Abstract—This paper presents a generalized direct synthesis technique for lossy reciprocal and nonsymmetrical microwave filters. The procedure handles multiple complex coupling among resonators and finite-$Q$ resonators. A coupling matrix model with complex entries is used as the basis for the analysis. New mathematical definitions are introduced to find the necessary properties of the complex coupling matrix. Synthesis verification examples include symmetrical, asymmetrical and different loss level lossy functions. A lossy four-pole Chebyshev filter in $Ku$-band has been designed, fabricated and tested using mixed combine and microstrip technologies for the first time. This design has the advantage of having the signal passing through at least two resonators for any possible signal path, which eliminates unwanted source to load coupling to a great extent.

Index Terms—Coupling matrix, lossy filters, matrix rotation, microwave filter synthesis, predistortion.

I. INTRODUCTION

MICROWAVE filters play a major role in various communications systems, particularly communications satellites, earth stations, wireless base stations, and repeaters. Direct synthesis of lossless microwave filters has been the focus of the microwave engineers for years since it is the fundamental circuit level approach to design filters. The state-of-the-art lossless synthesis of microwave filters can now be declared as “matured” to some extent with any filter transfer function being synthesized to the desired filter topology using exact synthesis procedures [1]–[3].

With the increase in the demand for high-performance and small microwave filters to meet the evolving standards in communication systems, designing filters with high $Q$ factor and small size at the same time has been always a challenge. Higher $Q$ technologies often result in increased resonator sizes, while lower $Q$ filters suffer from degraded insertion loss performance. Accepting some additional insertion loss and knowing the fact that the loss can be compensated for by the already-existing microwave amplifiers in the system [4], [5], the sharp response of the filter transmission can be restored using different design techniques. One approach to improve the selectivity for such filters is optimizing the filter elements, which has been practically done using adaptive predistortion techniques [6]. These techniques assume no change to the filter topology and thus result in degraded return loss performance. To compensate for the return loss performance, these filters are practically used with nonreciprocal devices such as isolators and circulators, which again increase the overall size. One recently used approach is to improve the response using lossy circuit extraction techniques [7], [8]. These methods improve the return loss using nonuniform dissipation and modified topologies with added loss. For designs with transmission zeros, Triplet sections and Brune sections were used in conjunction with the analysis. Use of resistive cross coupling and hyperbolic rotation were also introduced in [7] to improve the filter response and to distribute loss among the resonators. The disadvantages of the synthesis method and circuit realizations in [7], [8] can be summarized as follows.

• The synthesis method is limited to symmetrical filter networks using even and odd mode analysis.
• In the cases of higher order filters, empirical design approaches were used due to the complexity involved with the direct-synthesis techniques [7].
• All the realizations were done in planar technology.
• The even and odd mode pre-distortion technique presented in [7] result in filter realizations passing through only one resonator from source to load, which usually results in degraded out-of-band performance.

In this paper, a systematic synthesis approach to deal with finite-$Q$ resonators and complex coupling is presented based on a generalized multiple-coupled cavity model. The essential properties for direct synthesis of lossless coupling matrix as presented in [2] are no longer valid for dissipative filters as they result in non-Hermitian complex coupling matrices. This paper assumes new mathematical definitions to find new properties for the synthesis of reciprocal lossy filters. A four-pole microwave filter in mixed combine and microstrip technologies in $Ku$-band is also designed, fabricated and tested. Unlike the lossy realizations resulting from even and odd mode predistortion, all signal paths go through at least two resonators eliminating unwanted source to load coupling especially at higher frequencies. The approach is based on the recently introduced one in [9] with more detailed theoretical and practical expansion along with measurement results.

II. LOSSY FILTER SYNTHESIS STEPS

The synthesis of lossy filters is composed of different steps starting from the design requirements to a possible circuit design. Fig. 1 shows a stepwise approach for a generalized synthesis technique. Based on these steps, two types of synthesis are necessary: polynomial synthesis (Step 1) and circuit synthesis (Step 3). The circuit model considered in this paper is a coupling matrix with complex entries (Step 2). The focus of
the theoretical synthesis of this paper is on the circuit synthesis rather than polynomial synthesis.

The theoretical progress for the direct synthesis of lossy polynomials in the literature [7] has been so far limited to multiplying both reflection and transmission polynomials of a lossless function by a constant attenuation factor of \( K < 1 \) as

\[
S_{11, \text{lossy}} = KS_{11, \text{lossless}} \\
S_{21, \text{lossy}} = KS_{21, \text{lossless}}.
\]  

(1)

This is the simplest lossy scattering polynomial resulting in same-order admittance polynomials and thus is most commonly used. From the network perspective, this special case is equivalent to placing two identical matched attenuators at input and output ports of a lossless filter with attenuation factors of \( \sqrt{K} \), as in Fig. 2.

The two identical matched attenuators can then be made using different kinds of resistive networks such as PI or TEE resistive sections [10]. By doing some network transformations, the resistive sections can be placed as two shunt resistors at source/load as well as the first/last resonator of the lossless filter. This method is also suitable for an initial lossy coupling matrix synthesis for cases where both reflection and transmission coefficients are multiplied by \( K \).

This case, however, is not the most general form of lossy polynomials. The shape of these functions may be different from standard filtering functions but still satisfy the design specifications (see the sixth order lossy example in [7] obtained by optimization). In some other practical cases, different loss levels for reflection and transmission functions may be required if a lossy filter is to be realized, such as the channel-dropping input multiplexers (IMUX), where a 0-dB out-of-band reflection is crucial (see [3, p. 629]). These types of lossy circuits cannot be universally synthesized using the attenuator configuration in Fig. 2. Therefore, a more general theory for the synthesis of lossy polynomials is desired. The direct derivation of such lossy functions without leading to higher number of resonators in the process of \( S \) to \( Y \) transformation requires further research, and is not the focus of this paper. With the assumption that the lossy polynomials are given, the lossy coupling matrix synthesis presented in the next sections can be used. It covers all scenarios, as there is no theoretical limit for the choice of the lossy polynomials in the derivations. In other words, given any set of lossy admittance polynomials, the network can be synthesized. Due to the theoretical limitations of lossy polynomials, for experimental purposes, we have used a same-level-loss design using the generalized approach, which can also be synthesized using the attenuator approach.

For theoretical verification, three different cases are considered to cover symmetrical, nonsymmetrical and different-loss-level filtering functions in the next sections. The same-loss-level examples can also be verified using the attenuator approach explained earlier in this section. The general synthesis method presented in the next sections can be viewed as an extension of the method proposed by Atia et al. [2] to the lossy case.

III. COUPLING MATRIX REPRESENTATION

Consider the coupling matrix circuit model proposed in [2], with the difference that all the resonators include resistors and the coupling values are complex values, as shown in Fig. 3, with the following \( Z \) matrix:

\[
Z(P) = PI_n + jM
\]  

(2)

where \( I \) is the unity matrix and \( P \) is the bandpass frequency variable

\[
P = p + \frac{1}{p} \quad p = j\omega
\]  

(3)

and \( M \) is the coupling matrix with all complex entries.

This is the most general form of a coupling matrix useful for synthesizing lossy filters. For a realizable passive reciprocal circuit, the imaginary parts of the diagonal elements need to be negative to result in positive resistors at resonators.

IV. ADMITTANCE MATRIX REPRESENTATION

Using eigen decomposition theorem, any square complex matrix \( M \) can be diagonalized to its complex eigenvalues as

\[
M = T\Lambda T^{-1}
\]  

(4)

where \( T \) is a square matrix composed of eigenvectors of \( M \) and \( \Lambda \) is a diagonal matrix containing eigenvalues of \( M \). Therefore, the loop impedance matrix \( Z \) can be derived as

\[
Z_l(P) = PI_n + jT\Lambda T^{-1} = T(PI_n + j\Lambda)T^{-1}.
\]  

(5)

Then the \( Y \)-matrix can be written as

\[
Y_l(P) = T(PI_n + j\Lambda)^{-1} T^{-1} = T \text{diag} \left[ \frac{1}{P + j\lambda_1}, \ldots, \frac{1}{P + j\lambda_n} \right] T^{-1}
\]  

(6)

where \( T \) matrix and \( \lambda \) values are all complex.
V. PROPERTIES OF THE EIGENVECTOR MATRIX

In this section, we define a modified dot product and orthogonality to find the properties of the complex coupling matrix. Different from lossless filter synthesis, the eigenvector matrix $T$ in the previous section does not follow the properties resulted from a real symmetric $M$ matrix since neither is it orthogonal, nor does $T^{-1} = T^*$ applies. For complex matrices, these properties only apply to Hermitian matrices, where the $M$ matrix is equal to its transpose conjugate resulting in real diagonal elements. However, the diagonal elements of $M$ are complex numbers for finite $Q$ resonators. Moreover, due to the condition of reciprocity for these circuit models, $M^t = M$ applies only when $t$ is defined as a simple transpose and not a conjugate transpose. Therefore, matrix $M$ is not Hermitian and the conventional orthogonal properties can not be used to reconstruct the $M$ matrix in a synthesis procedure.

In order to find out the properties of the $T$ matrix, the following definitions are necessary:

A. Modified Dot Product Definition

The original dot product for vectors $u$ and $v$ is defined as

$$u \cdot v = u_1v_1^* + u_2v_2^* + \cdots + u_nv_n^*$$

(7)

where $^*$ is the conjugate symbol. Let us define the modified dot product for vectors $u$ and $v$ as

$$u \circ v = u_1v_1 + u_2v_2 + \cdots + u_nv_n$$

(8)

which is similar to the original dot product except that there is no conjugate operator when multiplying the two vectors.

B. Modified Orthogonality Theorem

Let $X_1$ and $X_2$ be eigenvectors of the pure symmetric complex matrix $M$ with complex eigenvalues $\lambda_1 \neq \lambda_2$. Then $X_1 \circ X_2 = 0$.

Proof:

$$X_1^tMX_2 = X_1^t\lambda_2X_2 = \lambda_2(X_1 \circ X_2),$$

(9)

On the other hand, since

$$X_1^tMX_2 = X_1^tM^tX_2 = (MX_1)^tX_2 = \lambda_1(X_1 \circ X_2),$$

(10)

From (9) and (10) $\lambda_1(X_1 \circ X_2) = \lambda_2(X_1 \circ X_2)$, which is valid only if $X_1 \circ X_2 = 0$. Therefore, the properties of $T$ matrix can be summarized as follows.

- The column vectors are normal complex vectors, but not the row vectors (different from the lossless case).
- The column vectors are orthogonal by the “modified” dot product.

C. Inverse Eigen-Matrix Properties

Applying a pure transpose on both sides of (4) gives

$$M^t = M = (T^{-1})^t \Lambda T^t = (T^t)^{-1} \Lambda T^t, \quad (11)$$

Using the new properties explained in sections A and B, it could be easily found that

$$T^tT = D = \begin{bmatrix}
d_1 \\
d_2 \\
\vdots \\
d_n
\end{bmatrix}$$

(12)

with matrix $D$ becoming a diagonal matrix. From (12),

$$T^{-1} = D^{-1}T^t$$

(13)

which shows that the inverse of the eigen-matrix can be obtained by multiplying a diagonal matrix by its pure transpose. It is apparent that for the lossless case, matrix $D$ becomes unity.

VI. ADMITTANCE MATRIX PROPERTIES

With the new definitions introduced in the previous section, the properties of the $Y$ matrix can be determined. Let

$$W = T^{-1}.$$

(14)
Then from (6),

\[
Y_i(P) = T \text{diag} \left[ \frac{1}{P + j\lambda_1}, \ldots, \frac{1}{P + j\lambda_n} \right] W
\]

\[
= \begin{bmatrix}
T_{11} & T_{12} & \cdots & T_{1n} \\
T_{21} & T_{22} & \cdots & T_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
T_{n1} & T_{n2} & \cdots & T_{nn}
\end{bmatrix}
\begin{bmatrix}
\frac{1}{P + j\lambda_1} \\
\frac{1}{P + j\lambda_2} \\
\vdots \\
\frac{1}{P + j\lambda_n}
\end{bmatrix}
\begin{bmatrix}
W_{11} & W_{12} & \cdots & W_{1n} \\
W_{21} & W_{22} & \cdots & W_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
W_{n1} & W_{n2} & \cdots & W_{nn}
\end{bmatrix}
\]  \hspace{1cm} (15)

Using (12) and (13), the admittance matrix can also be derived as

\[
Y_i(P) = Z_t^{-1}(P)
\]

\[
= T \text{diag} \left[ \frac{1}{d_1(P + j\lambda_1)}, \ldots, \frac{1}{d_n(P + j\lambda_n)} \right] T^T. \hspace{1cm} (16)
\]

The two-port admittance matrix is obtained by calculating the 11, 1n, n1, and nn components of matrix as follows:

\[
Y_{11} = \frac{T_{11}W_{11}}{P + j\lambda_1} + \frac{T_{12}W_{21}}{P + j\lambda_2} + \cdots + \frac{T_{1n}W_{n1}}{P + j\lambda_n} \hspace{1cm} (17)
\]

\[
Y_{1n} = \frac{T_{11}W_{11}}{P + j\lambda_1} + \frac{T_{12}W_{22}}{P + j\lambda_2} + \cdots + \frac{T_{1n}W_{nn}}{P + j\lambda_n} \hspace{1cm} (18)
\]

\[
Y_{n1} = \frac{T_{n1}W_{11}}{P + j\lambda_1} + \frac{T_{n2}W_{21}}{P + j\lambda_2} + \cdots + \frac{T_{nn}W_{n1}}{P + j\lambda_n} \hspace{1cm} (19)
\]

\[
Y_{nn} = \frac{T_{n1}W_{11}}{P + j\lambda_1} + \frac{T_{n2}W_{22}}{P + j\lambda_2} + \cdots + \frac{T_{nn}W_{nn}}{P + j\lambda_n} \hspace{1cm} (20)
\]

From [2], the two-port matrix components are calculated as

\[
Y_{2\times2} = \begin{bmatrix}
n_1^2Y_{11} & n_1n_2Y_{1n} \\
n_1n_2Y_{1n} & n_2^2Y_{nn}
\end{bmatrix}. \hspace{1cm} (21)
\]

Then the two-port Y-matrix elements become

\[
\frac{Y_{11}}{n_1^2} = \frac{T_{11}W_{11}}{P + j\lambda_1} + \frac{T_{12}W_{21}}{P + j\lambda_2} + \cdots + \frac{T_{1n}W_{n1}}{P + j\lambda_n} \hspace{1cm} (22)
\]

\[
\frac{Y_{1n}}{n_1n_2} = \frac{T_{11}W_{11}}{P + j\lambda_1} + \frac{T_{12}W_{22}}{P + j\lambda_2} + \cdots + \frac{T_{1n}W_{nn}}{P + j\lambda_n} \hspace{1cm} (23)
\]

\[
\frac{Y_{n1}}{n_1n_2} = \frac{T_{n1}W_{11}}{P + j\lambda_1} + \frac{T_{n2}W_{21}}{P + j\lambda_2} + \cdots + \frac{T_{nn}W_{n1}}{P + j\lambda_n} \hspace{1cm} (24)
\]

\[
\frac{Y_{nn}}{n_2^2} = \frac{T_{n1}W_{11}}{P + j\lambda_1} + \frac{T_{n2}W_{22}}{P + j\lambda_2} + \cdots + \frac{T_{nn}W_{nn}}{P + j\lambda_n} \hspace{1cm} (25)
\]

Using the residue matrix representation as in [2],

\[
Y_{2\times2} = \sum_{k=1}^{n} K^{(k)} \frac{P - P_k}{P - P_k} \hspace{1cm} (26)
\]

where \( P_k = -j\lambda_k \), the 2 x 2 residue matrix is then calculated as

\[
K^{(k)} = \begin{bmatrix}
n_1^2T_{1k}W_{1k} & n_1n_2T_{1k}W_{nk} \\
n_1n_2T_{1k}W_{nk} & n_2^2T_{nk}W_{kn}
\end{bmatrix}. \hspace{1cm} (27)
\]

There mercy, \n
\[
T_{1k}W_{k1} = \frac{K^{(k)}}{n_1^2} \hspace{1cm} T_{nk}W_{kn} = \frac{K^{(k)}}{n_2^2} \hspace{1cm} (28)
\]

\[
T_{nk}W_{k1} = \frac{K^{(k)}}{n_1n_2} \hspace{1cm} T_{1k}W_{nk} = \frac{K^{(k)}}{n_2n_1}. \hspace{1cm} (29)
\]

Alternatively, these equations can also take the form

\[
T_{1k}d_k = \frac{K^{(k)}}{n_1^2} \hspace{1cm} T_{nk}d_k = \frac{K^{(k)}}{n_2^2} \hspace{1cm} (30)
\]

\[
T_{nk}d_k = \frac{K^{(k)}}{n_1n_2} \hspace{1cm} T_{1k}d_k = \frac{K^{(k)}}{n_2n_1}. \hspace{1cm} (31)
\]

These equations are more universal than the ones in [2] since \( W \) is no longer the transpose of \( T \). By multiplying the equations in (30) and (31), the following property for the lossy residue matrix is obtained:

\[
K^{(k)}_{11}K^{(k)}_{22} = K^{(k)}_{12}K^{(k)}_{21}. \hspace{1cm} (32)
\]

Therefore, one of the four equations becomes redundant. One more equation is required to be able to solve the four unknowns. Equation (14) is the fourth condition to solve the first and last rows of the \( T \) matrix.

VII. SYNTHESIS ALGORITHM

To synthesize the \( M \) matrix from the two-port \( Y \)-matrix polynomial, the following procedure is used.

Step 1) Let \( W_{k1} = T_{1k} + \alpha_k, W_{nk} = T_{nk} + \beta_k \) or \( W_{k1} = T_{1k}d_k, W_{nk} = T_{nk}d_k \).

Step 2) Let \( W = T^T \). Then, the first and last rows of \( T \) equal the first and last rows of \( W(\alpha_k = \beta_k = 0, d_k = 1) \).

Step 3) Using similar formulas for the conventional synthesis [2], calculate the first and last rows of \( T \).

Step 4) Choose initial row vectors for the rest of the rows and use the modified Gram-Schmidt procedure to make all the row vectors orthogonal and normal using the new properties.

Step 5) Calculate inverse of \( T \) to get \( W \) and compute the difference between the first (last) row of \( T \) and the first (last) column of \( W \) from step one. If the new \( \alpha_k \) and \( \beta_k \) are equal to the previous ones, stop the algorithm.
Step 6) Modify the first and last rows of $T$ considering the difference as:

$$T_{1k}(T_{1k} + \alpha_k) = \frac{K_{11}^{(k)}}{n_1^2} \quad T_{nk}(T_{nk} + \beta_k) = \frac{K_{22}^{(k)}}{n_2^2}$$

or

$$T_{1k} = \sqrt{n_1^2 d_k} \quad T_{nk} = \sqrt{n_2^2 d_k}.$$  

Step 7) Apply the new Gram–Schmidt procedure on the column vectors of the $T$ matrix based on the new orthogonality conditions defined.

Step 8) Go to Step 5).

VIII. SYNTHESIS EXAMPLES

In order to show the validity of the synthesis technique, three examples of filter synthesis are considered to cover different lossy design cases. These include: 1) symmetrical response with same return and insertion loss levels; 2) asymmetrical response with same return and insertion loss levels; and 3) symmetrical response with different return and insertion loss levels. The first example also shows more detailed derivations to be also instructational. The second example shows the validity of the approach for nonsymmetrical filter networks. The third example examines the case where only the insertion loss response is attenuated compared to the lossless filter function, which represents a case where the attenuator synthesis method in Section II cannot be used.

A. Symmetrical Response With Same Loss Levels

Consider the synthesis of a lossy four-pole quasi-elliptic filter with the following scattering parameter polynomials and normalized frequency response shown in Fig. 4. The response is obtained from the lossless $S$-matrix polynomials with a return loss of 25 dB shifted down by 6 dB and transmission zero locations at $\pm j2$. The lossy $S$ polynomials become

$$S_{21}(p) = \frac{-j(0.3194 p^2 + 1.2778)}{p^4 + 2.5719 p^3 + 4.3432 p^2 + 4.3166 p + 2.535}$$

$$S_{11}(p) = \frac{0.5012 p^4 + 0.5192 p^2 + 0.072}{p^4 + 2.5719 p^3 + 4.3432 p^2 + 4.3166 p + 2.535}.$$  

Using $S$-matrix to $Y$-matrix transformation and with a characteristic impedance of 1, the $Y$ parameters become

$$Y_{21}(p) = \frac{j(0.6389 p^2 + 2.5555)}{2.2536 p^4 + 1.9258 p^3 + 6.4725 p^2 + 3.2323 p + 3.3388}$$

$$Y_{11}(p) = \frac{0.7488 p^4 + 3.2179 p^3 + 3.2522 p^2 + 5.4009 p + 1.9121}{2.2536 p^4 + 1.9258 p^3 + 6.4725 p^2 + 3.2323 p + 3.3388}.$$  

By partial fraction expansion of the $Y$ parameters and following the steps in Section V, the initial $T$ and $M$ matrices are calculated as shown in (37)-(39) at the bottom of this page. The final $T$ and $M$ matrices after seven iterations are calculated as

$$R_1 = \sum_{k=1}^{n} K_{11}^{(k)} = 1.144, \quad R_2 = \sum_{k=1}^{n} K_{22}^{(k)} = 1.144.$$  

The partial fraction expansion of the admittance functions also results in two shunt resistors at source and load with conductance values of 0.3323. The response corresponding to this coupling matrix matches that of Fig. 4 showing the validity of the approach.
matrix polynomial coefficients are shown in Table I. To transformer the transformer, the transformer matrix coefficients are polynomials. This matrix can be further calculated as shown in Table II. As it is clear from these tables, the entries at source and load of the matrix representing the shunt resistors appearing as constant numbers from the partial fraction expansion of \( Y \) polynomials. This matrix can be further rotated to obtain other solutions.

### B. Asymmetrical Response With Same Loss Levels

Consider the synthesis of a lossy asymmetrical filter with two imaginary zeros at 1.3217 and 1.8082, lossless return loss of 22 dB and a loss factor of 6 dB as shown in Fig. 5.

The \( S \)-matrix polynomial coefficients are shown in Table I. Using an \( S \) to \( Y \) transformation, the \( Y \)-matrix coefficients are calculated as shown in Table II. As it is clear from these tables, the coefficients of admittance polynomial are complex numbers resulting in complex eigenvalues and eigenvectors for the coupling matrix. The synthesis algorithm is then applied starting from the \( Y \) polynomials following the steps in Section VI. The initial and final \( T \) matrices after 14 iterations are calculated as shown in (43) and (44) at the bottom of this page, with the \( N+2 \) coupling matrix shown in Fig. 6.

The entries at source and load of the \( N+2 \) matrix represent shunt resistors appearing as constant numbers from the partial fraction expansion of \( Y \) polynomials. This matrix can be further rotated to obtain other solutions.

### C. Symmetrical Response With Different Loss Levels

Consider the design of a lossy two-pole Chebyshev filter with the same return loss response as the lossless case, while shifting the insertion loss down by 6 dB compared to the lossless response. The lossless response has an in-band return loss of 25 dB. By a \( S \)-matrix to \( Y \)-matrix transformation, the \( Y \) polynomials become of order 4, making a four-resonator filter. This example cannot be synthesized using the attenuator approach explained in Section II.

\[
T = \begin{bmatrix}
0.3304 + j0.1004 & 0.3523 - j0.0340 & 0.6115 - j0.0282 & 0.6115 + j0.0289 \\
0.08618 + j0.082 & 0 & 0.4953 + j0.0725 & 0 \\
-0.3904 - j0.0083 & 0.3523 + j0.0340 & -0.6115 + j0.0282 & 0.6115 + j0.0289 \\
\end{bmatrix}
\]

(40)

\[
M = T^T A T^{-1} = \begin{bmatrix}
-j0.4273 & 0.6891 & 0.6891 & -0.2478 \\
0.6891 & 0.8536 & 0 & 0.6891 \\
0.6891 & 0 & -0.8536 & -0.6891 \\
-0.2478 & 0.6891 & -0.6891 & -j0.4273 \\
\end{bmatrix}
\]

(41)

\[
T_0 = \begin{bmatrix}
0.5482 - j0.0325 & 0.2571 + j0.0771 & 0.4389 + j0.0406 & 0.6586 - j0.0302 \\
0.9219 - j0.0849 & 0 & 0.7744 - j0.0922 & 0 \\
-0.5482 + j0.0325 & 0.2571 + j0.0771 & -0.4389 - j0.0406 & 0.6586 + j0.0302 \\
0.5478 + j0.0652 & 0.2939 + j0.0649 & 0.4407 + j0.0407 & 0.6530 + j0.0299 \\
\end{bmatrix}
\]

(43)

\[
T = \begin{bmatrix}
0.5482 - j0.0325 & 0.2571 + j0.0771 & 0.4389 + j0.0406 & 0.6586 - j0.0302 \\
0.9219 - j0.0849 & 0 & 0.7744 - j0.0922 & 0 \\
-0.5482 + j0.0325 & 0.2571 + j0.0771 & -0.4389 - j0.0406 & 0.6586 + j0.0302 \\
0.5478 + j0.0652 & 0.2939 + j0.0649 & 0.4407 + j0.0407 & 0.6530 + j0.0299 \\
\end{bmatrix}
\]

(44)
Fig. 6. $N+2$ coupling matrix for the lossy asymmetrical four-pole Chebyshev filter example.

Fig. 7. $N+2$ coupling matrix for the lossy two-pole Chebyshev filter.

Fig. 8. Four-resonator filter synthesized from the lossy two-pole Chebyshev filter function.

As can be seen from Fig. 7, loss has appeared only at the second and third resonators and there are no shunt resistors present at source and load.

Similarly, the same method is applied to a three-pole lossy Chebyshev filter with a return loss of 20 dB and the insertion loss shifted down by 6 dB. The synthesized diagram after matrix rotations to a folded format is depicted in Fig. 11. The corresponding $N+2$ coupling matrix is shown in Fig. 12, with the response depicted in Fig. 13 after nine iterations. The synthesized circuits here have become of order $2N$. The reconfiguration of the synthesized circuit and/or the polynomial may be required to get practical designs, which is yet to be done by researchers.
IX. MATRIX ROTATION AND CIRCUIT IMPLEMENTATION

In order to make the coupling matrix realizable, we need to simplify the matrix using matrix rotations. Consider the quasi-elliptic lossy four-pole filter example in the previous section. By annihilating the coupling elements $M_{13}$ and $M_{24}$, the $N + 2$ coupling matrix is simplified to the one in Fig. 14.

The loss is present at the first and last resonators in addition to the shunt resistors at input/output nodes. It is usually preferred to have the loss distributed evenly among all the resonators for ease of fabrication. A hyperbolic rotation makes this possible [7]. Presenting the matrix in the form of $N + 2$ [3], inserting two nonresonating nodes, distributing the loss evenly among the resonators and proper scaling result in the following coupling matrix representation, shown in (47) at the bottom of this page.

The nonresonating nodes are originally created by adding two unity J-inverters at input and output, which does not change the amplitude response of the filter. This matrix in (47) is obtained by hyperbolic rotations of $-0.1258$ radians on pivots [1]–[4] and scaling the source and load nodes by a factor of 0.4061. The circuit topology corresponding to this matrix is depicted in Fig. 15.

X. RESISTIVE COUPLING REALIZATION FOR CAVITY RESONATORS

The resistors connecting two resonators or connecting one resonator to a nonresonating node such as the ones in Fig. 15 need to be further manipulated for practical microwave realizations. Firstly, microwave resistors come with a phase shift, which would cause the response to deviate from the designed one. Secondly, there is no direct realization for a resistor connecting to a microwave resonator compared to the circuit model. Thirdly, it is also usually preferred to have 50-$\Omega$ transmission lines in the design. Fig. 16 shows the circuit equivalents of a resistor and the steps to get a realizable circuit with transmission lines and negative coupling values at the two ends. The equivalent shown in Fig. 16(c) is obtained by connecting a pair of unity admittance inverters twice along with proper scaling. This circuit model is very appropriate for realization of the resistors connecting two resonators due to the following reasons.

$$
M_{Ellip} = \begin{bmatrix}
0 & -0.4061 & 0 & 0 & 0 & 0 & 0 \\
-0.4061 & -j0.0548 & -0.4378 & j0.0548 & 0 & 0 & 0 \\
0 & -0.4378 & -j0.1863 & 0.9512 & -j0.2410 & j0.1401 & -0.2654 \\
0 & j0.0548 & 0.9512 & -j0.2410 & 0.8712 & j0.1401 & 0 \\
0 & 0 & j0.1401 & 0.8712 & -j0.2410 & 0.9512 & -j0.1863 \\
0 & 0 & -0.2654 & j0.1401 & 0.9512 & -j0.1863 & -0.4378 \\
0 & 0 & 0 & 0 & j0.0548 & -0.4378 & -j0.0548 \\
0 & 0 & 0 & 0 & 0 & -0.4061 & 0
\end{bmatrix}
$$

(47)
Fig. 16. Resistive coupling realization. (a) Plain resistor. (b) Equivalent circuit with J-inverters. (c) Final equivalent model with $\lambda/4$ transmission lines and negative coupling values.

Fig. 17. Three-port network with a nonresonating node and resistive coupling. (a) Original network with two coupling elements and a resistor connected to the nonresonating node. (b) Equivalent network with added pairs of unity J-inverters. (c) Final equivalent network used for microwave realization.

1) The extra electrical length associated with a microwave resistor can be absorbed in the $\lambda/4$ transmission lines.
2) The capacitive (negative) coupling values at the two sides are easier to implement and favorable for cavity resonators as they can be easily adjusted for tuning purposes.
3) A series resistor is often easier to implement especially using planar technologies avoiding the use of via holes.
4) The coupling value at both ends can be arbitrarily selected for a more reasonable realization based on the physical conditions.

The realization of the resistors connecting to a nonresonating node (see Fig. 15, node 1) is also challenging as the node needs to be connected to transmission lines with proper coupling values to the resonators. The steps to find a circuit model suitable for realization are shown in Fig. 17. First, pairs of unity J-inverters with negative signs are introduced in the circuit, which can be looked at as $360^\circ$ transmission lines. Then, a series of scaling would easily result in the final realization.

XI. SIMULATED AND MEASURED RESULTS

A lossy four-pole Chebyshev filter design is considered for experimental purposes. It has a center frequency of 12 GHz and bandwidth of 120 MHz. The equivalent lossless response has a return loss of 25 dB. The lossy case has the same response with both $S_{11}$ and $S_{12}$ shifted down about 2.9 dB making the $Q$ of each resonator approximately 2000 after proper loss distribution. The resulting $N + 2$ coupling matrix is shown in Fig. 18, with additional input/output coupling values of 0.41. Based on this coupling matrix the two normalized resistors are 35.97 and 21.69.

The circuit design was simulated using ANSOFT Designer. The filter is realized with mixed combline and microstrip technologies. The resistive cross coupling elements are designed using microstrip technology with chip resistors, while the resonators are designed using combline technology, which has better quality factor compared to microstrip technology. The chip resistors have about 16 of phase length, which are absorbed into the adjacent microstrip lines. Fig. 19 shows the HFSS model of the filter along with its circuit node diagram. The corresponding optimized EM simulation response is depicted in Fig. 20. The synthesis response has the best match with the EM simulation response at a bandwidth of 110 MHz. An image of the fabricated filter with a quarter coin is shown in Fig. 21. The housing is silver-plated with chip resistors and coupling wires all soldered in place. The microstrip boards are attached using conductive epoxy. The lossy filter cavities have a size of 6.35 mm (0.25 x 0.25 in$^2$). This indicates that
the lossy filter size can be reduced by about a factor of 4 by embedding the microstrip circuits into the cavity walls [11]. The tuned measured response at 12 GHz is shown in Fig. 22. The response does not match the expected synthesis response. Using circuit extraction, it becomes apparent that the main source of the problem comes from increased transmission line lengths by about 14°. The 360° folded microstrip transmission lines contribute to 10° error with a 127 μm (5 mil) manufacturing tolerance from each axis. Dielectric constant and the microstrip width tolerance each have a share of 2° error resulting in a 14° overall error.

Since the microstrip lines are not tunable in the current design, an alternative solution is to redesign the circuit at a lower frequency, which would give the desired transmission line electrical lengths. With the existence of resonator and coupling screws, this becomes possible. The filter naturally tuned to a lower center frequency at 11.18 GHz. Fig. 23 shows the final tuned response of the lossy filter compared to synthesis with a bandwidth of 124 MHz. As could be seen from the response, the rejection in lower frequency band has slightly degraded, while the rejection in the upper frequency band has improved. Using circuit extraction, it becomes apparent that this is due to unwanted 1–3 and 2–4 small coupling values ($r_{13} = r_{24} = 0.045$).

The lossy circuit extraction using optimization techniques is quite tedious due to the added complexity of the design compared to the lossless case. Therefore, all the circuit extraction in this paper has been done using manual circuit model tuning with expert knowledge. Therefore, these types of filters may be good candidates for circuit extraction using fuzzy logic techniques [12].

The measured passband ripple is approximately 0.5 dB due to a lower actual $Q$ of about 1300 compared to 2000. The equivalent $Q$ obtained is about 3500. In order to compensate for the discrepancy between the actual $Q$ and the one used for synthesis, a new synthesis for the actual $Q$ of 1300 is needed, which results in different resistor and coupling values.

Since the chip resistors are not tunable, the ripple cannot be further improved. Use of a tunable resistor can solve this problem. It is also clear from the results that the lengths of the transmission lines play an important role especially to get the desired center frequency. This can also be overcome by choosing fabrication technologies with less tolerance or by introducing a phase tuning mechanism.
XII. OUT-OF-BAND PERFORMANCE

In order to observe how the filter performs at higher frequencies, a measurement from 10 to 40 GHz has been done. Fig. 24 shows the measured insertion loss and return loss out-of-band response.

The first spurious signal is at around 13.53 GHz with 42 dB insertion loss. The spurious performance for out-of-band is degraded compared to conventional combline filters. This is due to the effect of the additional microstrip lines. Although the 42 dB spurious magnitude can be acceptable, it can be improved by redesigning the microstrip lines to have smaller lengths, which would increase the spurious frequency.

The EM simulation response for out-of-band is depicted in Fig. 25. The spurious signal is present at a slightly higher frequency compared to the measured results. This discrepancy is due to the increased length of the lines due to manufacturing tolerances (about 254 μm or 10 mil), as well as an additional length of 406 μm (16 mil) for the resistors, resulting in about 660 μm (26 mil) of line increase. This causes about 2.5 GHz of resonance shift making the original EM simulation spurious signal at around 16 GHz.

XIII. CONCLUSION

A general and systematic synthesis algorithm has been presented for lossy filters with finite resonator Q values and arbitrary complex coupling values. The method does not assume circuit symmetry nor has any limitation for the choice of the lossy transfer functions, and thus is valid for any reciprocal lossy filter, provided that it can be modeled using a lossy coupling matrix. The synthesis technique has been verified for various filtering functions including symmetrical, asymmetrical, and different loss level functions. A four-pole Chebyshev filter in K-band has been designed, fabricated and tested successfully using mixed combline and microstrip technologies for the first time. The design has the advantage of having all input-output paths going through more than one resonator, which minimizes unwanted source to load coupling especially at high frequencies.

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REFERENCES

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