous, i.e.,

$$\overline{\lim_{n\to\infty}} I_1^{X_k^m}(y_n) \leq I_1^{X_k^m}(y) \text{ for any } y_n \to y \tag{III.12}$$

Let x be such that $g_k^m(x) = y$ for which the equation (III.11) is minimized; i.e. $I_1^{X_k^m}(y) = I_1^X(x)$. Now take any y' such that $d(y', y) < \delta_1$ for small enough δ_1. If we can find x' satisfying $g_k^m(x') = y'$ and $I_1^X(x') < I_1^X(x) + \varepsilon$, then by letting $\varepsilon \to 0$, we get (III.12). From the definition of T, we can find x' (not necessarily unique) and δ_2 such that $d(x', x) < \delta_2$ and $g_k^m(x') = y'$. But since $I_1^X(.)$ was assumed to be continuous, $I_1^X(x') < I_1^X(x) + \varepsilon$ is true for small enough $\delta_2 = \delta_2(\delta_1, \varepsilon)$.

Having obtained the LD asymptotics of flows in the network, we now consider the buffer overflow asymptotics.

Now by using (III.7) and (III.10),

$$\mathbb{P}\{\mathcal{F}_k^m\} \leq \mathbb{P}\left\{f_m(\mathbf{g}_k(\mathbf{Z}_t^N), NC_k) - o(N) < g_k^m(\mathbf{Z}_t^N)\right\}$$

and hence from the continuity of f_m and g_k^n's,

$$\begin{aligned}&\overline{\lim}_{N\to\infty} \tfrac{1}{N} \log \mathbb{P}\{\mathcal{F}_k^m\} \leq \\ &-\inf\{I_1^X(x) : f_m(g_k^1(x), \ldots, g_k^M(x), C_k) < g_k^m(x_m)\} \\ &= -\inf\{I_1^X(x) : m \notin \mathcal{A}(g_k^1(x), \ldots, g_k^M(x), C_k)\}\end{aligned}$$

Now we look at the lower bound. Similar to the upper bound part,

$$\mathbb{P}\{\mathcal{F}_k^m\} \geq \mathbb{P}\left\{f_m(\mathbf{g}_k(\mathbf{Z}_t^N), NC_k) + o(N) < g_k^m(\mathbf{Z}_t^N)\right\}$$

and thus

$$\begin{aligned}&\underline{\lim}_{N\to\infty} \tfrac{1}{N} \log \mathbb{P}\{\mathcal{F}_k^m\} \geq \\ &-\inf\{I_1^X(x) : f_m(g_k^1(x), \ldots, g_k^M(x), C_k) < x_m\} = \\ &-\inf\{I_1^X(x) : m \notin \mathcal{A}(g_k^1(x), \ldots, g_k^M(x), C_k)\}\end{aligned}$$

□

Remark 3.1: It is in general difficult to identify the function g_k^m explicitly. When the network is feedforward, it can be written as a composition of function f_n's recursively. However, $g_k^m(x)$ can be numerically computed at every point $x \in \mathbb{R}^M$. A simple algorithm would be to perform an exhaustive check for every possible scenario of whether x_m should go unchanged or scaled down along the nodes of type m's path. Note that for every given x, only one of these scenarios (unique fixed point in Ω_x) can happen as shown in the above proof. The validation of a scenario can be done by looking at the input-output relationships (functions f_n's) Since f_n's are linear, it can be verified whether the corresponding linear system for a scenario has a solution. In the worst case, this has to be performed for every possible scenario, taking exponential in $\sum_{m=1}^M l_m$ amount of time.

The logarithmic asymptotics for overflow probability can be improved when $X^{m,N}$ is the sum of N i.i.d. processes. For this purpose, we will use the sharper LD asymptotics in $\mathbb{R}^M$ given in [13]. W.l.o.g. assume that $M \in \mathcal{M}_k$. Let $\Phi_k : \mathbb{R}^{M-1} \to \mathbb{R}$ s.t.

$$\sum_{i=1}^{M-1} I_1^{X^i}(x_i) + I_1^{X^M}(\Phi_k(x_1, \ldots, x_{M-1})) = \mathbf{I}_k^m$$

and $\Psi_k : \mathbb{R}^{M-1} \to \mathbb{R}$ be such that

$$\{x_M = \Psi_k(x_1, \ldots, x_{M-1})\} = \\ \{x \in \mathbb{R}^M : m \notin \mathcal{A}(g_k^1(x), \ldots, g_k^M(x), C_k)\}$$

Let $V = (Hessian(I_1^X))^{-1}$ and let $H_{\Phi_k}(H_{\Psi_k})$ be the Hessian of Φ_k (Ψ_k). Take x_k^* to be the unique (by assumption) point where Proposition(3.1) result is minimized and $\alpha_k = \nabla I_1^X(x_k^*)$.

From the proof of Proposition 3.1, we know that

$$\mathbb{P}\left\{\mathcal{F}_k^m\right\} \sim \mathbb{P}\left\{\mathbf{Z}_t^N \in N\Gamma_k^{m,N}\right\}$$

where $\Gamma_k^{m,N} = \{x \in \mathbb{R}^M : m \notin \mathcal{A}(\mathbf{g}_k(x), C_k \pm o(1))\}$. Here $a_n \sim (\preceq)\ b_n$ means that $a_n/b_n \to (\leq)\ 1$ as $n \to \infty$.

Assume that $H_{\Psi_k} - H_{\Phi_k} > 0$. Then from Theorem 1.4 in [13],

$$\mathbb{P}\left\{\mathbf{Z}_t^N \in N\Gamma^{m,N}\right\} = \frac{e^{-N\mathbf{I}_k^m} d_0}{\sqrt{2\pi N Det(V(x_k^*))}\ |(\alpha_k)_M|} \\ \times \frac{1 + O(\frac{1}{N})}{(Det(|(\alpha_k)_M|(H_{\Psi_k}(x_k^*) - H_{\Phi_k}(x_k^*)))^{1/2}} \tag{III.13}$$

for $d_0 \preceq e^{o(N)}$ where $Det(.)$ is the determinant function. Here the exponent term $o(N)$ is at the order of buffer sizes.

It is difficult to analytically check or prove the positivity of $H_{\Psi_k} - H_{\Phi_k}$ in general but numerical evidence suggests that it is always verified.

We can also find the joint distribution of overflows in each buffer by using the vector version of the contraction principle. We only state the result since the proof follows mutatis mutandis as above.

Proposition 3.2: For any set of nodes $\mathcal{S} \subseteq \{1, \ldots, K\}$ and set of flow types $\mathcal{R}_k \subseteq \mathcal{M}_k$ for $k \in \mathcal{S}$,

$$\lim_{N\to\infty} \frac{1}{N} \log \mathbb{P}(\text{overflow for } m \in \mathcal{R}_k \text{ at nodes k} \in \mathcal{S}) \\ = -\inf\{I_1^X(x) : m \in \mathcal{R}_k, m \notin \mathcal{A}(\mathbf{g}_k(x), C_k), k \in \mathcal{S}\}$$

IV. Loss Ratio and Acceptance Region

In this section, we consider the problem of characterizing the admissible region defined by the loss asymptotics in a network which is accessed by a large number of independent flows. As mentioned in the introduction, this is one of the main motivations for the development of the asymptotics.

Let us first define the quantities of interest. At the modeling level, we consider the granularity at the level of bits since we are working with a discrete fluid model. Let $Q_{k,t}^{m,N}$ denote the number of bits of type m in the buffer at node k and at time t^+ and define $Q_{k,t}^N = \sum_{m=1}^M Q_{k,t}^{m,N}$. Note that $Q_{k,t}^N \leq B_k(N) \leq NC_k$ for large enough N. From Lemma 2.1, at time t, there exists a random set $\mathcal{A}_{k,t} \subseteq \mathcal{M}$ containing the flow types which are completely served. Thus $Q_{k,t}^{m,N} = 0$ for $m \in \mathcal{A}_{k,t}$. If $m \notin \mathcal{A}_{k,t}$ and its traffic exceeding the service capacity it receives is less than $B_k^m(N)$, then all of the excess traffic is buffered. Otherwise, the rest which cannot be buffered is lost. Under this scheme, the buffer content of type $m \notin \mathcal{A}_{k,t}$ at node k evolves according to

$$Q_{k,t}^{m,N} = \min\Big\{B_k^m(N), X_{k,t}^{m,N} + Q_{k,t-1}^{m,N} \\ -(NC_k - \textstyle\sum_{n\in\mathcal{A}_{k,t}}[X_{k,t}^{n,N} + Q_{k,t-1}^{n,N}])\frac{\phi_m}{\sum_{n\notin\mathcal{A}_{k,t}}\phi_n}\Big\}$$

For each input flow of type m, let $\mathbf{L}^{m,N}$ be the total loss ratio (LR), defined as the ratio of the expected value of lost bits (number of bits which arrive when the buffer is full) at all nodes along a route to the mean of input traffic in bits. Define $\mathbf{r}^m = \{\pi_i^m : i = 1, \cdots, l_m\}$ to be the set of nodes on the route of type m flow. Then

$$\mathbf{L}^{m,N} = \frac{\sum_{k \in \mathbf{r}^m} L_k^{m,N}}{\mathbb{E}[X_t^{m,N}]} \tag{IV.14}$$

where $L_k^{m,N}$ is the expected number of packets lost at node k for type m traffic and given by

$$L_k^{m,N} = \mathbb{E}\left[\left(X_{k,t}^{m,N} + Q_{k,t-1}^{m,N} - C_{k,t}^{m,N} - Q_{k,t}^{m,N}\right)^+\right] \tag{IV.15}$$

where $C_{k,t}^{m,N}$ is the capacity used by type m at node k and time t.

We then have the following result characterizing the asymptotic loss corresponding to each flow which is identified by the route it takes through the network. Note that we assume each flow is routed from its ingress to destination along a unique route with no loops in the route. This is a realistic assumption in MPLS type of architectures.

Proposition 4.1:

$$\lim_{N \to \infty} \frac{1}{N} \log \mathbf{L}^{m,N} = - \min_{k \in \mathbf{r}^m} \mathbf{I}_k^m \tag{IV.16}$$

Proof: Let $L_k^{m,N}$ be the expected value of packet loss at node k for type m traffic as defined above. Then,

$$\log \mathbf{L}^{m,N} = \log(\sum_{k \in \mathbf{r}^m} L_k^{m,N}) - \log(\mathbb{E}[X_t^{m,N}]).$$

Now

$$L_k^{m,N} = \mathbb{E}[\max(0, X_{k,t}^{m,N} + Q_{k,t-1}^{m,N} - C_{k,t}^{m,N} - Q_{k,t}^{m,N})]$$

$$\leq Ny\mathbb{P}\left\{\mathcal{F}_k^m\right\} + \mathbb{E}[X_{k,t}^{m,N} \mathbb{1}_{\{X_{k,t}^{m,N} > Ny\}}]$$

To find an upper bound for the second term on right hand side, first note that $X_{k,t}^{m,N} \leq X_t^{m,N} + o(N)$. Then

$$\mathbb{E}[X_{k,t}^{m,N} \mathbb{1}_{\{X_{k,t}^{m,N} > Ny\}}] \leq$$

$$\sum_{x=y}^{\infty} (Nx + 1)\mathbb{P}\left\{Nx \leq X_0^{m,N} - o(N) < Nx + 1\right\}$$

Thus

$$\overline{\lim_{N \to \infty}} \frac{1}{N} \log \mathbb{E}[X_{k,t}^{m,N} \mathbb{1}_{\{X_{k,t}^{m,N} > Ny\}}] \leq - \inf_{x \geq y} I_1^{X^m}(x)$$

Since $I_1^{X^m}$ has compact level sets, $I_1^{X^m}(x) \to \infty$ as $x \to \infty$ and therefore we can choose y such that $I_1^{X^m}(x) > \mathbf{I}_k^m$ for $x > y$. Thus we get

$$\overline{\lim_{N \to \infty}} \frac{1}{N} \log L_k^{m,N} \leq -\mathbf{I}_k^m$$

Now we consider the lower bound for $\mathbf{L}_k^{m,N}$.

$$\mathbb{E}[\max(0, X_{k,t}^{m,N} + Q_{k,t-1}^{m,N} - C_{k,t}^{m,N} - Q_{k,t}^{m,N})] \geq$$

$$\mathbb{E}[X_{k,t}^{m,N} - f_m(\mathbf{Z}_{k,t}^N, NC_k) - B_k(N)] \geq$$

$$Ny\mathbb{P}\left\{X_{k,t}^{m,N} - f_m(\mathbf{Z}_{k,t}^N, NC_k) > B_k(N) + Ny\right\} \geq$$

$$Ny\mathbb{P}\left\{X_{k,t}^{m,N} - f_m(\mathbf{Z}_{k,t}^N, NC_k) > 2Ny\right\} \geq$$

$$Ny\mathbb{P}\left\{X_{k,t}^{m,N} > f_m(\mathbf{Z}_{k,t}^N, NC_k + 2Ny/\phi_m)\right\}$$

where $y > 0$ and N is large enough to make $Ny > B_k(N)$. Thus

$$\underline{\lim}_{N\to\infty} \tfrac{1}{N} \log \mathbf{L}_k^{m,N} \geq$$

$$-\inf\{I_1^X(x) : f_m(\mathbf{g}_k(x), C_k + 2y/\phi_m) < x_m\}$$

From the continuity of $I_1^{X_k^{m,N}}$ and f, by letting $y \to 0$, we get

$$\underline{\lim}_{N\to\infty} \frac{1}{N} \log \mathbf{L}_k^{m,N} \geq -\mathbf{I}_k^m$$

Adding up $L_k^{m,N}$ and using $\lim_{N\to\infty} \frac{1}{N} \log \mathbb{E}[X_t^{m,N}] = 0$ gives

$$\overline{\lim}_{N\to\infty} \frac{1}{N} \log \mathbf{L}^{m,N} = -\min_{k\in\mathbf{r}^m} \mathbf{I}_k^m$$

□

Now assume that $X^{m,N}$ is the sum of Nn_m i.i.d. processes. In this situation, we define the notion of the admissible or acceptance region denoted by $\mathcal{D}$. This corresponds to the mix or collection $\{n_m\}_{m=1}^M$ of sources or flows, which when present in the network, results in each class meeting a QoS constraint on the loss rates. Note that a flow of type *m* is specified by the route π^m that it takes through the network. Specifically,

$$\mathcal{D} = \{(n_m), m = 1, \ldots, M : \lim_{N\to\infty} \frac{1}{N} \log \mathbf{L}^{m,N} < -\gamma_m\} \tag{IV.17}$$

for $\gamma_m > 0$. Note that $\mathbf{L}^{m,N}$, as defined in equation (IV.14) in terms of the aggregate traffic $X^{m,N}$, is also the average number of packets or bits lost per each source of type m, i.e., we assume that losses of type m are uniformly distributed over all type m sources.

Corollary 4.1: Let $\mathcal{D}$ be the acceptance region for (n_m) defined above. Consider the fictional system where $X^{m,N}$ goes to a node on its path without being affected by the upstream nodes until that node and let $\bar{\mathcal{D}}$ be the acceptance region for this case. Then

$$\bar{\mathcal{D}} \subseteq \mathcal{D}$$

Furthermore if $\gamma_m = \gamma$ for all m, then

$$\bar{\mathcal{D}} = \mathcal{D}$$

Proof: Consider a node k and type $m \in \mathcal{M}_k$. Let $\bar{\mathbf{I}}_k^m$ to be the LD rate function for the overflow probability of type m flow at node k in the fictional system. First note that $X_{k,t}^{n,N} \leq X_t^{n,N} + o(N)$. Then

$$\mathbb{P}\left\{\mathcal{F}_k^m\right\} = \mathbb{P}\left\{f_m(\mathbf{Z}_{k,t}^N, NC_k) < X_{k,t}^{n,N}\right\} \leq$$

$$\mathbb{P}\left\{f_m(\mathbf{Z}_0^N, NC_k + o(N)) < X_0^{n,N}\right\}$$

Thus it follows that $\bar{\mathbf{I}}_k^m \leq \mathbf{I}_k^m$ and we get $\bar{\mathcal{D}} \subseteq \mathcal{D}$.

Now take $(n_m) \in \mathcal{D}$. Then $\mathbf{I}_k^m > \gamma$ for every $k = 1, \ldots, K$. By definition $\bar{\mathbf{I}}_k^m = \sum_{m \in \mathcal{M}} I_1^{X^m}(\bar{x}_m)$ for some $\bar{x} = (\bar{x}_m)$. Assume $g_k^m(\bar{x}) < \bar{x}_m$ for some $m \in \mathcal{M}_k$. Then from the way g_k^m has been defined, there must exist a node k' for which $\sum_{m \in \mathcal{M}_{k'}} g_{k'}^m(\bar{x}) > C_{k'}$. Otherwise this will imply $g_k^m(\bar{x}) = \bar{x}_m$ for all $m \in \mathcal{M}_k$. Therefore using Proposition 3.1 again, we get $\sum_{m=1}^{M} I_1^{X^m}(\bar{x}_m) = \bar{\mathbf{I}}_k^m > \mathbf{I}_{k'}^n$ for some $n \in \mathcal{M}_{k'}$. Since $(n_m) \in \mathcal{D}$, $\mathbf{I}_{k'}^n > \gamma$ and hence $\bar{\mathbf{I}}_k^m > \gamma$.

If $g_k^m(\bar{x}) = \bar{x}_m$ for all $m \in \mathcal{M}_k$, then $\sum_{m \in \mathcal{M}_k} g_k^m(\bar{x}) = \sum_{m \in \mathcal{M}_k} \bar{x}_m > C_k$. In this case at least one queue must overflow at node k and therefore $\bar{\mathbf{I}}_k^m \geq \mathbf{I}_k^n > \gamma$ for some $n \in \mathcal{M}_k$. Thus $\bar{\mathbf{I}}_k^m > \gamma$ for all m, k and therefore (n_m) is also in $\bar{\mathcal{D}}$. This implies that $\mathcal{D} \subseteq \bar{\mathcal{D}}$ and completes the proof.

□

Remark 4.1: Above corollary shows that when the QoS (in terms of the packet loss) is required to be the same across all classes (end-to-end routes), end-to-end loss can be computed by considering the initial statistical characteristics of the flows as they enter the network.

V. Two Node Case and Numerical Results

In order to illustrate the results, in particular how the functions $g(.)$ are determined, we consider two examples of a simple network with two nodes. The first example is a feedforward network while the second example is a network where individual routes have no loops but the network is not feedforward.

Example 1: There are three classes of traffic, one of which uses resources from both nodes. The schema is illustrated in the figure below. Let us find the overflow probability for type 3 flow at the second node.

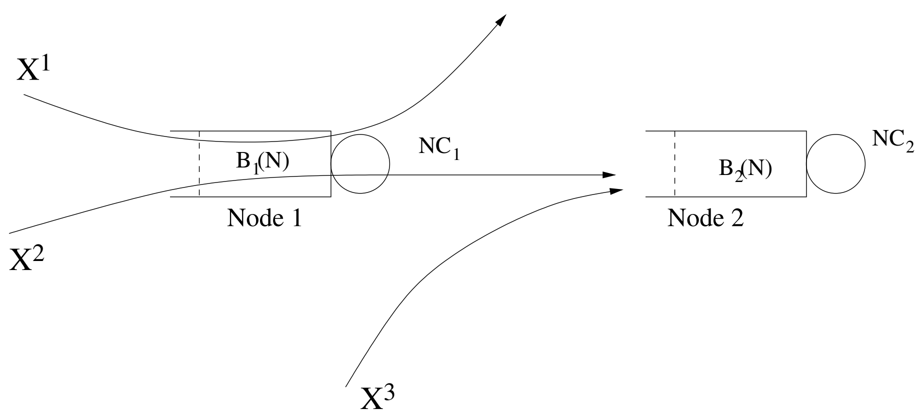

Fig. 2. Two node feedforward network

To this end, note that for $x = (x_1, x_2, x_3) \in \mathbb{R}^3$, $g_2^1(x) = 0$ and $g_2^3(x) = x_3$. For type 2 flow, we have

$$g_2^2(x) = \begin{cases} x_2 & x_1 + x_2 \leq C_1 \ or \ x_2 \leq \frac{\phi_2}{\phi_1+\phi_2} C_1 \\ \frac{\phi_2}{\phi_1+\phi_2} C_1 & otherwise \end{cases}$$

Thus we get

$$\mathbf{I}_2^3 = \inf\{I_1^X(x) : x_3 > \frac{\phi_3}{\phi_2+\phi_3}C_2,\ x_3 + g_2^2(x) > C_2\}$$

To use the sharper asymptotics in (III.13), assume that $X^{m,N}$ is the sum of N i.i.d. processes. Assume $\mathbf{I}_2^3 = I_1^X(x^*)$. Then we can compute

$$|\alpha| = \{\textstyle\sum_{i=1}^3 [(I_1^{X^i})'(x_i^*)]^2\}^{\frac{1}{2}}$$

$$Det(V(x^*)) = 1/\textstyle\prod_{i=1}^3 (I_1^{X^i})''(x_i^*)$$

$$\psi_2(x_1,x_2) = \max\{\tfrac{\phi_3}{\phi_2+\phi_3}C_2, C_2 - g_2^2(x)\}$$

$$\textstyle\sum_{i=1}^2 I_1^{X^i}(x_i) + I_1^{X^3}(\phi_2(x_1,x_2)) = \mathbf{I}_2^3 = I_1^X(x^*)$$

Without further assumptions on the input traffic it is difficult to check that the Hessian $H_{\psi_2}(x^*) - H_{\phi_2}(x^*)$ is positive definite; nevertheless it can be numerically verified.

The simulation for this example has given consistent results with the formula (III.13). These results (stated in terms of a 90% confidence interval) along with the estimate given through equation (III.13) are presented in Table I and II. The results given represent $\frac{1}{N}\log_{10}\mathbb{P}\{\text{overflow for type 3 at node 2}\}$.

The traffic sources were taken as the sum of N i.i.d. ON-OFF processes with periods (Tper), probability of being ON (p), peak rate (r) and weight factor (ϕ) as given below:

Flow	Tper	p	r	ϕ
1	30	0.33	3	0.2
2	40	0.40	2	0.3
3	45	0.55	3	0.5

The results in Table I correspond to the situation when type 2 flow does not overflow at node 2. Hence $g_2^2(x^*) = x_2$. In this case, the optimum point was found to be $x^* = (1, 1.162, 2.445)$. Node 1 and 2 had capacity $2.3N$ and $3.6N$ respectively and the buffers were taken to be of the order $\sqrt{N}$. For the results in Table II, type 2 flow overflows at the first node as well. To get this scenario, some of the input parameters were changed as follows: $\phi_1 = 0.3, \phi_2 = 0.2$ and $C_1 = 1.9, C_2 = 3.2$. In this case, $x^* = (1, 1.030, 2.316)$. As can be seen, the estimation technique is fairly accurate when the scaling factor is 100 or more which

TABLE I
RESULTS WHEN NO OVERFLOW AT NODE 1

N	Formula (III.13)	Simulation 90% conf.
10	-0.77923	(-1.65935, -0.87308)
20	-1.25845	(-2.62248, -1.69771)
50	-2.80292	(-3.86659, -3.22508)
70	-3.51323	(-4.20805, -3.67714)
80	-3.73507	(-4.24211, -3.77006)
100	-4.18452	(-4.59805, -4.00401)
120	-4.53184	(-4.99378, -4.35273)
150	-5.14357	(-5.55611, -4.72269)

can be found in many realistic cases.

Example 2: In this example, we consider a non-feedforward network with two nodes and two types of traffic as illustrated below. We will find the overflow asymptotics for type 2 flow at node 2. Let $x = (x_1, x_2)$. Then

$$\mathbf{I}_2^2 = \inf\{I_1^X(x) : x_2 > \phi_2 C_2,\ x_2 + g_2^1(x) > C_2\}$$

TABLE II
RESULTS WHEN OVERFLOW AT NODE 1

N	Formula (III.13)	Simulation 90% conf.
10	-0.77566	(-1.32077, -1.21675)
20	-1.20184	(-1.71778, -1.54059)
50	-2.22781	(-2.70593, -2.39074)
70	-2.85220	(-3.27359, -2.82593)
80	-3.15686	(-3.51278, -2.90836)
100	-3.75665	(-4.17732, -3.42352)
120	-4.34758	(-4.76554, -4.02275)
150	-5.12302	(-5.35258, -4.71349)

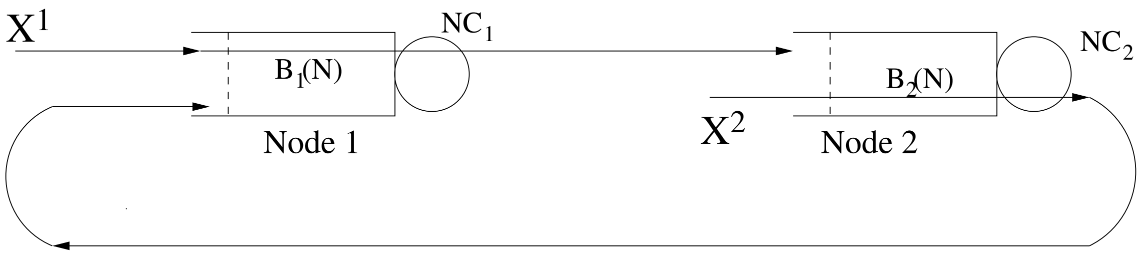

Fig. 3. Two node non-feedforward network

g_2^1 can be expressed as:

$$g_2^1(x) = \begin{cases} x_1 & x_1 \le \phi_1 C_1 \ or \ x_1 + g_1^2(x) \le C_1 \\ \phi_1 C_1 & otherwise \end{cases}$$

Similarly

$$g_1^2(x) = \begin{cases} x_2 & x_2 \le \phi_2 C_2 \ or \ x_2 + g_2^1(x) \le C_2 \\ \phi_2 C_2 & otherwise \end{cases}$$

From these, the set of (x_1, x_2) in calculating $\mathbf{I}_2^2$ can be obtained as:

$$\begin{aligned} \{x_2 > \phi_2 C_2, x_2 + g_2^1(x) > C_2\} = \ & \{x_2 > \phi_2 C_2, x_2 + x_1 > C_2, x_1 + \phi_2 C_2 \le C_1\} \cup \\ & \{x_2 > \phi_2 C_2, x_2 + \phi_1 C_1 > C_2, x_1 + \phi_2 C_2 > C_1\} \end{aligned}$$

VI. DISCUSSION

The results can be extended in several directions:

1) We have assumed that the weight factors ϕ_m were same at each node. Similar arguments and results apply when weight factors at each node are different.
2) Continuous time fluid models can be handled for the bufferless case with the assumption that for every $m = 1, \dots, M$
$$\lim_{t \to 0} \frac{X^{m,N}(0,t)}{t} = \bar{X}^{m,N} \ a.s.$$
for some r.v. $\bar{X}^{m,N}$. Then this limit r.v. can be taken as the instantaneous fluid input rate for the analysis.
3) When there is traffic entering the network with total rate of $o(N)$, above results still hold. These sources will use a negligible amount of the capacity and LD rate functions of these sources as well as their packet loss will be 0 at 0 and infinite elsewhere. Therefore they will not have any impact on the LD rate functions for overflow.

When the buffers are large such as $B_k(N) \ge O(N)$, the time scale for overflow will be bigger than 1. In this case, it is again necessary to find the large deviation rate functions of outputs in terms of the inputs but this is a very difficult problem.

It is not always easy to find or measure the LD rate functions for a general traffic process. When the input traffic has a known peak rate and a mean rate, we can find a lower bound for the rate function and hence use this bound for the admission control purposes. Assume $X^{m,N}$ is the sum of Nn_m i.i.d. sources. Let $\tilde{X}^m$ be one of the Nn_m sources which make up $X^{m,N}$ and $\tilde{X}^m_t$ be its rate at time t. Assume that $\tilde{X}^m_t \in (0, \pi_m)$. Using Hoeffding's Inequality,

$$\mathbb{E}[\exp(\theta \tilde{X}^m_t)] \leq \frac{\rho_m}{\pi_m} e^{\theta \pi_m} + \frac{\pi_m - \rho_m}{\pi_m}$$

Therefore,

$$I_1^{X^m}(x) = \sup_\theta \{\theta x - \log \mathbb{E}[\exp(\theta \tilde{X}^m_t)]\} \geq$$
$$\frac{x}{\pi_m} \log\left(\frac{x(\pi_m - \rho_m)}{\rho_m(\pi_m - x)}\right) - \log(\frac{\pi_m - \rho_m}{\pi_m - x})$$

We can find an on-off source for which the lower bound of the rate function is achieved. For example, choose $\tilde{X}^m_t$ to be the stationary version of the following periodic function (in the discrete time, time intervals must be chosen small enough to make the approximation better):

$$\mathbf{Z}_t = \begin{cases} \pi_m & 0 \leq t < \rho_m/\pi_m \\ 0 & \rho_m/\pi_m \leq t < 1 \end{cases}$$

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